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Modelling of live loads in the problem of reliability of highway bridges

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Discrete models for describing the live bridge load caused by a traffic flow is considered. A few situations of load localization on the bridge, which give the maximal structure response in a long-term exploitation are analyzed. The probabilistic characteristics of the extreme loads, which are the most important ones while the reliability treated as the first crossing problem is estimated are determined.

Key words: *highway bridges, live load, extreme load.*

1. Introduction

One of the most difficult problems in the estimation of bridge reliability is elaborating an appropriate theoretical model of loads. Vehicles moving along the bridge act on the structure dynamically and cause coupling vibrations of the system: *structure – vehicle* [1]. Furthermore, the structure elements weights, dimensions, equipments and also the vehicles weights, the velocity of their movement, the distances between them all are variable quantities of a random type. This is the reason that for modelling the bridge loads the random variables and also the stochastic processes should be used. In general, the bridge loads can be divided into following types:

- the dead load following from the structures weight and equipment,
- live loads due to the traffic flow, which can act statically or dynamically,
- climatic loads such as wind load, snow or icing load, thermal load,
- non-mechanical loads, for example the imperfection of the structure such as the crakes in the concrete, and
- incidental loads which follow for example from collisions of vehicles.

The problem of describing and modelling the bridges loads has been considered in many papers [1-13]. Two groups of load models can be distinguished. The first group form the discrete models and the second one pertains to the continuous models of the loads. In the discrete models each vehicle or the pressure on the bridge of each axis is presented as a point force. In the continuous models the loads caused by a single vehicle are not distinguished but replaced by a stochastic process of known probabilistic characteristics. In most cases the normal stochastic process is assumed as a continuous model of the load. The theoretical model of the load depends on many factors such as the type of the bridge: short, medium or long-span, the type of the load: static or dynamic, the type of the limit condition used in the reliability estimation: the first crossing condition in the ultimate limit state or in the serviceability limit state or the fatigue limit condition. In all this cases another parameters of the acting load play the greatest role, for example in estimating the bridge reliability treated as the first crossing problem in the ultimate limit state and in the serviceability limit state the most important role play the heaviest vehicles and in estimating the fatigue reliability very important is the knowledge of the whole traffic flow, as it is needed for calculating the cumulative damage.

In the presented work we focused our attention on the discrete models for describing the live bridge loads caused by a traffic flow. The following components of the load are taken into account: the characteristics of the particular class of the vehicles, the structure of the traffic flow, i.e. the percentage part of each vehicle class in the whole stream, the distances between the vehicles, and also their velocities, in the dynamic case. The probabilistic characteristics of the extreme loads, which are the most important ones while the reliability is treated as the first crossing problem, are determined. The probability density function of the weight of the heaviest vehicles of the truck or trailer type is calculated. In the reliability analysis the exploitation time is replaced by the expected number of vehicles which pass the bridge in a given time. For calculating the probability density function of the load, which causes the extreme value of the structure response in the effort or displacement state of the bridge a few extreme situations are considered. They include:

- the passage of the bridge by a heaviest vehicle in a given time,
- the selection of all events in which two or more (depending on the bridge span) successive vehicles of the truck type arrive on a single traffic line, which results in the maximal load,
- the events when, in any time, two heavy vehicles are present simultaneously on corresponding (parallel) traffic lanes.

Below the basic quantities constituting the presented load model are described.

2. Model of load caused by a single vehicle

The vehicles in a traffic flow can be divided into a few classes. In general, one can distinguish more than ten classes of vehicles, but most often only five classes are distinguished [7] (Fig. 1): (I) cars, (II) small trucks, (III) two-axis large trucks, (IV) three-axis trucks, (V) four-axis trucks (in [8] five-axis, so called semi-trailer truck).

We assume that the vehicle dimensions are deterministic variables and the vehicle weight Q is a random variable. From the investigations of the vehicle weight follows that it can be described by a random variable the probability density function of which is of the lognormal shifted distribution type [7], the truncated lognormal distribution or the beta distribution.

