3.31 — teoria fal elektromagnetycznych

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30/1990

WARSZAWA 1990

ISSN 0208-5658

Praca wpłynęła do Redakcji dnia 7 lutego 1990

N a   p r a w a c h   r ę k o p i s u

Instytut Podstawowych Problemów Techniki PAN

Nakład 120 egz. Ark.wyd.1 Ark.druk. 1,25
Oddano do drukarni w sierpniu 1990 r.
Nr zamówienia 308/90

Warszawska Drukarnia Naukowa, Warszawa,
ul.Śniadeckich 8

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TRANSFER AND SCATTERING MATRIX ANALYSES OF REFLECTION AND TRANSMISSION AT MULTILAYERED MEDIA

ABSTRACT

The transfer and scattering matrix methods are implemented in the analysis of plane wave interactions with multilayered dielectric structures. The explicit expressions of specular reflectivity and transmittivity of such structures are presented. Some relations based on the reciprocity and energy conservation laws are given to be used in the final verification of the numerical results.

1. INTRODUCTION

The purpose of this paper is to derive and compare two matrix methods of the analysis of reflection and transmission of electromagnetic plane waves at multilayered dielectric structures. In the first method, named as the transfer matrix method[1)], the relations between field amplitudes in two arbitrary points of a homogeneous medium are expressed in a matrix form. That expression, together with the boundary conditions on the interfaces and the radiation conditions at infinity, relates the field amplitudes in the first and the last layers of the multilayered structure by the use of the transfer matrix of the whole structure. Subsequently, the reflection and transmission coefficients can be effectively calculated. In the scattering matrix formalism[2)], the fields in each layer are divided into components propagating forwards (in the direction perpendicular to the layers) and those propagating backwards. While the transfer matrix formalism relates only the total field values, the second approach saves both forward and backward field amplitudes throughout the numerical computation. In some cases[3)] one of those two amplitudes is many orders of magnitude smaller than the other. On the other hand it contains important physical information about the evanescent modes, which may be lost when using the transfer matrix method. The presented formalism is of fundamental importance in the analysis of more realistic interaction phenomena between planar multilayered structures and the light beams of finite transverse cross-sections. It seems also to be an appropriate starting point in the evaluation of the electromagnetic field reflected or transmitted at nonlinear waveguiding structures [10, 11]. In the context of the paper the waveguiding structure is understood as the structure which can support normal, surface or leaky modes of electromagnetic field propagating along the structure. Such modes can be excited during reflection or transmission of the laser beam interacting with the structure.

2. TRANSFER MATRIX FORMULATION OF THE FIELD SOLUTION

The Maxwell's equations of the transverse magnetic (TM) electromagnetic field for locally reacting, time invariant, linear, isotropic, dielectric and homogeneous medium in a sourceless, two dimensional case ($\partial/\partial y=\partial_y=0$) are[1,4,5)]:

$$\tilde{E}_x = (k_0 n)^{-1} Z_0 Z \beta \tilde{H}_y \ ,$$

$$\tilde{E}_z = (k_0 n)^{-1} Z_0 Z i \cdot \partial_x \tilde{H}_y \ , \qquad (1)$$

$$(\partial_x^2 + \alpha^2)\tilde{H}_y = 0 \ ,$$

where z-axis points to the direction of the propagation and

$$\alpha^2 = k_0^2 n^2 - \beta^2 \ ,$$

$$Z_0^2 = \mu_0/\varepsilon_0 \ ,$$

$Z^2 = \mu/\varepsilon$.

In (1) μ , ε are relative permeability and permittivity of the medium, μ_0 and ε_0 are vacuum permeability and permittivity, α and β are propagation constants in x- and z-directions, respectively, k_0 is a wavenumber in vacuum, n is an index of refraction, Z_0 is a vacuum wave impedance and Z is a relative wave impedance of a medium. As usual, the field can be Fourier decomposed with respect to time and z-variable

$$H_y(x,z,t) = (2\pi)^{-2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{H}_y(x,\beta,\omega)\,\exp(-i\omega t+i\beta z)\,d\beta\,d\omega \quad ; \tag{2}$$

where ω stands for the angular frequency. Let $\Phi(x,x_0)$, $\Psi(x,x_0)$ denote the standardized solutions of Maxwell's equations, specified by the following boundary conditions at $x=x_0$:

$$\tilde{H}_y(x) = \Phi(x,x_0) \tag{3a}$$

$$\Phi(x_0,x_0) = 1 \quad ; \quad i(k_0 n)^{-1}Z_0 Z\cdot\partial_x\Phi(x,x_0)\big|_{x=x_0} = 0$$

$$\tilde{H}_y(x) = \Psi(x,x_0) \tag{3b}$$

$$\Psi(x_0,x_0) = 0 \quad ; \quad i(k_0 n)^{-1}Z_0 Z\cdot\partial_x\Psi(x,x_0)\big|_{x=x_0} = 1 \quad .$$

