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Computer-aided design of structures for optimal reliability

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New methodology of design is introduced. All possible load arrangements, load combinations and load effect interactions are taken into account thanks to the new matrix procedure. Global load vector, load-load effect influence array, load combination matrix and load effect interaction matrix are defined. The most serious load cases can be selected that not always could be discovered while the ordinary design procedure was applied. A non-conventional probabilistic method of design is presented. It is called κ-method and it reflects quite different design philosophy unlike the well-known β-method has done. Maximum safety, $\max q(\beta \mid S_d = R_d)$, is required for the κ-method with load effect S_d and structural resistance R_d design values at the ultimate limit states. Maximum failure frequency $\max dq/dS$ is revealed as an objective function of the β-method. The measure κ is defined as the relative hazard ratio. A value κ shall be constant in design of construction works of the same reliability class. A constant safety index β has been supposed as the adequate measure for the determination of design values. A theorem has been derived within the framework of the κ theory that the hazard ratios κ_S and κ_R shall be equal for random variables S and R and split reliability indices β_S and β_R shall be decreasing functions of sensitivity coefficients α_S, α_R. The split indices β_S, β_R according to the β-method are increasing functions of the sensitivity coefficients. The design values S_d and R_d are not coupled according to the κ method and structural design never gives negative or imaginary solutions for structural dimensions as it may happen if the β-method is applied.

1. Introduction

Both semi-probabilistic design and probabilistic design need new methodology unlike the deterministic allowable stress method of design. The *semi-probabilistic method* is meant as it has been used since 40 years in Central and East-European countries and it is recommended now for countries of the European Union. A *probabilistic method* of design is supposed to be implemented in next generation of standards and its format is not definite yet [8].

Structural analyses shall be done separately for the *serviceability limit states* (SLS) and for the *ultimate limit states* (ULS). When a semi-probabilistic method is applied, *characteristic values* of basic variables F_k, R_k are inserted to equations of the design algorithm for verification of the SLS and factored *design values* $F_d = \gamma_f F_k$, $R_d = R_k/\gamma_m$ are inserted for verification of the ULS. *Basic variables* are: F – random loads and environmental influences, R – random material properties, but geometrical quantities are not necessarily random. The results of the two analyses are not proportional provided that the load effect S is a combination of independent loads F_f and/or the resistance R is relative to a composite structural member because various partial factors are used for particular loads and materials and combination factors for loads. When a probabilistic structural analysis is applied, the *central values* (mean or median) shall be inserted to analyze both the SLS and the ULS conditions, but additional analysis is necessary for the ULS such that *standard deviations* of the basic variables are inserted to linear or linearized equations. Standard deviations of load effect, σ_S, and standard deviation of the resistance σ_R can be easily evaluated if relations of independent basic variables are linear.

As a matter of fact structural analysis has to be repeated much more than twice. The extreme values of load effects can be reached for various load options (arrangements) $v = 1, 2, \ldots, n_v$, and load combinations $c = 1, 2, \ldots, n_c$, and load effect interactions $e = 1, 2, \ldots, n_e$.

Example 1. Let the number of independent variable loads F_f be $n_f = 4$, where the subscript $f = 1, 2, 3, 4$ indicates: F_1 – live load, F_2 – snow, F_3 – wind, F_4 – temperature action. Let each variable load have $n_v = 3$ options (including the zero option, i.e. absent action); permanent load G may have two options G_{sup}, G_{inf} (in semi-probabilistic design). If the permutation rule is applied for load combinations [9] (but simultaneous snow and summer temperature are excluded), the number of load cases will be $(3^4 - 1) \times 2 \times 4! = 3840$.

The steel design standard [10] requires that the ULS of steel members shall be verified taking into account $n_e = 6$ formulae of M-N-V interaction. So, the structural analyses shall be repeated $3840 \times 6 = 23040$ times in order to select the extreme scalar load effects $S_{\max}$ and $S_{\min}$, the absolute value of which should not exceed the design resistance R_d. Such high number of repeated analyses is not acceptable by an ordinary designer. He has to guess the most serious load cases.

