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Application of response surface method to reliability-based optimization of laminated composite plate subject to in-plane strength

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In this study, the response surface method is applied to the reliability-based optimum design of a balanced symmetric laminated composite plate consisting of 0°, ±45° and 90° plies subject to in-plane loads, where material properties and applied loads are treated as random variables. In order to improve the calculation efficiency, the reliability-based optimization algorithm is transformed from a nested iteration into a single iteration by making use of the response surface of the reliability in the lamination parameter space. The improvement of the calculation efficiency is demonstrated through numerical examples of the thickness minimization design under the reliability constraint.

Key words: *Response Surface Method, Reliability-Based Optimization, First Order Reliability Method, Laminated Composite Plate, Lamination Parameter, First Ply Failure, Tsai-Wu Criterion.*

1. Introduction

The first order reliability method (FORM) [1] is widely used for the reliability-based optimization. Since the FORM is formulated as a nonlinear programming problem, the reliability-based optimization problem is formulated as a nested iteration problem which takes much computational cost.

For the reliability-based optimum design of the symmetric balanced laminate subject to in-plane strength criterion under variations of material properties and applied loads [2], the authors' research [3] demonstrates that the calculation efficiency is improved by adopting the lamination parameters [4] as design variables. However, the improvement is not sufficient because of the nested iteration algorithm.

In this study, the response surface method (RSM) [5] is adopted to improve the computational efficiency by transforming the nested iteration optimization problem into a single iteration problem. The efficiency is demonstrated through the thickness minimization design under the reliability constraint.

2. Strength Analysis of Laminated Composite

2.1. In-plane stiffness and lamination parameter

Consider a balanced symmetric laminated composite plate subjected to in-plane load $\boldsymbol{N} = (N_1, N_2, N_6)^T$ as shown in Fig. 1, where the subscripts 1, 2 and 6 correspond to the longitudinal, lateral and shear directions, respectively. For the balanced symmetric lay-up, the in-plane and flexural responses can be separated. Additionally, the in-plane tension-shear coupling terms are eliminated. Accordingly, the in-plane strain $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \epsilon_6)^T$ is obtained by the in-plane stress-strain relationship as follows:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix}. \tag{2.1}$$

where $\boldsymbol{A}$ is the in-plane stiffness matrix.

The stiffness matrix of the composite plate with thickness h is expressed in terms of material invariants U_i $(i = 1, \cdots, 5)$ and the in-plane lamination parameters V_j^* $(j = 1, \cdots, 4)$ as follows:

$$\begin{aligned} A_{11} &= h(U_1 + U_2V_1^* + U_3V_2^*), \qquad & A_{22} &= h(U_1 - U_2V_1^* + U_3V_2^*), \\ A_{12} &= h(U_4 - U_2V_2^*), & A_{66} &= h(U_5 - U_2V_2^*). \end{aligned} \tag{2.2}$$

The in-plane lamination parameters are defined in terms of the i-th ply orientation angle θ_i, volume ratio v_i and the number of plies N as follows:

$$\{V_1^* \quad V_2^*\}^T = \sum_{i=1}^{N} v_i \{\cos 2\theta_i \quad \cos 4\theta_i\}^T. \tag{2.3}$$

Equation (2.3) indicates that the lamination parameters corresponding to an angle-ply laminate $[\pm\theta]_s$ lie on the parabola, $V_2^* = 2V_1^{*2} - 1$. For any

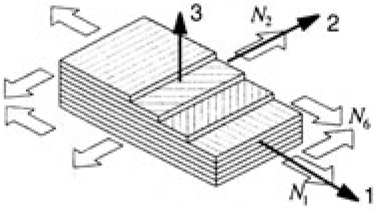


FIGURE 1. Symmetric laminated composite plate.