Then, assuming that the tangent field components are a priori known at the surface $x=x_0$, the field expressions at the arbitrary level x can be cast into the matrix form:

$$\begin{bmatrix}\tilde{H}_y(x)\\ \tilde{E}_z(x)\end{bmatrix} = M(x,x_0)\cdot\begin{bmatrix}\tilde{H}_y(x_0)\\ \tilde{E}_z(x_0)\end{bmatrix} \tag{4a}$$

$$M(x,x_0) = \begin{bmatrix}\cos[\alpha(x-x_0)] & -iZ_x^{-1}\sin[\alpha(x-x_0)]\\ -iZ_x\sin[\alpha(x-x_0)] & \cos[\alpha(x-x_0)]\end{bmatrix} \tag{4b}$$

where $Z_x=\alpha(k_0n)^{-1}Z_0Z$ is a wave impedance along x-direction and $M(x,x_0)$ is a transfer matrix to be evaluated. Defining for TM field the transverse field vector

$$\tilde{F}(x) = \begin{bmatrix} \tilde{H}_y(x) \\ \tilde{E}_z(x) \end{bmatrix}$$

we can write

$$\tilde{F}(x) = M(x,x_0)\tilde{F}(x_0) . \tag{5}$$

For TE field we get the same equation[2] with

$$\tilde{F}_{TE}(x) = \begin{bmatrix} \tilde{E}_y(x) \\ \tilde{H}_z(x) \end{bmatrix}$$

and with Z_x being replaced with a wave admittance along x-direction $Y_x=\alpha(k_0n)^{-1}Y_0Y$, where $Y_0=Z_0^{-1}$ and $Y=Z^{-1}$ stand for the vacuum wave admittance and the relative wave admittance of the medium, respectively.

3. REFLECTION AND TRANSMISSION AT A STEP-INDEX PLANAR WAVEGUIDE STRUCTURE

The planar waveguide srtucture consisting of planar homogeneous layers $D_2, D_3, \ldots, D_{N-1}$, a semi-infinite substrate D_N and a semi-infinite superstrate D_1 is shown on Fig. 1. The total field $\tilde{F}_t(x)$ may be decomposed into the sum of the incident field $\tilde{F}_i(x)$ and the scattered field $\tilde{F}(x)$:

$$\tilde{F}_t(x) = \tilde{F}_i(x)+\tilde{F}(x) . \tag{6}$$

We assume that the incident field exists in domains $D_1,\ldots,D_s$, where it satisfies the field equations (1) and continuity conditions at x_1 ,...,x_s. The scattered field obeys (5) within each layer D_j:

$$\tilde{F}_j(x_j) = M_j(x_j, x_{j-1})\tilde{F}_j(x_{j-1}) \tag{7}$$

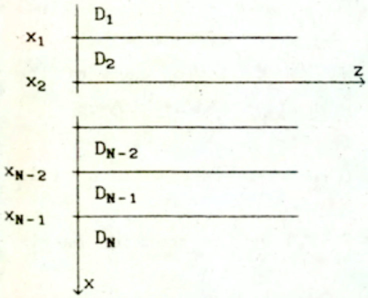

Fig. 1. Configuration of the multilayered structure.

and the following continuity relations on boundaries :

$$\tilde{F}_s(x_s)+\tilde{F}^1_s(x_s) = \tilde{F}_{s+1}(x_s) \tag{8a}$$

and

$$\tilde{F}_{j+1}(x_j)-\tilde{F}_{\ }(x_j) = 0 \quad \text{for } j \neq s \quad . \tag{8b}$$