2. New methodology of design

If even a designer can guess which load options v and combinations c and load effect interactions e are not important, a remarkable number of repeated analyses still have to be done. Therefore the conventional design procedure shall be replaced by computer-aided matrix procedure. It may be arranged as follows:

1. *Global force vector* $\mathbf{F}_f$ contains design values or central values of each load F_f, $f = 0, 1, 2, \ldots, n_f$ (e.g. $F_f = G, Q, Sn, W, T$ etc.). The definition of global permanent load $F_0 = G$ is simple, it is the overall weight of construction works. The definitions of global load of variable actions may be left to the designer (e.g. it may be the sum of horizontal forces with or without drag coefficients for the wind action $\mathbf{W}$). The characteristic values and central values of global loads shall be either increased or decreased if anticipated lifetime of the structure is either smaller or larger than the standard reference period (usually 50 years for buildings).
2. *Matrix of combination forces* $\psi\mathbf{F}_{cf} = \psi'_{cf} \cdot \mathrm{diag}(\mathbf{F}_f)$ is derived taking into account a load combination matrix ψ_{cf}, $c = 1, 2, \ldots, n_c$, defined according to a load combination rule. Different combination rules are recommended by national or regional design standards [7, 9] etc. Preferably a stochastic rule [5] may be applied that will give save upper bound estimates of combination loads.
3. *Load-load effect influence array* $\mathbf{c}_{vjf}$ will be determined as results of structural analyses repeated $n_f \times n_v$ times for "normed" loads $\mathbf{F}_f / F_f$ in their spatial configuration. Influence coefficients c_{vjf} change any load $\mathbf{F}_f$ in component load effects S_j, $j = 1, 2, 3, \ldots, n_j$, e.g.: $S_1 = M_f$ (bending moment), $S_2 = n_f$ (axial force), $S_3 = V_f$ (shear force) etc. Options (arrangements) $v = 0, 1, 2, \ldots, n_v$ are taken into consideration for each variable load F_f, $f > 0$.
4. *Interaction matrix* $\mathbf{rs}_{ej}$, with $e = 1, 2, \ldots, n_e$ rows, is defined for n_e linear interaction forms. If the interaction diagram is originally curvilinear, it shall be replaced by a piece-wise linear diagram with any required accuracy. Too many pieces increase the number n_e of interactions. *Equivalent effect* S_{eff} (moment M_{eff} or force n_{eff}) is defined so that inequality $M_{eff} < M_R$ or $n_{eff} < n_R$ gives the evidence of safety. M_{eff}, n_{eff} are functions of load and environmental influences; M_R, n_R are functions of material properties.
5. *Relative influence array* $\mathbf{crs}_{vef} = \mathbf{rs}_{ej} \cdot \mathbf{c}_{vjf}$ is reduced to two relative *extreme influence matrices*: $\mathbf{maxcrs}_{ef} = \max_v(\mathbf{crs}_{vef})$ and $\mathbf{mincrs}_{ef} = \min_v(\mathbf{crs}_{vef})$. In such a way the most serious load options v are selected for each load effect interaction e. "Relative" means that they are relative to the structural element which is being designed and its proportions.

6. *Equivalent effect matrices* are derived by multiplication of the matrix of combination forces $\psi\mathbf{F}_{cf}$ by the extreme influence matrices:

$$\begin{aligned} \mathbf{maxS}_{ce} &= \psi\mathbf{F}_{cf} \cdot \mathbf{maxcrs}_{fe}, \\ \mathbf{minS}_{ce} &= \psi\mathbf{F}_{cf} \cdot \mathbf{mincrs}_{fe}. \end{aligned} \tag{2.1}$$

The "lower case" extreme values $\mathbf{maxS}_{ce}$ and $\mathbf{minS}_{ce}$ take into account the most risky options v of variable loads for each load combination c.

