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Reliability analysis of sheet metal forming operations by response surface method

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Methodology developed for reliability calculations of structures is applied to estimate reliability of sheet metal forming operations. Forming Limit Curves (FLC) used in the industrial practice as a criterion of material breakage in the manufacturing process are treated as the limit state function for reliability analysis. We try to quantify intuitive terms of probability of failure/success of forming operations given some uncertainty of parameters characterizing a forming process like friction parameters, sheet thickness or material properties. Since the employment of the gradient-based reliability techniques is very limited due to numerical noise introduced by the explicit dynamic simulation of sheet stamping the Response Surface method was chosen for reliability assessment.

Key words: *metal forming, response surface, reliability analysis.*

1. Introduction

Methodology of reliability theory proves useful for studying significance of some parameters characterising forming processes, such as friction parameters, material properties, thickness and blankholding force. Effective Response Surface Method (RSM) developed for reliability calculations of structures is applied to estimate reliability of sheet forming operations. The RSM is based on adaptation of the hyperplane to the Forming Limit Curve (FLC is a boundary between the strain combinations, which separate the safe and the failure – necking and/or fracture – zones). Weight least squares method is combined with the design of simulations. Results estimated by RSM are compared with the probability of failure assessment by Advanced Monte Carlo (AMC) simulation technique. Although AMC usually provides good results it requires many simulations. Gradient method combined with response surface technique seems to be more promising for such problems. An example of practical significance is presented as an illustration of the approach discussed.

2. Finite element modelling of sheet forming operations

In a typical model of sheet forming operation the workpiece in the form of a blank and three basic tools: die, punch and blankholder, are considered. The tools can be treated as rigid bodies, which enforce the blank deformation by the contact action under prescribed kinematics. Both normal and frictional contact interaction between tools and sheet should be taken into account. Friction has great influence on the forming process. Sheet undergoes complex deformation processes characterized by large displacements and large strains.

In the present work the so-called explicit dynamic formulation has been used in the finite element simulation. Because of its efficiency in the analysis of large-scale systems the explicitly integrated dynamic approach has become very popular in sheet stamping simulation. The method is based on the solution of the discretized equations of motion written in the current configuration in the following form:

$$\mathbf{M}\ddot{\mathbf{a}} + \mathbf{C}\dot{\mathbf{a}} = \mathbf{p} - \mathbf{f}, \tag{2.1}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, $\ddot{\mathbf{a}}$ and $\dot{\mathbf{a}}$ are the vectors of nodal accelerations and velocities, $\mathbf{p}$ and $\mathbf{f}$ are the vectors of external loads and internal forces, respectively. The element internal force vector is calculated from the relation

$$\mathbf{f} = \int_V \mathbf{B}^T \boldsymbol{\sigma} \, \mathrm{d}V, \tag{2.2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor.

Contact forces due to interaction between the sheet and tools are included in the vector of external load $\mathbf{p}$. Contact constraints in the normal direction are enforced using the penalty method and the Coulomb law is employed to evaluate frictional forces.

3. Reliability assessment via RSM

Lack of the differentiability of the sheet metal forming limit state function (LSF) unfortunately eliminates any gradient-based optimization techniques, like FORM (and SORM, cf. [1]) method. Alternative classical methods insensitive to this requirement are the simulation methods (see [2] for a review).

The best for this type of LSF seems to be the Adaptive Monte Carlo (AMC) method. However for a small number of random variables the AMC method needs a number of LSF calls which appears too high.

To overcome this difficulty it has been decided to use RSM (for a review, see [3]). The technique is based on linear regression analysis. A model describing linear or quadratic relationship between vector of random variables $\mathbf{U} = [U_1, U_2, \ldots, U_n]$ and LSF value $h(\mathbf{U})$ of N-dimensional set $\{[\mathbf{u}_1, y_1 = h(\mathbf{u}_1)], [\mathbf{u}_2, y_2 = h(\mathbf{u}_2)], \ldots [\mathbf{u}_N, y_N = h(\mathbf{u}_N)]\}$ can be defined as follows

$$\mathbf{Y} = \mathbf{AB} + \epsilon \tag{3.1}$$

where $\mathbf{B}$ is a vector of p unknown coefficients, $\mathbf{Y}$ is an N-dimensional vector of LSF values and ϵ is an error vector. Matrix $\mathbf{A}_{N\times p}$ is filled by the vector $\mathbf{u}_i$ components or their squares accordingly to the structure of $\mathbf{B}$ (the index i is the simulation counter). The error sum of squares $\epsilon^{\mathrm{T}}\epsilon$ minimization yields the least squares estimation of $\mathbf{B}$ which is the value $\mathbf{b}$. The LSF approximation can be then obtained from the so-called *normal equations* of the form

$$(\mathbf{A}^{\mathrm{T}}\mathbf{A})\,\mathbf{b} = \mathbf{A}^{\mathrm{T}}\mathbf{Y} \quad \longrightarrow \quad \mathbf{b} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Y}. \tag{3.2}$$