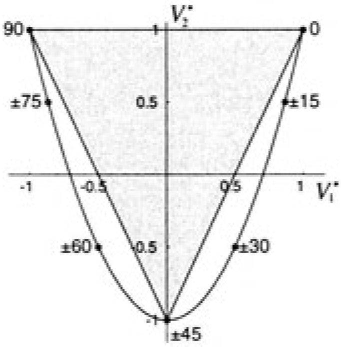


FIGURE 2. Lamination parameter space of a balanced symmetric laminate. Hatched region corresponds to $[0°, \pm 45°, 90°]_s$ laminate.

balanced symmetric laminates with two or more ply orientation angles, the lamination parameter space is described as follows [4]:

$$V_2^* \geq 2V_1^{*2} - 1, \quad V_2^* \leq 1. \tag{2.4}$$

When the laminate consists of the three ply orientation angles; 0°, ±45°, and 90°, the feasible region is limited to the hatched area in Fig. 2.

$$V_2^* \geq 2V_1^* - 1, \quad V_2^* \geq -2V_1^* - 1, \quad V_2^* \leq 1. \tag{2.5}$$

The three vertices of the feasible triangle in this formula, $(1, 1)$, $(0, -1)$, and $(-1, 1)$ correspond to 0°, ±45°, and 90°, respectively. Each ply volume ratio corresponding to the lamination parameters is evaluated through the following equation resulting from Eq. (2.3):

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} v_{0°} \\ v_{45°} \\ v_{90°} \end{Bmatrix} = \begin{Bmatrix} V_1^* \\ V_2^* \\ 1 \end{Bmatrix}. \tag{2.6}$$

2.2. Strength analysis

In this study, the ply strength is evaluated by the Tsai-Wu criterion in the strain space [6], because the criterion in the strain space is known to have better calculation efficiency for the reliability analysis than that in the stress space [7]. The criterion is described as follows:

$$G_{xx}{\epsilon_x}^2 + 2G_{xy}\epsilon_x\epsilon_y + G_{yy}{\epsilon_y}^2 + G_{ss}{\epsilon_s}^2 + G_x\epsilon_x + G_y\epsilon_y - 1 = 0 \tag{2.7}$$

where the i-th ply strain $(\epsilon_x, \epsilon_y, \epsilon_s)_i^T$ with ply orientation angle θ_i is obtained by transforming the plate strain $(\epsilon_1, \epsilon_2, \epsilon_6)^T$. The strength parameters $G_{ij}, G_i\,(i, j = x, y, s)$ are defined by the ply stress-strain relationship $Q_{ij}(i, j = x, y, s)$ and the material strength, X_t, X_c, Y_t, Y_c and S, where X, Y and S denote the axial strength along the fiber direction and the lateral direction, and the shear strength, respectively. The subscripts t and c indicate the tensile and the compression side, respectively.

In this study, the first ply failure (FPF) criterion [6] is adopted. Thus, the plate is regarded as in failure when the weakest ply fails. It is modeled by the strength ratio R_i which is defined as the ratio between the ply failure strain ϵ_F and the i-th ply strain $\epsilon_i = (\epsilon_x, \epsilon_y, \epsilon_s)_i^T$ under the proportional loading assumption:

$$\epsilon_F = R_i \epsilon_i. \tag{2.8}$$

R_i is obtained by solving the quadratic equation formulated by substituting Eq. (2.8) into Eq. (2.7):

$$\left(G_{xx}{\epsilon_x}^2 + 2G_{xy}\epsilon_x\epsilon_y + G_{yy}{\epsilon_y}^2 + G_{ss}{\epsilon_s}^2\right) R_i^2 + \left(G_x\epsilon_x + G_y\epsilon_y\right) R_i = 1. \tag{2.9}$$

In this study, the first ply failure criterion is adopted. That is, the plate strength ratio is represented by the smallest ply strength ratio:

$$R_{\min} = \min_i R_i. \tag{2.10}$$

The plate is regarded as in failure when $R_{\min}$ is less than the unity as shown in Fig. 3.

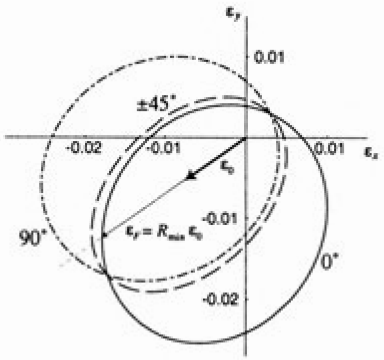


FIGURE 3. Strength ratio.