7. *Extremal effects* $MaxS = \max(\mathbf{maxS}_{ce})$ and $MinS = \min(\mathbf{minS}_{ce})$ ("upper case" maximum and minimum values) are selected for the most serious load combinations c and load effect interactions e and the positions (c, e) are determined (e.g. in order to evaluate eccentricity M/N for the same load case). Sometimes, both extreme effective moments are necessary to design (e.g. reinforced concrete members) and sometimes the larger absolute value only (e.g. steel members with bi-symmetrical cross-sections)

$$S = \max(MaxS, |MinS|) \leqslant S_R. \tag{2.2}$$

If the semi-probabilistic method is applied, the resistance M_{Rd} will be divided by an *importance factor* γ_n which depends on reliability class of the construction works. If a probabilistic design is applied, both values M_{eff} and M_R shall be evaluated with a safety measure appropriate to the *reliability class*.

Example 2. Let a single interaction function involves bending moment M and axial force N

$$M + r_N N < M_R$$

with:

$r_N = Z/A = 0.5\,\mathrm{m}$ – cross-section core (Z – section modulus, A – area of the cross-section);

$M_{Rd} = 19.5\,\mathrm{kNm}$ – design resistance of the member in condition of simple bending;

$Q_d = 50\,\mathrm{kN}$, $W_d = 20\,\mathrm{kN}$ – design global loads with two positive options of either load, the subscripts $v = 1, 2$ indicate the possible options: Q_1, Q_2 and W_1, W_2;

$\psi = 0.9$ – simultaneity factor from Ref. [9] for a non-dominant action in any load combination.

Two linear load effect combinations are taken into account in conventional design. The first load effect combination happens when M_d is maximum and associated n_d occurs; the second load effect combination happens when n_d is maximum and associated M_d occurs. An experienced designer can guess appropriate load combinations: $Q_{d2} + \psi W_{d1}$ with Q as the dominant action and the second load combination $\psi Q_{d1} + W_{d1}$ with W as the dominant action and he will analyze the structure twice. The details of analysis being not important here, the results only are shown as follows:

$$M = 14.3\,\text{kNm} \quad \text{and} \quad N = 10.1\,\text{kN}$$
– from the first analysis when $M = maxM$,

$$M = 12.4\,\text{kNm} \quad \text{and} \quad N = 12.6\,\text{kN}$$
– from the second analysis when $N = maxN$;

$$14.3 + 0.5 \cdot 10.1 = 19.35 < 19.5\,[\text{kNm}],$$
$$12.4 + 0.5 \cdot 12.6 = 18.7 < 19.5\,[\text{kNm}].$$

The safety condition is satisfied in either case.

The new algorithm uses the matrix procedure:

$$\mathbf{F}_f = \begin{bmatrix} 50 \\ 20 \end{bmatrix}, \quad \psi_{cf} = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix} \quad \longrightarrow \quad \psi\,\mathbf{F} = \psi_{cf} \cdot \text{diag}(\mathbf{F}_f) = \begin{bmatrix} 50 & 18 \\ 45 & 20 \end{bmatrix},$$
where $f = 0, 1$ for Q, W;

$$\mathbf{c}_{vj0} = \begin{bmatrix} q & q \\ 0.12 & 0.08 \\ 0.16 & 0.04 \end{bmatrix}, \ \mathbf{c}_{vj1} = \begin{bmatrix} 0 & 0 \\ 0.35 & 0.45 \\ 0.15 & 0.25 \end{bmatrix}$$
from the elastic analysis with
$$\mathbf{r}_j = \begin{bmatrix} 1 \\ r_N \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$ gives:

$$\mathbf{cr}_{v0} = \mathbf{cr}_{vj_0} \cdot \mathbf{r}_j = \begin{bmatrix} 0 \\ 0.16 \\ 0.18 \end{bmatrix}, \qquad \mathbf{cr}_{v1} = \mathbf{cr}_{vj_1} \cdot \mathbf{r}_j = \begin{bmatrix} 0 \\ 0.575 \\ 0.275 \end{bmatrix};$$

$$\mathbf{maxcr}_v = \begin{bmatrix} \max(\mathbf{cr}_{v0}) \\ \max(\mathbf{cr}_{v1}) \end{bmatrix} = \begin{bmatrix} 0.180 \\ 0.575 \end{bmatrix}, \quad \mathbf{mincr}_v = \begin{bmatrix} \min(\mathbf{cr}_{v0}) \\ \min(\mathbf{cr}_{v1}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\mathbf{maxM}_c = \psi\mathbf{F}_{cf} \cdot \mathbf{maxcr}_f = \begin{bmatrix} 19.35 \\ 19.60 \end{bmatrix}, \quad \mathbf{minM}_c = \psi\mathbf{F}_{cf} \cdot \mathbf{mincr}_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$MaxM_{eff} = \max(\mathbf{maxM}_c) = 19.6 > M_{Rd} = 19.5\,[\text{kNm}],$$

$$MinM_{eff} = \min(\mathbf{minM}_c) = 0\,[\text{kNm}].$$

The safety condition is not satisfied.