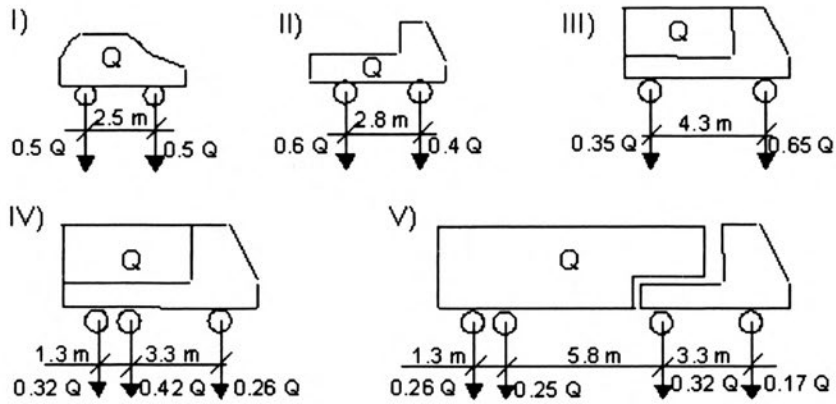

FIGURE 1. Classes of vehicles.

3. Model of traffic flow

For describing the traffic flow various models depending on the type of the move – free or forced move (Fig. 2) – are used.

The most frequently applied traffic models are: the regular stream (Fig. 3), binomial (Fig. 4) or Poisson stochastic process (Fig. 5), renewal stochastic process with Erlang distribution of the distances between the vehicles, gamma stochastic process and others. In the figures below each vehicle is presented

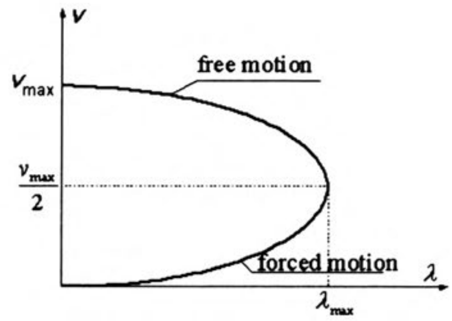

FIGURE 2. Theoretical relationship between the motion intensity and the mean velocity of the vehicles.

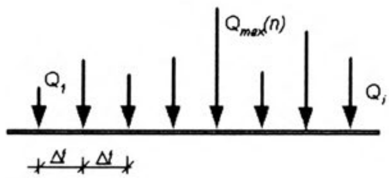

FIGURE 3. Regular model of traffic flow.

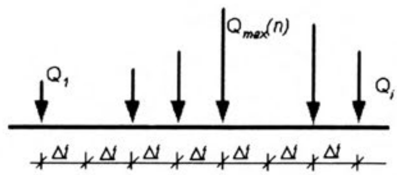

FIGURE 4. Binomial model of traffic flow.

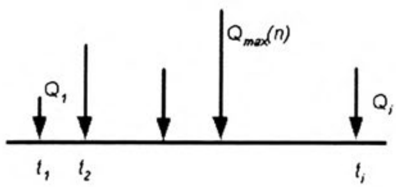

FIGURE 5. Poisson model of traffic flow.

as one point force and the models describe the cases when, in a given time period between n vehicles, only one of them is of the heaviest type.

4. Distribution of maximal quantities

For different models of traffic flow such as regular stream, binomial stream or Poisson stochastic process one can show that the probability density function $f_{\max}(q,n)$ for the maximum value of vehicle weight of a subset of n vehicles is

$$f_{\max}(q,n) = (n-1)\, f_Q(q)\, \exp\big[-(n-1)\big(1-F_Q(q)\big)\big] \tag{4.1}$$

where $f_Q(q)$ and $F_Q(q)$ are the probability density function and the cumulative distribution function of a single vehicle, respectively. For the Poisson model of the traffic flow the symbol n denotes the mean number of vehicles in a given time period.

In Figs. 6 and 7 the cumulative distribution function and the probability density function of the maximum value of vehicle weight for the shifted lognormal distribution of the vehicle weight and different number of vehicles are presented. The calculations have been done for the data taken from [7]: $q_{\min} = 51\,\text{kN} \leqslant q < \infty$ and $E[Q] = 219\,\text{kN}$, $\sigma_Q = 84\,\text{kN}$.

In practice, both values: the lower as determined minimal values $q_{\min}$ and higher as determined by maximal values $q_{\max}$ loads are unrealistic. Therefore, the more adequate distribution for the vehicle weight seems to be a truncated ($q_{\min} \leqslant Q \leqslant q_{\max}$) lognormal distribution or the beta distribution.