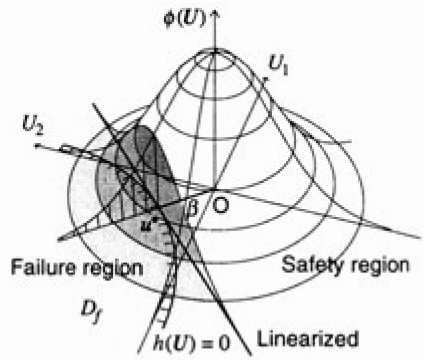


FIGURE 4. First order reliability method.

3. Reliability Analysis

The system reliability of the composite plate is evaluated by modelling the plate as a series system consisting of each ply failure.

Each ply failure probability is evaluated by the FORM [1]. The reliability index of the i-th ply failure is evaluated by the following problem:

$$\begin{aligned} \text{Minimize}: &\quad \beta_i = \sqrt{\boldsymbol{u}^T \boldsymbol{u}} \\ \text{subject to}: &\quad g_i(\boldsymbol{u}) = R_i - 1 = 0 \end{aligned} \tag{3.1}$$

where the random vector $\boldsymbol{u}$ consists of the applied load and the material properties and R_i is the strength ratio of the i-th ply.

It is known that the problem (3.1) has multiple local optima, because the limit state function is strongly nonlinear. Premature convergence will yield the overestimation of the reliability. In order to avoid the premature convergence, a global optimization method should be recommended for the numerical searching. In this study, the modified tunnelling method suitable for the FORM developed by the authors [8] is used as one of the global optimization method.

The system reliability is approximated by Ditlevsen's upper bound [9]:

$$\beta_U = -\Phi^{-1}(P_U), \qquad P_U = \sum_{i=1}^{m} P_i - \sum_{i=2}^{m} \max_{j<i} P_{ij} \tag{3.2}$$

where P_i is the i-th ply failure probability, P_{ij} is the joint probability of the i-th and j-th ply failures, and m is the number of failure mode. In this study, the number is set to 4 corresponding to the number of the ply orientation angles, 0°, +45°, −45° and 90°.

4. Reliability-Based Optimization

The thickness minimization design under the reliability constraint is formulated in terms of the lamination parameters as follows [3]:

$$\begin{aligned} \text{Minimize}: &\quad h \\ \text{subject to}: &\quad \beta_U(V_1^*, V_2^*, h; \boldsymbol{u}) \geq \beta_a \\ &\quad h > 0 \\ &\quad V_2^* \geq 2V_1^* - 1 \\ &\quad V_2^* \geq -2V_1^* - 1 \\ &\quad V_2^* \leq 1 \end{aligned} \tag{4.1}$$

where β_a is the allowable lower reliability limit, which is set to 3.0 in this study. The last three constraints indicate the feasible region of the lamination parameters.

As a numerical procedure, the sequential quadratic programming (SQP) method is adopted [10].

5. Response Surface Method (RSM)

In order to transform the reliability-based optimization problem into a single iteration scheme, the RSM [5] is adopted to approximate the reliability in the lamination parameter space. The RSM is originally developed to fit the experimental response in terms of design variables based on statistics. Recently, this method is widely expanded to the deterministic optimization problem to reduce the computational cost by making use of an approximation of the structural response. It is also applied to the deterministic composite laminate configuration design problem [11].

5.1. Construction of response surface using least-squares

The response y is generally approximated by the polynomial form in terms of the design factor x_i $(i = 1, 2, \cdots, k)$ as follows:

$$y \approx b_0 + \sum_{i=1}^{n} b_i x_i + \epsilon \tag{5.1}$$

where $b_i (i = 0, 1, \cdots, k)$ and ϵ indicate the unknown coefficient and the approximation error, respectively.