Example 2 shows that the new procedure and the conventional procedure can give different results. The example is given for the semi-probabilistic method of safety verification. Such a disagreement could be shown also for the probabilistic method but other discrepancies could not allow to see the difference of results due to different procedures of the critical load case (v, c, e) selection.

3. Partial factors and split factors

Code writers of the semi-probabilistic Eurocode draft standards (CEN) and the LRFD specifications (USA) try to explain that partial factors γ_f, γ_m can be derived from a given *safety index* $\beta =$ const treated as a unique safety measure for a reliability class of construction works. But the partial factors and design values $\gamma_f F_k$, $f = 0, 1 \ldots, n_f$, and $\gamma_m R_k$, $m = 1, 2 \ldots, n_m$, are necessary for forensic investigation in order to see who is guilty for structural failure. Too many involved individuals $n_f + n_m$ make the responsibility problem fuzzy. Three and only three responsibility fields were suggested by the author [2] for building. They were related to responsibility of designer (design error C), contractor (structural resistance R) and owner (applied load S). The random variables C, R, S have been called *coordinates of state* of the structure [2]. Authors of the LRFD specifications have followed this idea, they have defined three *split factors*: the analysis factor, material factor and load factor.

The number of responsibility fields can be still reduced but at least two must be left, they are related to the load effect S and resistance R of the

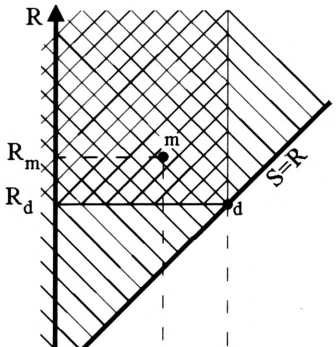

FIGURE 1. Mean point m and design point d on the S-R plane.

structural member (Fig. 1):

$$S < R. \tag{3.1}$$

Exactly two coordinates of state have been defined by the Eurocode [7]. The global safety index β has been split in two components for a uncoupled semi-probabilistic design format by means of an arbitrary rule:

$$\beta_S = 0.7\beta, \qquad \beta_R = 0.8\beta \tag{3.2}$$

with:

$\beta =$ const for any reliability class;

$\beta = 3.8$ has been recommended [7] for buildings of normal reliability class.

Two split (or component) safety factors are defined. They depend on the coefficients of variation (COV) $v_S = \sigma_S/S_m$, $v_R = \sigma_R/R_m$ of the coordinates of state S and R and they are applied to their mean values S_m and R_m:

$$\gamma_S = 1 + \beta_S\, v_S, \qquad \gamma_R = (1 - \beta_R\, v_R)^{-1}. \tag{3.3}$$

The split factors (3.3) and the design values $S_d = \gamma_S S_m$, $R_d = R_m/\gamma_R$ can not be derived from the sole condition $\beta =$ const. Any point (R_d, S_d) on the limit states line $S = R$ gives exactly the same β, interpreted as the standardized *safety margin* $g = R - S$,

$$\beta = \frac{R_m - S_m}{\sqrt{\sigma_R^2 + \sigma_S^2}}. \tag{3.4}$$

The probability p_c of $R > S$ event (i.e. $g > 0$) used to be derived under the condition that the safety margin $g(R, S)$ is characterized by an unlimited Gauss probability function without any tail truncation in effect of reliability control:

$$\begin{aligned} p_c &= \mathrm{Prob}(R > S) = \Phi(\beta), \\ p_c &= 7 \cdot 10^{-5} \quad \text{for} \quad \beta = 3.8. \end{aligned} \tag{3.5}$$

If a full probabilistic theory is concerned, an objective function is necessary to get the optimal solution for definitions of design values R_d, S_d.