Besides, the radiation conditions (which require vanishing of $\tilde{F}(x)$ at infinity) must be imposed, what gives the following relations in the lower medium ($x \in D_N$):

$$\mathrm{Im}(\alpha_N) \equiv \mathrm{Im}[(k_0^2 n_N^2 - \beta^2)^{1/2}] \geq 0 \quad , \tag{9a}$$

$$\tilde{E}_z(x) = -Z_{x,N}\tilde{H}_y(x) \quad \text{for } x \in D_N, \tag{9b}$$

$$Z_{x,N} = \alpha_N(k_0 n_N)^{-1} Z_0 Z_N \quad , \tag{9c}$$

and in the upper medium ($x \in D_1$)

$$\mathrm{Im}(\alpha_1) \equiv \mathrm{Im}[(k_0^2 n_1^2-\beta^2)^{1/2}] \geq 0 \tag{10a}$$

$$\tilde{E}_z(x) = +Z_{x,1}\tilde{H}_y(x) \quad \text{for } x\in D_1, \tag{10b}$$

$$Z_{x,1} = \alpha_1(k_0 n_1)^{-1} Z_0 Z_1 , \tag{10c}$$

where Im means an imaginary part. Then we get the following form of the continuity relation at the interface $x=x_s$ between the s and s+1 layers

$$\tilde{F}_s(x_s)-\tilde{F}_{s+1}(x_s) = -\tilde{F}_s^{\,\mathrm{l}}(x_s) \quad , \tag{11}$$

where the field vector $\tilde{F}_s(x_s)$ is related to $\tilde{F}_1(x_1)$ by the transfer matrix equation (5):

$$\tilde{F}_s(x_s) = M_s(x_s,x_{s-1})\cdot M_{s-1}(x_{s-1},x_{s-2})\ldots M_2(x_2,x_1)\cdot\tilde{F}_1(x_1) \tag{12}$$

with

$$\tilde{F}_1(x_1) = \begin{bmatrix} \tilde{H}_{y,1}(x_1) \\ +Z_{x,1}\cdot\tilde{H}_{y,1}(x_1) \end{bmatrix}$$

and

$$\tilde{F}_{s+1}(x_s) = M_{s+1}(x_s,x_{s+1})\cdot M_{s+2}(x_{s+1},x_{s+2})\ldots M_{N-1}(x_{N-2},x_{N-1})\cdot\tilde{F}_N(x_{N-1}) \tag{13}$$

with

$$\tilde{F}_N(x_{N-1}) = \begin{bmatrix} \tilde{H}_{y,N}(x_{N-1}) \\ -Z_{x,N}\cdot\tilde{H}_{y,N}(x_{N-1}) \end{bmatrix}$$

We also use the proportionality between $\tilde{H}_y(x_s)$ and $\tilde{E}_z(x_s)$ on both sides of the $x=x_s$ surface:

$$\tilde{E}_{z,s}(x_s) = +\bar{Z}(x_s)\cdot\tilde{H}_{y,s}(x_s) \quad , \tag{14}$$

where $\overleftarrow{Z}(x_s)$ is the input (surface) impedance for TM fields looking into the upper (n=s) side of the boundary $x=x_s$; and

$$\tilde{E}_{z,s+1}(x_s) = -\vec{Z}(x_s)\cdot\tilde{H}_{y,s+1}(x_s) \quad , \tag{15}$$

where $\vec{Z}(x_s)$ is the input (surface) impedance for TM fields looking into the lower (n=s+1) side of the boundary $x=x_s$. The substitution of equations (14) and (15) in the continuity relation (11) leads to

$$\tilde{H}_{y,s}(x_s)-\tilde{H}_{y,s+1}(x_s) = -\tilde{H}^i_{y,s}(x_s) \tag{16a}$$

$$\vec{Z}\cdot\tilde{H}_{y,s+1}(x_s) + \overleftarrow{Z}\cdot\tilde{H}_{y,s}(x_s) = -\tilde{E}^i_{z,s}(x_s) \tag{16b}$$

what, after some manipulations gives

$$\tilde{H}_{y,s}(x_s) = \frac{-\vec{Z}\cdot\tilde{H}^i_{y,s}(x_s) - \tilde{E}^i_{z,s}(x_s)}{\overleftarrow{Z} + \vec{Z}} \quad , \tag{16c}$$

$$\tilde{H}_{y,s+1}(x_s) = \frac{+\overleftarrow{Z}\cdot\tilde{H}^i_{y,s}(x_s) - \tilde{E}^i_{z,s}(x_s)}{\overleftarrow{Z} + \vec{Z}} \quad . \tag{16d}$$