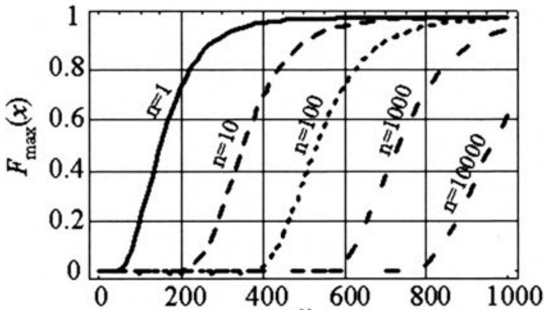

Figure 6. Cumulative distribution functions.

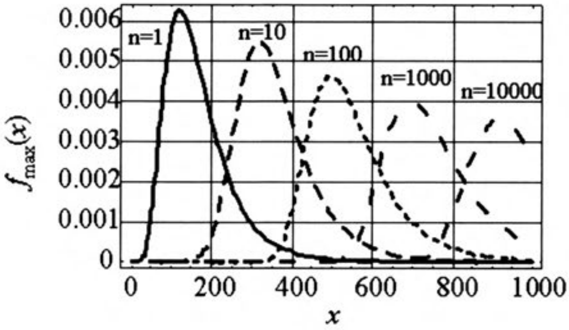

FIGURE 7. Probability density functions.

For the first case the cumulative distribution function and the probability density function for the heaviest vehicles are equal to:

$$F_{\max}[n, q] = \frac{1}{e^{n\left(1-\frac{F[q-q_{\min}]}{a}\right)}}, \tag{4.2}$$

$$f_{\max}[n, q] = \frac{n\, f[q-q_{\min}]}{a\, e^{(n-1)\left(1-\frac{F[q-q_{\min}]}{a}\right)}}, \tag{4.3}$$

where

$$a = \int\limits_{q_{\min}}^{q_{\max}} f[q]\, dq = \int\limits_{0}^{q_{\max}-q_{\min}} f[q-q_{\min}]\, dq.$$

In Figs. 8 and 9 the cumulative distribution function and the probability density function of the maximum value of vehicle weight for the truncated lognormal distribution of the vehicle weight and different number of vehicles are shown. The results have been obtained for $q_{\min} = 51\,\text{kN} \leqslant q \leqslant q_{\max} = 557\,\text{kN}$ and $E[Q] = 219\,\text{kN}$, $\sigma_Q = 84\,\text{kN}$ [7].

From Fig. 9 follows that when the number of vehicles tends to be infinite ($n \to \infty$) the probability density function $f_{\max}(q, n)$ tends to the one-point distribution at the point $q_{\max}$. Therefore, an important question is how to predict the maximal value $q_{\max}$ for a large number ($n > 10^6$) of vehicles, which can appear in a long time period of the bridge exploitation. The quantity $Q_{\max}$ can be treated as a random value. Hence, the cumulative distribution function and probability density function of the maximum value of vehicle weight can be determined as conditional characteristics, i.e. $F_{\max}(q_{\max}, n|_{Q_{\max}})$ and $f_{\max}(q_{\max}, n|_{Q_{\max}})$ and the non-conditional proba-

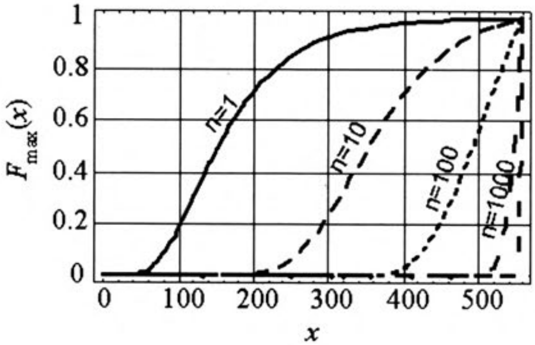

FIGURE 8. Cumulative distribution functions.