4. Optimization of structural safety

There are two optimization problems in the probability-based reliability [2]:

1. how to define component factors γ_R, γ_S if the global safety factor γ is given or how to define component indices β_R, β_S, if the safety index β is given for safety verification;

2. how to define the overall cost of construction works and the appropriate reliability measure γ or β including risk of failure.

It is necessary to define *structural failure* more precisely [2]. It happens not only as a *collapse*, i.e. sudden unexpected destruction of the structure or its important element but also as a demolition at imminent collapse and/or reconstruction in effect of *decommissioning.* Statistical records [2, 6] show that structural failures are not so rare, $p = 10^{-3} \div 10^{-2}$, it is much more than the global safety index $\beta = 3.8$ would give. Statistics of structural failures cover evidently also the decommissioning events. Structural failures are due to human errors in either case. The conditional probability p_c of collapse accidents (3.5) is due to insufficient safety control. The conditional probability p_d of decommissioning (4.1) is relative to events when the safety control turned out effective

$$p_d = 1 - \mathrm{Prob}(S < X,\, R > X). \tag{4.1}$$

If the coordinates of state R, S are independent, p_d may be formulated as a product of reliabilities $p_d = \mathrm{Prob}(R < X)\,\mathrm{Prob}(S > X)$. The specified value X would be recognized by the inspection units as the legal limit between responsibility fields. The theorem of the *total probability* gives the unconditional probability of failure

$$p = (1 - \eta)p_c + \eta p_d, \qquad 0 < \eta < 1, \tag{4.2}$$

where η is the effectiveness of control; the value η will not be important in optimization analysis.

Figure 1 shows the locus where the probability density function $f(R, S)$ shall be integrated in order to get the probability of failure p. It shall be integrated with a weight η over the quadrant $R > R_d$, $S < S_d$ and with the weight $(1 - \eta)$ over the half-plane $S < R$.

Two optional objective functions have been define in order to get optimal values R_d, S_d:

- either the *probability density function* PDF attains its maximum at a point R_d, S_d on the ultimate limit states line $R = S$ (Fig. 1):

$$f(R_d, S_d \mid R = S) = \max; \tag{4.3}$$

- or the *cumulative probability function* (CPF) attains its maximum not necessarily at the same point $(R_d,\, S_d)$ as it may be found for criterion (4.3):

$$q(R_d, S_d \mid R = S) = \max\,. \tag{4.4}$$

The objective function (4.3) is fundamental for probabilistic method of safety which is commonly recognized in the world. Let it call the β-method. The objective function (4.4) is fundamental for a new so called κ-method [2–5].

The β-method assures the maximum frequency of failure (max vulnerability) and it hardly can be treated as an optimal solution from the general point of view. The design values R_d, S_d according to the β-method are coupled (see e.g. [1, 6]). The split factors γ_R, γ_S increase with their coefficients v_R, v_S of variation (COV):

$$R_d = R_m - \alpha_R \, \beta \, \sigma_R, \qquad S_d = S_m + \alpha_S \, \beta \, \sigma_S, \tag{4.5}$$

where: $\alpha_R = \sigma_R / \sqrt{\sigma_R^2 + \sigma_S^2}$, $\alpha_S = \sigma_S / \sqrt{\sigma_R^2 + \sigma_S^2}$ – the probabilistic sensitivity coefficients; R_m, σ_R^2, S_m, σ_S^2 – the first- and second-order moments treated as the Gaussian parameters.

As a matter of fact the design values R_d, S_d are not necessary to safety verification. The ULS condition is sufficient to dimension any structural element

$$\frac{R_m - S_m}{\sqrt{(v_R R_m)^2 + (v_S S_m)^2}} = \beta. \tag{4.6}$$

The κ-method assures maximum reliability of structures to an extent that selection of design values can attribute. The design values R_d, S_d are uncoupled and they make hazards of crossing design level equal for contractor and owner:

$$R_d \, h(R_d) = S_d \, h(S_d) = \kappa, \tag{4.7}$$

where: $h(R) = f(R)/(1 - F(R))$, $h(S) = f(S)/F(S)$ – the hazard functions; κ – the nondimensional *hazard ratio*. The derivation may be found in [2, 4].