For s=1 w arrive at the case of reflection from the upper surface. From (14) we get

$$\tilde{E}_{z,1}(x_1) = +\overleftarrow{Z}(x_1)\cdot\tilde{H}_{y,1}(x_1) \quad , \tag{17}$$

and generally in the first domain, from (10b) we get as well

$$\tilde{E}_{z,1}(x_1) = Z_{x,1}\cdot\tilde{H}_{y,1}(x_1) \quad . \tag{18}$$

Therefore, the right sides of (17) and (18) are of the same value, what yields

$$\overleftarrow{Z} = Z_{x,1} \quad .$$

The incident field components in the first domain satisfy

$$\tilde{E}_z^i(x_1) = -Z_{x,1} \cdot \tilde{H}_y^i(x_1) \tag{19}$$

from which we get

$$\tilde{H}_{y,1}(x_1) = \frac{+Z_{x,1} \cdot \tilde{H}_y^i(x_1) - \vec{Z} \cdot \tilde{H}_y^i(x_1)}{Z_{x,1} + \vec{Z}} = \frac{Z_{x,1} - \vec{Z}}{Z_{x,1} + \vec{Z}} \tilde{H}_y^i(x_1) \; . \tag{20}$$

Defining the amplitude-based reflection coefficient R_1

$$\tilde{H}_{y,1}(x_1) = R_1 \tilde{H}_y^i(x_1) \tag{21}$$

we get

$$R_1 = \frac{Z_{x,1} - \vec{Z}}{Z_{x,1} + \vec{Z}} \; . \tag{22}$$

In order to evaluate the reflectivity R_1 the surface impedance $\vec{Z}$ must be determined. To this end, from (15)

$$\tilde{E}_{z,2}(x_1) = -\vec{Z} \cdot \tilde{H}_{y,2}(x_1) \tag{23}$$

and from (13)

$$\tilde{F}_2(x_1) = M(x_2, x_{N-1}) \cdot \tilde{F}_N(x_{N-1}) \tag{24}$$

we get

$$\tilde{H}_{y,2}(x_1) = M_{11} \tilde{H}_{y,N}(x_{N-1}) + M_{12} \tilde{E}_{z,N}(x_{N-1}) \; . \tag{25}$$

Then, the substitution of (9b) in (25) leads to

$$\tilde{H}_{y,2}(x_1) = (M_{11} - Z_{x,N} M_{12}) \cdot \tilde{H}_{y,N}(x_{N-1}) \; . \tag{26}$$

Along the same lines we can get

$$\tilde{E}_{z,2}(x_1) = (M_{21}-Z_{x,N}M_{22})\cdot\tilde{H}_{y,N}(x_{N-1}) = \frac{M_{21} - Z_{x,N}M_{22}}{M_{11} - Z_{x,N}M_{12}}\,\tilde{H}_{y,2}(x_1) \quad . \tag{27}$$

The comparison of (23) and (27) gives the explicit form of the surface impedance $\vec{Z}$:

$$\vec{Z} = -\,\frac{M_{21} - Z_{x,N}M_{22}}{M_{11} - Z_{x,N}M_{12}} \quad . \tag{28}$$

From (22) and (28) we finally obtain

$$R_1 = \frac{M_{21} + (Z_{x,1}M_{11} - Z_{x,N}M_{22}) - Z_{x,1}Z_{x,N}M_{12}}{-M_{21} + (Z_{x,1}M_{11} + Z_{x,N}M_{22}) - Z_{x,1}Z_{x,N}M_{12}} \quad . \tag{29}$$

In the limit case of the boundary between two homogeneous media (N=3 and W=0, $W=x-x_0$ being the width of the layer), the transfer matrix M becomes unit matrix and

$$R_1 = \frac{Z_{x,1} - Z_{x,3}}{Z_{x,1} + Z_{x,3}} \tag{30}$$

becomes the well known reflection coefficient of the interface. The same result is obtained by assuming $n_2=n_3$ in (29) for N=3.

The transmittivity T_N of the structure for TM field is defined by

$$\tilde{H}_{y,N}(x_{N-1}) = T_N\cdot\tilde{H}^i_y(x_1) \tag{31}$$

and along the same lines as for the reflection coefficient we can get

$$T_N = \frac{2Z_{x,1}}{-M_{21} + (Z_{x,1}M_{11} + Z_{x,N}M_{22}) - Z_{x,1}Z_{x,N}M_{12}} \quad . \tag{32}$$

what for the single interface resolves into

$$T = \frac{2\,Z_{x,1}}{Z_{x,1} + Z_{x,3}} \quad . \tag{33}$$

The foregoing derivations from the equation (16) have been presented for TM polarization. For TE field we get the similar equations:

$$\tilde{E}_{y,s}(x_s)-\tilde{E}_{y,s+1}(x_s) = -\tilde{E}^{i}_{y,s}(x_s) \tag{34a}$$

$$\vec{Y}(x_s)\tilde{E}_{y,s+1}(x_s) + \overleftarrow{Y}(x_s)\tilde{E}_{y,s}(x_s) = -\tilde{H}^{i}_{z,s}(x_s) \tag{34b}$$

and

$$\tilde{E}_{y,s}(x_s) = \frac{-\vec{Y}\cdot\tilde{E}^{i}_{y,s}(x_s) - \tilde{H}^{i}_{z,s}(x_s)}{\overleftarrow{Y} + \vec{Y}} , \tag{34c}$$

and for s=1, that is for the incident field in D_1:

$$\tilde{H}^{i}_{z}(x_1) = -Y_{x,1}\tilde{E}^{i}_{y}(x_1) . \tag{35}$$

Finally,

$$R^{TE}_{1} = \frac{Y_{x,1} - \vec{Y}}{Y_{x,1} + \vec{Y}} , \tag{36}$$

and

$$\vec{Y} = - \frac{M^{TE}_{21} - Y_{x,N}M^{TE}_{22}}{M^{TE}_{11} - Y_{x,N}M^{TE}_{12}} , \tag{37}$$

where M^{TE}_{ij} is a transfer matrix for TE field.

4. SCATTERING MATRIX APPROACH

The electromagnetic field inside each layer of the stratified medium can be divided into waves propagating in the positive x-direction and in the negative x-direction[3,6,7]. For TM field there are two independent field components in mth layer:

$$\tilde{H}_y(x,\beta,\omega) = a_m(\beta,\omega)\exp(+i\alpha_m(x-x_{m-1})) + b_m(\beta,\omega)\exp(-i\alpha_m(x-x_{m-1})) = \tilde{H}_m^+ + \tilde{H}_m^- \tag{38}$$

and

$$\tilde{E}_z(x,\beta,\omega) = -Z_{x,m}a_m(\beta,\omega)\exp(+i\alpha_m(x-x_{m-1})) + Z_{x,m}b_m(\beta,\omega)\exp(-i\alpha_m(x-x_{m-1})) = \\ = \tilde{E}_m^+ + \tilde{E}_m^- \quad , \tag{39}$$

where $\tilde{H}_y(x,\beta,\omega)$ and $\tilde{E}_z(x,\beta,\omega)$ are coefficients of a plane wave decomposition (or Fourier transforms) of H_y and E_z defined by equation (2), α_m is a wavenumber x-direction component in mth layer, a_m is the amplitude of the wave traveling in the positive x-direction, b_m is the amplitude of the wave traveling in the negative x-direction, and in the upper half-space D_1 we assume that $x_0=x_1$. The scattering matrix of the system couples the amplitudes of the incoming waves of the system with the amplitudes of the outgoing waves. Let us consider a subsystem consisting of the first n layers of the N-layer system. The incoming waves of the subsystem are the waves propagating in the positive x-direction in the first medium and the waves propagating in the negative x-direction in the nth medium. The outgoing waves are those traveling in the negative x-direction in the first medium and those traveling in the positive x-direction in the nth medium. Therefore, the scattering matrix equation of the subsystem is defined by

$$\begin{bmatrix} b_1 \\ a_n \end{bmatrix} = S(1,n)\begin{bmatrix} a_1 \\ b_n \end{bmatrix} , \tag{40}$$

where S(1,n) is a 2 x 2 scattering matrix. The field amplitudes in D_n and D_{n+1} domains are coupled by the equation:

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = I(n+1)\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} , \tag{41}$$

with the modified transfer matrix I(n+1) to be found. Eliminating a_n and b_n from equations (40) and (41) we get the scattering matrix equation for the n+1-layer subsystem:

$$\begin{bmatrix} b_1 \\ a_{n+1} \end{bmatrix} = S(1,n+1) \begin{bmatrix} a_1 \\ b_{n+1} \end{bmatrix} \qquad (42)$$

with

$$\begin{aligned} S_{21}(1,n+1) &= (I_{11} - S_{22}I_{21})^{-1}S_{21} \\ S_{22}(1,n+1) &= (I_{11} - S_{22}I_{21})^{-1}(S_{22}I_{22} - I_{12}) \\ S_{11}(1,n+1) &= S_{12}I_{21}S_{21}(1,n+1) + S_{11} \\ S_{12}(1,n+1) &= S_{12}I_{21}S_{22}(1,n+1) + S_{12}I_{22} \end{aligned} \qquad (43)$$