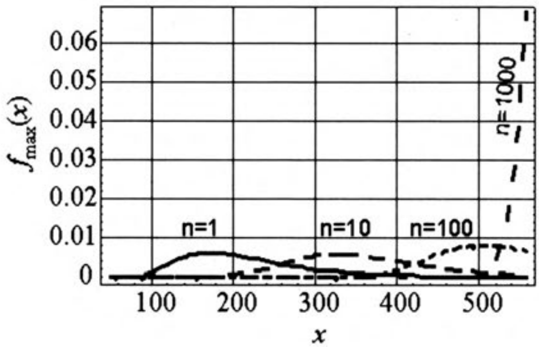

FIGURE 9. Probability density functions.

bility density function of these can be determined from the formula

$$f_{\max}(q_{\max}, n) = \int_Q f(q_{\max}|_Q, n)\, f_{Q_{\max}}(q)\, dq. \tag{4.4}$$

The question appears how to calculate the probability density function $f_{Q_{\max}}(q)$ while the empirical data from the future are not known. It can be done only by extrapolating the increase of vehicles load in a recent time period. Let us remind that the quantity $Q_{\max}$ and hence $f_{Q_{\max}}(q)$ depend on the predicted time of bridge exploitation, and indirectly on the number n of vehicles moving during this time along the bridge.

From the investigations follows that the beta distribution is a good one for describing empirical data. Therefore in Figs. 10 and 11 we show also the cumulative distribution function and the probability density function of the

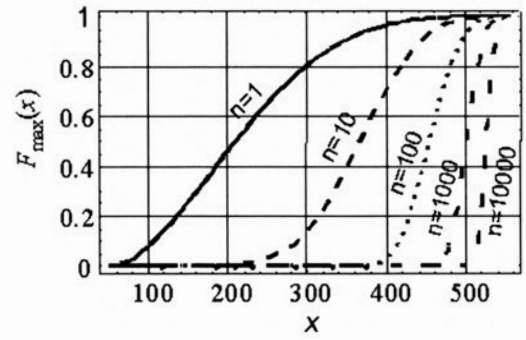

FIGURE 10. Cumulative distribution functions.

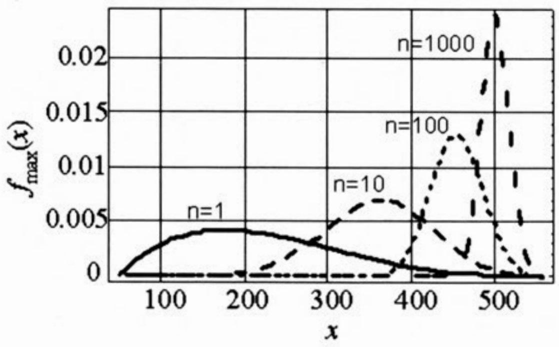

FIGURE 11. Probability density functions.

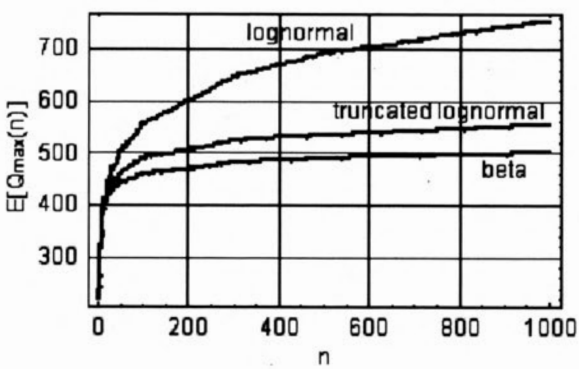

FIGURE 12. Expected value of the maximal value of the vehicle weight.

maximum value of vehicle weight when the vehicle weight is described by the beta distribution with parameters 2 and 4.

5. Load models for two traffic lane

Until now we have considered only the load of a single traffic lane. Consider now two traffic lanes in the same traffic direction. Let us consider a short or medium-span bridge of the length L when, in any time, each traffic lane could be occupied by one heavy vehicle – "truck" (Fig. 13).

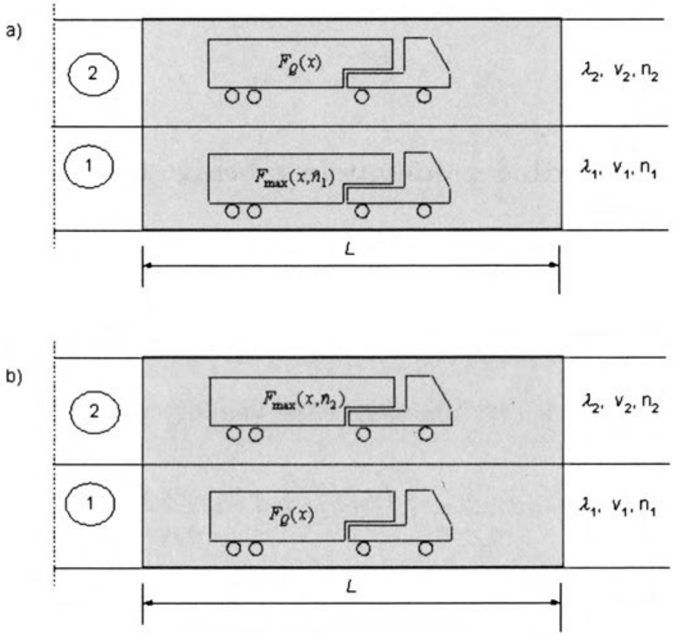

FIGURE 13.