The probability functions $F(X)$, $f(X)$, $h(X)$ are not necessarily normal; however, the log-normal functions for the resistance R and the Gauss-normal functions for the load effect S are preferable. These distribution facilitate the evaluation of the logarithmic ν_R and normal v_S COV of the coordinates of state R, S when the COV of basic variables have been assumed. The log-normal function do not admit negative values of R.

5. Features of the κ-method

A crucial question is how much the hazard ratio κ is for a reliability class of structures. The second problem of optimization needs too many econometric data [2], therefore a commonly recognized standard design value is accepted as the optimal one and it is compared to experimental data and the hazard ratio κ is specified for the normal class of reliability. The most representative statistical data are available for the yield point of common structural steel. The median value depends on thickness and kind of steel products, $R_m = 290\,\text{MPa}$ has been estimated for $t = 16\,\text{mm}$ thick bars [2]. The logarithmic COV $\nu_R = 0.10$ has been enhanced because of random variations of dimensions of steel members. The standard design strength is $R_d = 215\,\text{MPa}$ for $t < 16\,\text{mm}$ thick products. The component factor, the component index and the log-normal hazard ratio are as follows:

$$\gamma_R = \frac{290}{215} = 1.34, \qquad \beta_R = \frac{\ln(1.34)}{0.10} = 2.96, \qquad \kappa = \frac{\phi(2.96)}{0.10} = 0.05.$$

The hazard ratio $\kappa = 0.05$ is less than $\kappa = 2/3$ which was estimated in earlier publications [2, 3], where three coordinates C, R, S were taken into consideration. The index 2.96 is close to 3.0 which was taken for:

- extreme error in surveying by Gauss (1777-1855),
- calibration of design strengths by Working Group for Unification of Design in USSR, 1951,
- $0.8 \cdot 3.8 = 3.04$ – the split index β_R for the resistance by the Eurocode 1 from Eq. (3.2), 1993.

Bias factors and coefficients of variation of other basic variables may be identified, supposing that the semi-probabilistic partial factors were calibrated by means of the "$3\,\sigma$ rule" Revision of the statistical parameters will be perhaps useful but it is an independent problem from the probabilistic format of design.

The results of the semi-probabilistic design and the maximum reliability design ($\kappa = \text{const}$) will agree for a "simple" design situation, i.e. when there is a single load and single material of a structural member; however, a difference will occur if the member is composite and/or more independent loads are applied. The results of the probabilistic $\kappa = \text{const}$ design and $\beta = \text{const}$ design will disagree even in simple design situations. The component safety index β_R for the resistance R decreases with increasing ν_R according to the κ-method and the component factor γ_R increases but not so fast as it will according to the β-method. Similar trends are for the load effect S.

The component safety factors γ_R, γ_S relative to the κ-method may be derived from Eqs. (4.7) with the assumption of the log-normal distribution of the resistance R and the Gauss normal distribution of the load effect S. The *global safety factor* will be $\gamma = \gamma_S\,\gamma_R$:

$$\begin{aligned} &\frac{1}{\nu_R}\,\phi\left(\frac{\ln(\gamma_R)}{\nu_R}\right) - \kappa = 0 \quad \Rightarrow \quad \gamma_R, \\ &\frac{\gamma_S}{v_S}\,\phi\left(\frac{\gamma_S - 1}{v_S}\right) - \kappa = 0 \quad \Rightarrow \quad \gamma_S. \end{aligned} \tag{5.1}$$

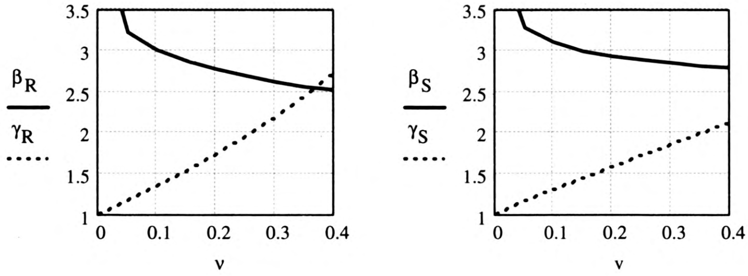

FIGURE 2. Component indices β_R, β_S and component factors γ_R, γ_S according to the κ-method.