where I means I(n+1) and S without parentheses means S(1,n). Using the above procedure with the matrix

$$S(1,1)=\begin{bmatrix} 0, & 1 \\ 1, & 0 \end{bmatrix}$$

at the starting point, we can generate subsequent matrices S(1,2), S(1,3)... and the final matrix S(1,N) is the scattering matrix of the whole structure:

$$\begin{bmatrix} b_1 \\ a_N \end{bmatrix} = S(1,N) \begin{bmatrix} a_1 \\ b_N \end{bmatrix}. \qquad (44)$$

The reflection and transmission coefficients R_1 and T_N can be calculated by choosing $a_1=1$ and $b_N=0$, then $R_1=S_{11}(1,N)$ and $T_N=S_{21}(1,N)$. The explicit form of the modified transfer matrix I(n+1) can be derived from the transfer matrix equations (4) coupling fields in x_n and x_{n+1}, and from continuity conditions at x_n

$$H_n^+(x_{n-1})+H_n^-(x_{n-1}) = -iZ_{x,n}^{-1}\sin(D)(E_{n+1}^+(x_n)+E_{n+1}^-(x_n))+ \\ +\cos(D)(H_{n+1}^+(x_n)+H_{n+1}^-(x_n)) \qquad (45)$$

where $D=\alpha_n W_n$ is the phase change on passing the nth layer and $W_n=x_n-x_{n-1}$ is the nth layer width. Using impedance equations:

$$E^{+}_{n+1}(x_n) = -Z_{x,n+1}H^{+}_{n+1}(x_n)$$
$$E^{-}_{n+1}(x_n) = +Z_{x,n+1}H^{-}_{n+1}(x_n) \qquad (46)$$

and substituting sin and cos functions by exponential functions we get:

$$H^{+}_{n}(x_{n-1})+H^{-}_{n}(x_{n-1}) = 1/2\exp(+iD)(H^{+}_{n+1}(x_n)(1+Z) + H^{-}_{n+1}(x_n)(1-Z))+ \\ +1/2\exp(-iD)(H^{+}_{n+1}(x_n)(1-Z) + H^{-}_{n+1}(x_n)(1+Z)) \qquad (47)$$

with $Z=Z_n^{-1}Z_{n+1}$. According to definitions of H^{+} and H^{-} we can mark the elements proportional to $\exp(+iD)=\exp(+i\alpha_n(x_n-x_{n-1}))= E\cdot\exp(-i\alpha_n x_{n-1})$ by a superscript (-) and those proportional to exp(-iD) by a superscript (+), or assign:

$$H^{+}_{n}(x_{n-1}) = 1/2\exp(-iD)\cdot(H^{+}_{n+1}(x_n)(1+Z) + H^{-}_{n+1}(x_n)(1-Z))$$
$$H^{-}_{n}(x_{n-1})=1/2\exp(+iD)\cdot(H^{+}_{n+1}(x_n)(1-Z) + H^{-}_{n+1}(x_n)(1+Z)) \qquad (48)$$

On the other hand, from the definitions of H^{+} and H^{-} we have $H^{+}_{n+1}(x_n)=a_{n+1}$, $H^{-}_{n+1}(x_n)=b_{n+1}$, and the explicit form of the transfer matrix I(n+1) is

$$I(n+1) = 1/2 \begin{bmatrix} (1+Z)\exp(-iD) & ; & (1-Z)\exp(-iD) \\ (1-Z)\exp(+iD) & ; & (1+Z)\exp(+iD) \end{bmatrix} \qquad (49)$$

For TE polarization we get the same results with $Z_{x,m}$ replaced by $Y_{x,m}$.