Let the traffic intensity on the first traffic lane be equal to λ_1 and the vehicles speed be equal to v_1 and on the second one λ_2 and v_2, respectively. Let the traffic flow on each traffic lane be of the binomial process type. The probability that within the time period $(t, t+dt)$ two such vehicles are present simultaneously on the corresponding (parallel) traffic lanes is equal to

$$P\{N_1(t_1) \cap N_2(t_2)\} = P\{dN_1(t_1)\} P\{N_2(t_2)\} = \lambda_1 \lambda_2 (dt)^2. \qquad (5.1)$$

Using the above relationship one can obtain the probability that just two vehicles are at the same time on the bridge as

$$P\left\{N_1\left(\frac{L}{v_1}\right)\cap N_2\left(\frac{L}{v_2}\right)\right\} = \lambda_1\,\lambda_2\,\frac{L}{v_1}\,\frac{L}{v_2}. \tag{5.2}$$

From the above considerations follows that for two traffic lanes, apart from the event that the vehicle of a maximal weight arrives on one of the traffic lines, two additional events shown in Fig. 13 should be considered. In these cases

$$\hat{n}_1 = \lambda_2\,\frac{L}{v_2}\,\lambda_1\,t = \lambda_2\,\frac{L}{v_2}\,n_1 \tag{5.3}$$

and

$$\hat{n}_2 = \lambda_1\,\frac{L}{v_1}\,\lambda_2\,t = \lambda_1\,\frac{L}{v_1}\,n_2. \tag{5.4}$$

6. Maximal bending moments in a bridge beam

In the case of the bridge loaded by two trucks the bending moment of the bridge beam in a given cross-section can be given in the form

$$M(x) = \alpha_1(x)\,Q_1 + \alpha_2(x)\,Q_2. \tag{6.1}$$

The probability density function of the bending moment is equal to

$$f_m(m) = \frac{1}{|\alpha_2(x)|}\int\limits_{q_{\min}}^{q_{\max}} f_q(q)\,f_q\left(\frac{m-\alpha_1(x)\,q}{\alpha_2(x)}\right)dq. \tag{6.2}$$

The expression (6.2) allows to calculate the probability density function of the maximal bending moment from the n_1 pairs of heavy vehicles (trucks) which arrive on the bridge among n different vehicles:

$$f_{m_{\max}}(m,n_1) = \frac{n_1}{|\alpha_2(x)|}\,\frac{\int\limits_{q_{\min}}^{q_{\max}} f_q(q)\,f_q\left(\frac{m-\alpha_1(x)\,q}{\alpha_2(x)}\right)dq}{\exp\big[(n_1-1)\big(1-F_m(m)\big)\big]}. \tag{6.3}$$

The probability density function of maximal bending moment in a given cross-section of the bridge beam can be determined also for two other situations.

The second situation is when the bridge is loaded by a pair of two trucks: one $Q_1 = Q$ and the second of maximal weight $Q_2 = Q_{\max}(n,q)$. In this case the probability density function of maximal bending moment is given by the formula

$$f_{m_{\max}}(n,m,x) = \frac{n}{|\alpha_2(x)|} \int\limits_{q_{\min}}^{q_{\max}} \frac{f_Q(q)\, f_Q\left(\frac{m-\alpha_1(x)\,q}{\alpha_2(x)}\right)}{\exp\left[(n-1)\left(1-F_Q\left(\frac{m-\alpha_1(x)\,q}{\alpha_2(x)}\right)\right)\right]}\, dq. \quad (6.4)$$