In order to estimate the $k+1$ unknowns, n $(n \geq k+1)$ experimental points are required to evaluate the response. Usually, the number of experimental points is set to twice more than that of variables [5]. The response formula is described in the following matrix form:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{b} + \epsilon. \tag{5.2}$$

The unknown coefficient $\hat{\boldsymbol{b}}$ is estimated by using the linear least-squares fit as follows:

$$\hat{\boldsymbol{b}} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{y}. \tag{5.3}$$

In this study, the reliability is approximated by the following third order polynomials in terms of the lamination parameters and the plate thickness after several numerical experiments:

$$\beta_U \approx b_0 + b_1 V_1^* + b_2 V_2^* + b_3 h + b_4 V_1^{*2} + b_5 V_2^{*2} + b_6 h^2 + b_7 V_1^* V_2^* + b_8 V_2^* h$$
$$+ b_9 V_1^* h + b_{10} V_1^{*3} + b_{11} V_2^{*3} + b_{12} h^3 + b_{13} V_1^{*2} V_2^* + b_{14} V_1^* V_2^{*2}$$
$$+ b_{15} V_2^{*2} h + b_{16} V_2^* h^2 + b_{17} V_1^* h^2 + b_{18} V_1^{*2} h + b_{19} V_1^* V_2^* h. \tag{5.4}$$

Replacing each polynomial term into the variable x_i, Eq. (5.4) can be transformated into the linear equation as Eq. (5.2). Then, the unknown coefficients can be estimated by Eq. (5.3).

The estimation error is described by the following covariance matrix:

$$\mathrm{Var}(\hat{\boldsymbol{b}}) = \sigma^2 (\boldsymbol{X}^T \boldsymbol{X})^{-1} \tag{5.5}$$

where σ^2 indicates an error variance. The accuracy of the estimated coefficients depends on both the error variance σ^2 of the response and the combination of the experimental points $\boldsymbol{X}$.

5.2. *D*-optimal criteria and genetic algorithms (GAs)

The following *D*-optimal criteria [5] is widely adopted to determine the experimental point selection in order to reduce the estimation error in Eq. (5.5):

$$\text{Maximize} : D_{\text{eff}} = \frac{\| \boldsymbol{X}^T \boldsymbol{X} \|^{1/(k+1)}}{n}. \tag{5.6}$$

This criteria implies the minimization of $\|(\boldsymbol{X}^T \boldsymbol{X})^{-1}\|$.

In this study, genetic algorithms (GAs) are adopted to determine the experimental point selection [12]. The candidates of the experimental point are combination of 221 grid points with every 0.1 spacing in the lamination parameters as shown in Fig. 5 and 16 thickness candidates between 0.7 mm and 1.075 mm with interval of 0.025, where the thickness bounds are determined from the engineering judgment.

The algorithm is based on a simple GAs with an elitist plan. The combination of 40 experimental design points which is twice as much as the number of the unknown coefficients is selected from the above 3536 (= 211 × 16) candidates, where the duplicated experimental point selection is prohibited by treating such combination as a lethal gene.

An example of the convergence history of the GAs is illustrated in Fig. 6. It shows that GAs required 428 iterations, where the population size is set to 50. However, the computational cost is negligible in comparison with the FORM, because the evaluation of the objective function (5.6) is inexpensive.

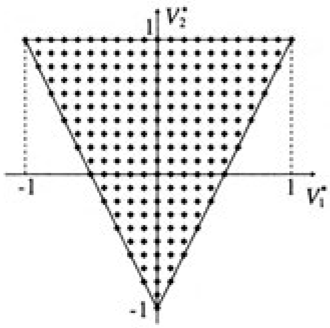


FIGURE 5. Candidates in lamination parameter space.

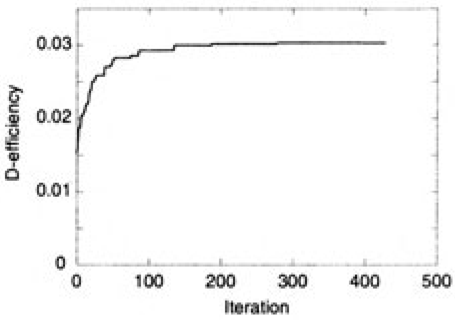


FIGURE 6. Iteration history of experimental points selection by GAs.