5. THE RECIPROCITY AND ENERGY CONSERVATION RELATIONS

The Lorenz reciprocity theorem[4] couples two linearly independent solutions of Maxwell's equations, given by the boundary condition fields $\tilde{F}_1=[\tilde{E}_1,\tilde{H}_1]$ and $\tilde{F}_2=[\tilde{E}_2,\tilde{H}_2]$. Assuming, that the medium is isotropic and linear, we have for any closed surface Σ:

$$\int_{\Sigma}(\tilde{E}_1\times\tilde{H}_2 - \tilde{E}_2\times\tilde{H}_1)\bar{n}\cdot d\Sigma = 0 \qquad (50)$$

where $\bar{n}$ is a unit vector perpendicular to Σ. For the multilayered planar structure described in previous sections, we choose the surface Σ consisting of two bottom parts Σ_1 and Σ_2, which are parallel to the layers and the lateral part Σ_3 perpendicular to the structure. Let $x=x_1$ on Σ_1 and $x=x_N$ on Σ_2. The integral on Σ_3 vanishes because of the symmetry of the field pattern. From eqs.(38) and (39) we find, that the fields at $x=x_1$ are as follows:

$$\tilde{H}_1(x_1) = (a_1+b_1)\bar{e}_y$$

$$\tilde{E}_1(x_1) = -Z_{x,1}(a_1-b_1)\bar{e}_z$$

$$\tilde{H}_2(x_1) = (c_1+d_1)\bar{e}_y \qquad (51)$$

$$\tilde{E}_2(x_1) = -Z_{x,1}(c_1-d_1)\bar{e}_z ,$$

and at $x=x_{N-1}$:

$$\tilde{H}_1(x_{N-1}) = (a_N+b_N)\bar{e}_y$$

$$\tilde{E}_1(x_{N-1}) = -Z_{x,N}(a_N-b_N)\bar{e}_z$$

$$\tilde{H}_2(x_{N-1}) = (c_N+d_N)\bar{e}_y \qquad (52)$$

$$\tilde{E}_2(x_{N-1}) = -Z_{x,N}(c_N-d_N)\bar{e}_z .$$

where $\bar{e}_y$ and $\bar{e}_z$ are unit vectors in y- and z-directions, the amplitudes of forward and backward waves of $\tilde{F}_1$ are a and b, and the amplitudes of forward and backward waves of $\tilde{F}_2$ are c and d, respectively. As the fields $\tilde{H}$ and $\tilde{E}$ depend only on x-coordinate, the integrals over Σ_1 and Σ_2 surfaces are proportional to the integrand. Substitution of the above equations in (50) leads to:

$$S_{12}Z_{x,1} = S_{21}Z_{x,N} \qquad (53)$$

or

$$T_{21}Z_{x,1} = T_{12}Z_{x,N} \quad (54)$$

The reciprocity theorem result (eq. 53) can be cast into the matrix form as well:

$$[P][S] = ([P][S])^T = [S]^T[P] \quad (55)$$

where

$$[P] = \begin{bmatrix} Z_{x,1} & ; & 0 \\ 0 & ; & Z_{x,N} \end{bmatrix} , \quad (56)$$

which means , that [P][S] is a symmetric matrix.

The generalized energy conservation law for two independent solutions $\tilde{F}_1=[\tilde{H}_1,\tilde{E}_1]$ and $\tilde{F}_2=[\tilde{H}_2,\tilde{E}_2]$ of stationary, sourceless Maxwell's equations is as follows[2)]

$$\int_\Sigma (\tilde{E}_1 \times \tilde{H}_2^* + \tilde{E}_2^* \times \tilde{H}_1)\bar{n}\cdot d\Sigma = 0 \quad (57)$$

where asterisk means a complex conjugate, Σ is a closed surface and $\bar{n}$ is a unit vector perpendicular to Σ. The components of $\tilde{F}_1$ and $\tilde{F}_2$ are given by eqs.(51) and (52). Introducing equations (51) and (52) into eq. (57), and using the property of proportionality of the integral to the integrand, we get:

$$Z_{x,1}(a_1c_1^* + a_1d_1^* - b_1c_1^* - b_1d_1^*) + Z_{x,N}(a_Nc_N^* + a_Nd_N^* - b_Nc_N^* - b_Nd_N^*) +$$
$$+Z_{x,1}^*(a_1c_1^* - a_1d_1^* + b_1c_1^* - b_1d_1^*) + Z_{x,N}^*(a_Nc_N^* - a_Nd_N^* + b_Nc_N^* - b_Nd_N^*) = 0 \quad . \quad (58)$$

Using equation (44) for the replacement of b_1 and a_N with a_1 and b_N, and for the replacement of d_1 and c_N with c_1 and d_N we can rearrange the equation (58) into the following matrix form

$$[a_1,b_N]([P_+] + [P_-][S]^* - [S]^T[P_-] - [S]^T[P_+][S]^*)[c_1,d_N]^{*T} = 0 \ , \quad (59)$$

where $[P_+]=[P]+[P]^*$, $[P_-]=[P]-[P]^*$ and [P] matrix is defined by eq.(56).