The third situation is when the bridge is loaded by two trucks each of maximal weight $Q_1 = Q_{\max}(n,q)$ and $Q_2 = Q_{\max}(n,q)$. In this case the probability density function of maximal bending moment is given by the formula

$$f_{m_{\max}}(n,m,x) = \frac{n^2}{|\alpha_2(x)|} \int\limits_{q_{\min}}^{q_{\max}} \frac{f_Q(q)\, f_Q\left(\frac{m-\alpha_1(x)\,q}{\alpha_2(x)}\right)}{\exp\left[(n-1)\left(2-F_Q(q)-F_Q\left(\frac{m-\alpha_1(x)\,q}{\alpha_2(x)}\right)\right)\right]}\, dq. \quad (6.5)$$

7. Example

As an example the reliability index has been calculated for the two-span beam bridge with the span length equal to 20 m, live load described by the beta distribution, dead load described by the normal distribution with parameters $E[g] = 12.7\,\mathrm{kN/m}$, $V[g] = 0.05$, beam capacity described by the normal distribution with parameters $E[M] = 2000\,\mathrm{kNm}$, $V[M] = 0.05$.

For the same data the probability density function and the cumulative distribution function of the maximal bending moment in the cross section over the middle support have been calculated and shown in Figs. 15 and 16.

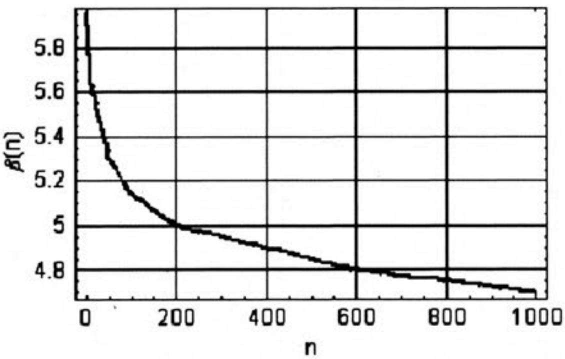

FIGURE 14. Reliability index of the bridge.

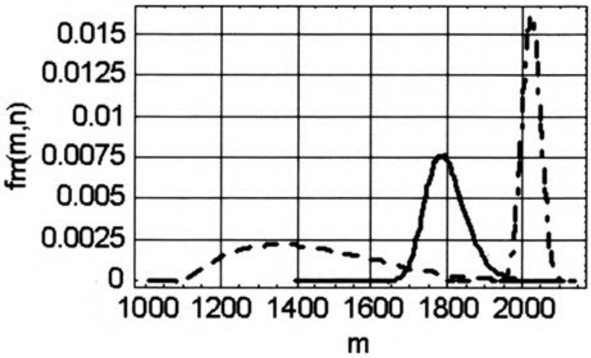

Figure 15. Probability density function of the maximal bending moment.

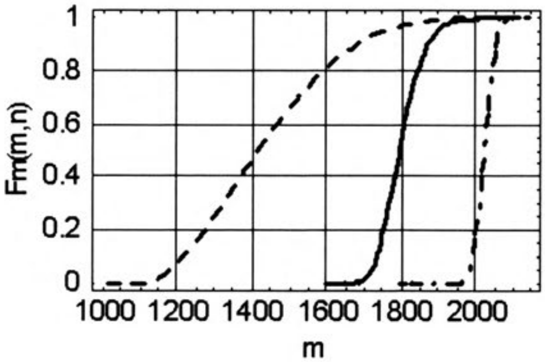

Figure 16. Cumulative probability density function of the maximal bending moment.

In Figs. 15 and 16 the dashed line has been obtained from Eq. (6.4), the solid one from Eq. (6.3) and the last one from Eq. (6.5).

8. Concluding remarks

In the algorithm for estimating the bridge reliability it is very important to build an appropriate load model based on real experimental data. Such a load model should take into account the overall traffic structure, the type of the bridge for which the model is being built and the type of the limit condition for which the reliability is considered. The investigations of traffic flow are made from the point of view of traffic engineering in which the structure of vehicle stream, the distances between them, the velocity of their move, the traffic intensity, and so on are considered. Results of investigations can be used for setting up a theoretical load model of the bridge, but such investigations, particularly in Poland, should be completed by investigation of the vehicles weights, distances between the vehicle axes, press of these axes onto the bridge etc.

The presented theoretical model of the live load can be a base for further investigations of the bridge load caused by the traffic flow, because it has been shown which additional parameters are needed. This model can be used for calculating the reliability of the bridge and its elements, when the first crossing limit reliability condition is applied and after some modifications also for calculating the reliability when the limit condition follows from the material fatigue.

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