Approximate formulae may be used for $0.05 < v < 0.40$ (Fig. 2):

$$\begin{aligned} \beta_R &= 3.20 - 2.0\,\nu_R, \qquad & \gamma_R &= \exp(\beta_R\,\nu_R), \\ \beta_S &= 3.17 - 1.2\,v_S, \qquad & \gamma_S &= 1 + \beta_S\,v_S. \end{aligned} \tag{5.2}$$

The evaluation of ν_R for a composite material needs usually linearization of design formulae. The evaluation of v_S for many loads needs application of the computer-aided matrix procedure presented in Sec. 2.

6. Final remarks

Maximum reliability method (when κ = const) is ready to be implemented as a practical method of design. It is:

- uncoupled – design values of variable loads may be determined without information what is material of the structure and vice versa;
- consistent – coefficients of variation remain the same although they are identified in different design situations;
- realistic – probability of safety is in the range $10^{-3} \div 10^{-2}$ as it has been confirmed by statistical investigations;
- economic – the dimensions of structural elements are never higher than nowadays standards would give;
- stable – different designers will obtain the same solutions when they verify a structural member;
- never absurd – no negative or complex dimensions of structural members can happen as results of design;
- designer-friendly – calculations are simpler and no iteration is necessary for the linear analysis of structures.

Unfortunately, the maximum failure frequency method (when $\beta = \text{const}$) has just the opposite properties. Seven counterexamples are given in [3], where the properties of β-method and the κ-method have been discussed. One of them is presented here.

Example 3. Let the safety condition for a structural member be as follows:

$$S < f\,A,$$

where: A [m^2] – unknown area of the cross-section; f [MPa] – random strength with the mean $F_m = 25\,\text{MPa}$ and COV $v_R = 0.30$; S [MN] – random load effect with the mean $S_m = 5\,\text{MN}$ and COV $v_S = 0.60$.

The β-theory requires that the standardized safety margin $g = A\,f - S$ will attain a specified value β at the ultimate limit state. Assuming $\beta = -3.8$ we have an algebraic equation (4.6) that can be reduced to a square equation

$$\frac{25 \cdot A - 5}{\sqrt{(0.30 \cdot 25 \cdot A)^2 + (0.60 \cdot 5)^2}} = -3.8$$

$$\Rightarrow \qquad 187.25A^2 + 250 \cdot A + 104.96 = 0.$$

The two solutions are complex: $A = -0.668 \pm 0.339i$. Complex area of a cross-section is absurd!

The κ-method requires that the hazard ratios κ_S and κ_R do not exceed a specified value κ at the ultimate limit state. Assuming $\kappa = 0.05$ we have two independent equations (5.1) involving the non-elementary Mills' function $\phi(\beta) = \varphi(\beta)/\Phi(\beta)$,

$$f_{lm} = \frac{25}{1 + 0.30^2} = 22.94\,[\text{MPa}],$$

$$\nu_R = \sqrt{\ln(1 + 0.30^2)} = 0.294, \quad \text{the logarithmic parameters,}$$

$$\frac{1}{0.294}\,\phi\left(\frac{\ln(\gamma_R)}{0.294}\right) = 0.05 \quad \Rightarrow \quad \gamma_R = 2.13,$$

$$\frac{\gamma_S}{0.60}\,\phi\left(\frac{\gamma_S - 1}{0.60}\right) = 0.05 \quad \Rightarrow \quad \gamma_S = 2.60.$$

The global safety factor $\gamma = 2.13 \cdot 2.60 = 5.52 \Rightarrow A = \frac{5 \cdot 5.52}{22.94} = 1.204\,[\text{m}^2]$ – the solution is real!

This and other counterexamples [4] show how misleading the β-method can be. That is why the κ-method deserves more attention.

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