5.3. Test for significance

The approximation error of the response surface also depends on the error variance σ^2 in Eq. (5.5). After estimating the unknown $\boldsymbol{b}$, the accuracy is clarified by applying the adjusted coefficient of multiple determination R^2_{ad}:

$$R^2_{ad} = 1 - \frac{SS_E/(n-k-1)}{S_{yy}/(n-1)} \tag{5.7}$$

where SS_E and S_{yy} are the residual sum of squares and the total sum of squares, respectively.

$$SS_E = \boldsymbol{y}^T\boldsymbol{y} - \boldsymbol{b}^T\boldsymbol{X}^T\boldsymbol{y}, \qquad S_{yy} = \boldsymbol{y}^T\boldsymbol{y} - \left(\sum_{i=1}^{n} y_i\right)^2/n. \tag{5.8}$$

Additionally, the significance of any individual coefficient is judged by t-test. The test statistics for the null hypothesis is defined as follows:

$$t_0 = \hat{b}_j\left(\hat{\sigma}^2 C_{jj}\right)^{-1/2} \tag{5.9}$$

where $\hat{b}_j$ and C_{jj} are the estimated j-th coefficient and the diagonal element of $(\boldsymbol{X}^T\boldsymbol{X})^{-1}$ corresponding to b_j, respectively. $\hat{\sigma}^2$ indicates an unbiased estimator of σ^2, which is defined as follows:

$$\hat{\sigma}^2 = SS_E/(n-k-1). \tag{5.10}$$

The null hypothesis is rejected if $|t_0| > t_{\alpha/2,m}$, where $t_{\alpha/2,m}$ is t-distribution with m degrees of freedom. In this study, the level of significance α is set to 0.05. If the null hypothesis is satisfied, the j-th term can be eliminated.

5.4. Two-stage approach

The approximation accuracy of the response surface is not sufficient in this study as discussed below. In order to overcome the situation, the response surface is reconstructed around the found optimum design using the above response surface. The feasible region of the lamination parameters is zoomed to the square of the length 0.3 with a center on the found optimum and the thickness region is set to ±0.07 mm to the found optimum. The combination of the experimental points is also selected based on D-optimal criterion by GAs. The candidates are combinations of 256 lamination parameters with every 0.02 spacing and 16 variables for the thickness.

Then, the second optimization is performed based on the reconstructed response surface in the small region around the first optimum design. That is, the RSM is applied in the two stages.

6. Numerical Examples

The problem is to find the laminate configurations of the balanced symmetric laminated composite plate consisting of 0°, ±45° and 90° plies which minimizes the plate thickness under the reliability constraint, $\beta_U \leq 3.0$. The material properties and the applied loads are treated as random variables. The former is assumed to be normally distributed, where the means and coefficients of variation are listed in Table 1. The latter is also assumed to be normally distributed, where the mean is set to $\bar{\boldsymbol{N}} = (0.1, 0.05, 0.04)$ [MN/m] and the standard deviation is set to $\boldsymbol{\sigma}(\boldsymbol{N}) = (0.03, 0.03, 0.03)$ [MN/m].

TABLE 1. Material properties of T300/5208.

	Stiffness [GPa]				Strength [MPa]				
	E_x	E_y	E_S	ν_x*	X_t	X_c	Y_t	Y_c	S
Mean	181.0	10.7	7.17	0.28	1500	1500	40	246	68
COV	0.05	0.05	0.05	0.01	0.1	0.1	0.1	0.1	0.1

(* dimensionless)

This optimum design obtained by the conventional nonlinear programming method [3] is listed in Table 2. The reliability distribution in the lamination parameter space at the optimum thickness plate is illustrated in Fig. 7. This plot is drafted from the reliability evaluations by the FORM at 221 grid points with every 0.1 spacing in the lamination parameter space. It is found that the reliability is distributed smoothly in the lamination parameter space. Hence, the optimum design is easily obtained. However, the searching takes

TABLE 2. Thickness-minimized design [3].

V_1^*		0.156
V_2^*		-0.356
h_{opt}	[mm]	0.814
h_0	[mm]	0.195
$h_{\pm 45}$	[mm]	0.552
h_{90}	[mm]	0.067