The amplitudes a_1, b_N, c_1, d_N are arbitrary, so the energy conservation condition is

$$[P_+] + [P_-][S]^* - [S]^T[P_-] - [S]^T[P_+][S]^* = 0 \qquad (60)$$

This is a set of four equations, of which two are identical, so the energy conservation law results in three equations coupling the scattering matrix elements:

$$2\mathrm{Im}(Z_{x,1})\mathrm{Im}(S_{11}) + (1 - S_{11}S_{11}^*)\mathrm{Re}(Z_{x,1}) - S_{21}S_{21}^*\mathrm{Re}(Z_{x,N}) = 0 \qquad (61)$$

$$2\mathrm{Im}(Z_{x,N})\mathrm{Im}(S_{22}) + (1 - S_{22}S_{22}^*)\mathrm{Re}(Z_{x,N}) - S_{12}S_{12}^*\mathrm{Re}(Z_{x,1}) = 0 \qquad (62)$$

$$S_{21}^*(i\mathrm{Im}(Z_{x,N}) - S_{22}\mathrm{Re}(Z_{x,N})) - S_{12}(i\mathrm{Im}(Z_{x,1}) + S_{11}^*\mathrm{Re}(Z_{x,1})) = 0 \quad . \qquad (63)$$

For propagating modes (Z_x real) in both the first and the last layers, [P] becomes a real matrix, $[P_+]=2[P]$, $[P_-]=0$, and from eq.(60) we get the generalized unitarity condition for [S] matrix:

$$[P_+][S]^{-1} = [S]^{*T}[P_+] \quad . \qquad (64)$$

Both matrix equations (55) and (60) have their close counterparts in the theory of scattering on waveguide junctions (see ref. [2]).

6. CONCLUSIONS

Both methods of the field evaluation at the multilayered structure, namely the transfer matrix approach and the scattering matrix approach were implemented in a numerical computation of the reflectivity and the transmittivity functions. Fig.2 displays an example of the numerical evaluation of the reflection coefficient at the three-layer structure, consisting of a superstrate D_1, layer D_2 and substrate D_3, with $\epsilon_1=1.0$, $\epsilon_2=3.0$, $\epsilon_3=1.5$, the width of the layer $W_2=10.0$, the vacuum wavelength $\lambda_0=2\pi/k_0=2.0$, $\mu_1=\mu_2=\mu_3=1.0$. The magnitude of the reflection coefficient is plotted as a function of the angle of incidence from D_1. As it was shown in ref.[10], each of the three minima, appearing on the plot, can be associated with a reflectance null-pole pair, which identifies leaky modes of the structure.

Although a number of numerical simulations have been carried out, none of them could confirm the existence of the differences between the two methods as reported in ref.[3]. In spite of that, due to the field division into the forward and backward components, it seems that the scattering matrix method is better suited for the field analysis at the multilayered structures. The application of this method in analysis of the field at nonlinear waveguiding structures [10, 11] will be reported in future.

The work was supported by CPBP 02.13/7.11 Programme.

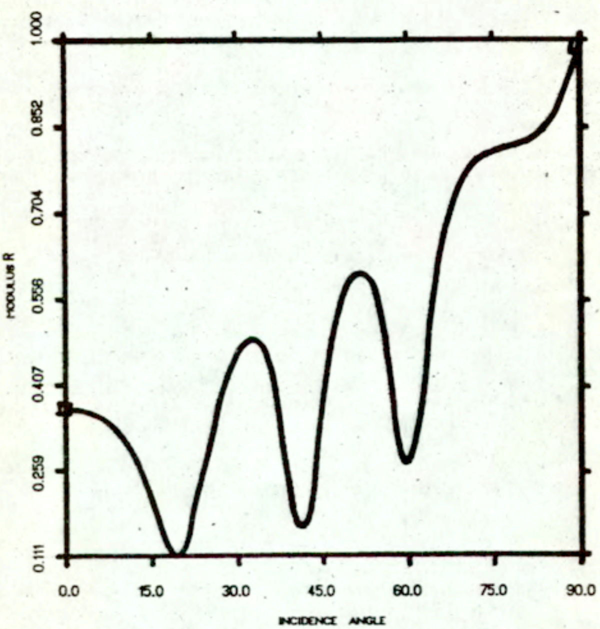

Fig.2. The magnitude of reflectivity of plane waves incident on the three-layer structure, plotted against the incidence angle.

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