RSM regression employed here is supported by the weighted least squares due to the standard normal space properties and an estimate of P_f. The method allows to assess importance of points $\{[\mathbf{u}_i, y_i = h(\mathbf{u}_i)]\}$ for the correct approximation of LSF in the vicinity of the design point. The unknown coefficients $\mathbf{b}$ can be obtained from the formula

$$\mathbf{b} = (\mathbf{A}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{Y} \tag{3.3}$$

where the variance matrix $\mathbf{V}_{N\times N}$ is diagonal and its inverse $\mathbf{V}^{-1} = \mathrm{diag}[w_1, w_2, \ldots w_n]$ is known as the weight matrix.

RSM procedure is based on the Rackwitz–Fiessler recursive formula [4]

$$\mathbf{u}^{(k+1)} = \frac{1}{\|\nabla h(\mathbf{u}^{(k)})\|^2}\left[\nabla h^T(\mathbf{u}^{(k)})\,\mathbf{u}^{(k)} - h(\mathbf{u}^{(k)})\right]\,\nabla h(\mathbf{u}^{(k)}) \tag{3.4}$$

with the following convergence criterion

$$\left|u_i^{(k+1)} - u_i^{(k)}\right| \le \varepsilon \quad \text{and} \quad |\,g(\mathbf{x}^*)\,| \le \varepsilon \quad \text{for all variables,} \tag{3.5}$$

where ε is the tolerance assumed and $\mathbf{x}^* = \mathbf{T}^{-1}(\mathbf{u}^*)$. At each step, instead of an unknown function $h(\mathbf{u})$ (and $\nabla h(\mathbf{u})$), the following least square linear approximation is used

$$\hat{h}(\mathbf{u}) = b_0 + \sum_{i=1}^{n} b_i u_i \quad \text{and} \quad \nabla\hat{h}(\mathbf{u}) = [b_0, b_1, \ldots, b_n]\,. \tag{3.6}$$

The approach is based on points placed at an assumed distance $\Delta^k \mathbf{u}$ around the central point and additional points from previous steps are taken from spherical vicinity of the k-th central point (Fig. 1).

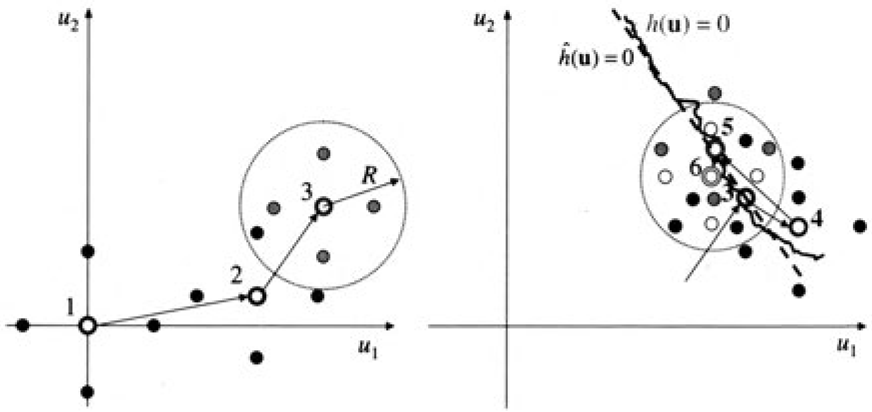

FIGURE 1. Idea of response surface method.

RSM estimates the LSF in the vicinity of the design point and then standard gradient reliability optimization method (FORM) is used to search for the *exact* design point on the LSF approximation. An additional improvement is provided by a quadratic weight approximation

$$\hat{h}(\mathbf{u}) = b_0 + \sum_{i=1}^{n} b_i u_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} u_i u_j \tag{3.7}$$

of LSF based on a set of simulations chosen from the searching stage and an additional sample of points specifically planned. The square of the multiple correlation coefficient R^2 defined as

$$R^2 = \frac{\sum_{i=1}^{N} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2} \quad \text{where} \quad \bar{y} = \frac{\sum_{i=1}^{N} y_i}{N}, \tag{3.8}$$

which measures the square of the correlation between $y = h(\mathbf{u})$ and $\hat{y} = \hat{h}(\mathbf{u})$, is used to select the important second order coefficients b_{ij} of equation (3.7), and accordingly to plan an additional sample of points. The probability of failure, therefore, can be estimated by the SORM method based on the second order approximation of LSF. The SORM method used in the paper has been implemented in COMREL-TI, which is a part of the STRUREL package, cf. [5], in the following version

$$P_{f2} \approx \Phi(-\beta) \prod_{i=1}^{n-1} \Big[1 + \kappa_i \frac{\varphi(-\beta)}{\Phi(-\beta)}\Big]^{-\frac{1}{2}} \tag{3.9}$$

where appropriate correction terms containing curvatures are included. The above formula can not be used if $|\beta| \leq 1$ and $\beta\kappa_i < -1$.