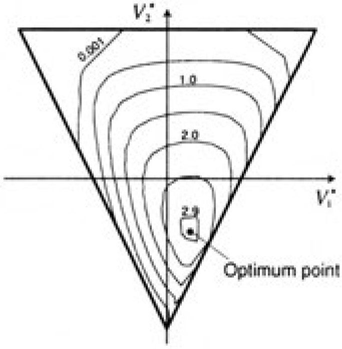
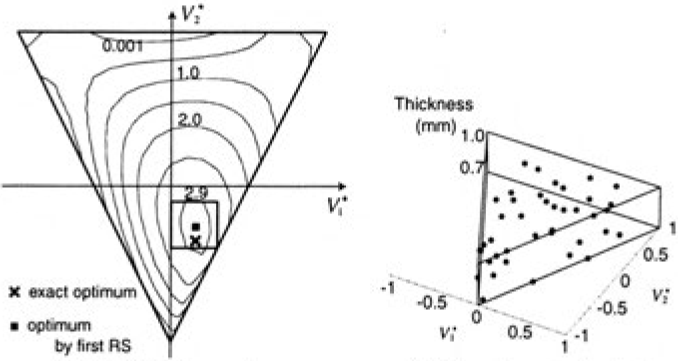


FIG. 7. Reliability distribution at the optimum thickness plate [3].

much computational cost because of the nested iteration formulation of the reliability-based optimization [3].

In order to reduce the computational cost, the RSM is applied to make the reliability approximation in terms of the design variables. Then, the reliability-based optimization is evaluated by using the response surface. The estimated coefficient in each step is listed in Table 3 (a), as well as the absolute value of the t_0 in Eq. (5.9). The mark of "—" means that the term is eliminated by the t-test.

The constructed reliability response surface at the optimum thickness is illustrated in Fig. 8 (a). The selected experimental points are also illustrated in Fig. 8 (b). In this first stage, the accuracy of the response surface in the whole lamination parameter space is not sufficient in comparison with Fig. 7. The optimum design disagrees with that by the previous study [3].



(a) Reliability contour (b) Experimental points

FIGURE 8. First response surface at the optimum thickness.

TABLE 3. Estimated unknown coefficients and t-test values.

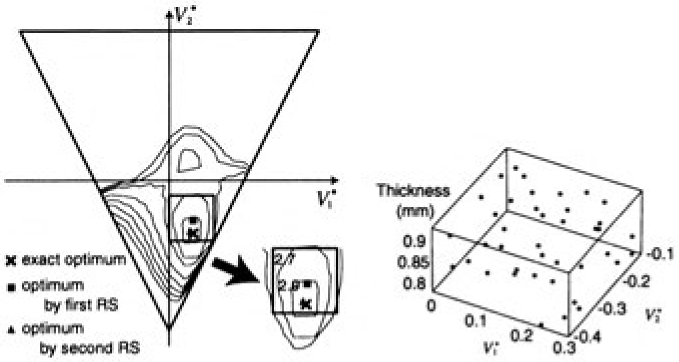
#	Term	(a) First step b_i	(a) First step $\|t_0\|$	(b) Second step b_i	(b) Second step $\|t_0\|$
1	1	-0.661	1.978	-0.601	1.997
2	V_1^*	—	—	—	—
3	V_2^*	—	—	6.123	3.309
4	h	—	—	—	—
5	V_1^{*2}	-6.452	19.09	9.741	6.354
6	V_2^{*2}	—	—	16.48	3.901
7	h^2	7.631	5.802	8.187	7.595
8	$V_1^* V_2^*$	-3.148	13.88	-15.89	29.73
9	$V_2^* h$	-1.867	5.993	-7.795	3.767
10	$V_1^* h$	4.647	12.08	—	—
11	V_1^{*3}	—	—	-7.719	3.500
12	V_2^{*3}	0.620	4.231	8.916	3.415
13	h^3	-3.643	3.75	-4.236	5.291
14	$V_1^{*2} V_2^*$	7.874	12.73	54.74	31.12
15	$V_1^* V_2^{*2}$	1.683	4.159	—	—
16	$V_2^{*2} h$	-2.919	29.48	-16.93	4.124
17	$V_2^* h^2$	0.795	2.829	—	—
18	$V_1^* h^2$	-2.987	7.496	—	—
19	$V_1^{*2} h$	—	—	-9.108	6.741
20	$V_1^* V_2^* h$	—	—	—	—

Then, the second response surface is reconstructed around the first optimum. The reliability contour and the selected experimental points are illustrated in Fig. 9. Also, the coefficient of the response surface is listed in Table 3 (b). The second optimization is performed in the limited design space as shown in Fig. 9 (b). The obtained optimum design is listed in Table 4. It is found that a sufficient accuracy is achieved by the two-step RSM.

TABLE 4. Thickness-minimized design.

	V_1^*	V_2^*	h_{opt} [mm]	β_{RSM}	β_{FORM}
Exact optimum [3]	0.156	-0.356	0.814	—	3.0
First RSM	0.160	-0.246	0.867	3.0	3.232
Second RSM	0.150	-0.349	0.815	3.0	3.002

Finally, the calculation cost is compared with the conventional method [3] that requires 241 times of the reliability analyses. The RSM requires 80 times



(a) Reliability contour (b) Experimental points

FIGURE 9. Second response surface at the optimum thickness.

of the reliability analyses, which is about three times better than the conventional method. However, the number of the reliability analyses is still large. Better improvement is expected by the other types of the formulation of the response surface.

7. Conclusion

The response surface method is applied to construct the reliability approximation for the reliability-based optimization of a laminated composite plate subject to the FPF criterion. In order to improve the calculation efficiency with keeping high accuracy, the response surface is constructed in two stages. In the first stage, the reliability is approximated in the whole lamination parameter space while in the second step it is applied only to the neighbours around the optimum in the first step. Then, the final optimum design is achieved based on the second response surface.

Through numerical examples, the efficiency of the RSM is demonstrated. However, the number of the reliability analyses is still large. Better improvement is required by applying the other types of the approximate formulation.

References

1. P. THOFT-CHRISTENSEN, Y. MUROTSU, *Application of Structural Systems Reliability Theory*, Springer-Verlag, Berlin 1986.

2. S. SHAO, M. MIKI, Y. MUROTSU, Optimization of Fiber Orientation Angle of Multiaxially Laminated Composites Based on Reliability, *AIAA J.*, Vol.31, pp.919-920, 1993.

3. N. KOGISO, S. NAKAGAWA, Y. MUROTSU, Lamination Parameters Applied to Reliability-Based Optimization of Composite Plate, *Proc. 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, AIAA-2002-5562, 2002.

4. M. MIKI, Y. SUGIYAMA, Optimum Design of Laminated Composite Plates Using Lamination Parameters, *AIAA J.*, Vol.31, pp.921-922, 1993.

5. R.H. MYERS, D.C. MONTGOMERY, *Response Surface Methodology Process and Product Optimization Using Design Experiments*, John Wiley & Sons, New York 1995.

6. S.W. TSAI, H.T. HAHN, *Introduction to Composite Materials*, Technomic, Lancaster 1980.

7. H. NAKAYASU, Z. MAEKAWA, A Comparative Study of Failure Criteria in Probabilistic Fields and Stochastic Failure Envelopes of Composite Materials, *Reliability Engineering and System Safety*, Vol.56, pp.209-220, 1997.

8. N. KOGISO, S. NAKAGAWA, Y. MUROTSU, Application of Tunneling Algorithm to Reliability Analysis of Laminated Composite Plate, *Proc. 9th IFIP WG 7.5 Working Conf. on Reliability and Optimization of Structural Systems*, (Nowak A.S. & Szerdesen M. eds.), pp.135-142, Univ. of Michigan, USA 2001.

9. O. DITLEVSEN, Narrow Reliability Bounds for Structural Systems, *J. Struct. Mech.*, Vol.7, pp.453-472, 1979.

10. T. IBARAKI, M. FUKUSHIMA, *FORTRAN 77 Optimization Programming*, [in Japanese], Iwanami-Shoten, Tokyo 1991.

11. A. TODOROKI, R.T. HAFTKA, Lamination Parameters for Efficient Genetic Optimization of the Stacking Sequences of Composite Panels, *Proc. 7th AIAA/USAF/NASA/ ISSMO Symposium on Multidisciplinary Analysis and Optimization*, AIAA-98-4816, 1998.

12. G. VENTER, R.T. HAFTKA, Minimum-Bias based Experimental Design for Constructing Response Surfaces in Structural Optimization, *Proc. 38th AIAA/ASME/ASCE/ AHS/ASC Structures, Structural Dynamics and Material Conference*, AIAA-97-1053, 1997.