RSM procedure saves the cost of the finite elements analysis and makes it possible even if LSF is not smooth enough. However, such a response surface technique is limited to the case of LSF with only one distinct design point.

4. Reliability analysis of sheet metal forming operations

There are different uncertainties in the sheet metal forming process and material parameters that lead to uncertainties in the results of practical realization and numerical analysis of a given process. Real values of virtually every such parameter, like material constants, friction coefficients, geometric dimensions, blankholder force, etc. are known to have a certain scatter around their nominal values – adopting the latter ones to describe the process may lead to essential deviations in terms of the simulation results to be obtained.

Possibility of material fracture in sheet metal forming operations is estimated in practice using forming limit diagrams (FLD). In a typical forming limit diagram (Fig. 2) major principal strain values are plotted against minor principal strain values. Usually logarithmic or engineering strain measures are used. Points representing strain states all over the deformed sheet are confronted with the forming limit curve (FLC). FLC is supposed to represent the boundary between the strain combinations which produce instability (above the curve) and/or fracture and those that are permissible in forming operations (below the curve).

Results of sheet forming operations exhibit significant scatter, therefore any forming limit curve can be regarded as bounding the safe zone with some probability only. The safe zone is considered as the one where failure is highly improbable, while the failure zone is regarded as the one defining strain states with a high probability of failure. Usually between the two zones, safe and failure, a critical zone (marginal zone) is introduced (Fig. 2), with the probability of failure high enough so that the strain state cannot be considered safe. The accepted standard is to define the marginal zone such that the vertical distance between FLC and the lower boundary of the marginal zone is 10% (cf. [6]).

Forming limit curves for various types of materials vary with respect to both their shape and position and their location on FLD. For one material

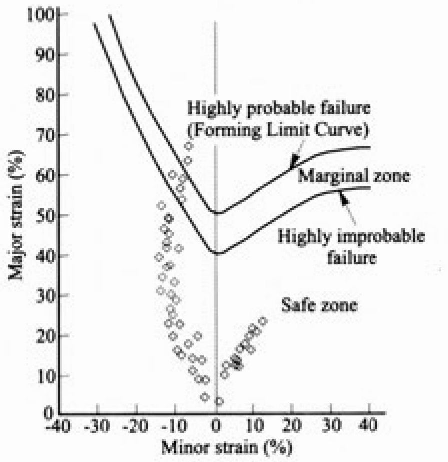

FIGURE 2. Typical forming limit diagram.

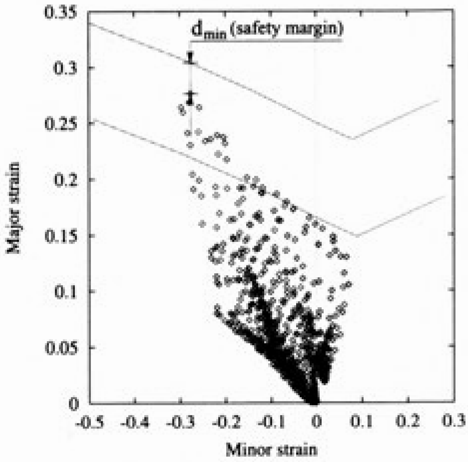

FIGURE 3. Definition of the LSF.

type FLC may also vary due to change of the material properties and sheet thickness. In this case we can assume that the shape of FLC remains constant and its position on FLD changes only, with its shift in the vertical direction (along the major strain axis) being the most significant [6]. The vertical position of FLC is dependent mainly on two factors: the hardening coefficient n and sheet thickness t. Increasing either of these parameters improves formability and is reflected by a shift of FLC upwards on the FLD. These considerations can be formulated by writing the first order approximation of the FLC shift in the following way:

$$h(n,t) = a(n - \bar{n}) + b(t - \bar{t})$$

where a and b are the sensitivities of the vertical position of FLC with respect to n and t, respectively. The symbols $\bar{n}$ and $\bar{t}$ stand for the mean values of n and t.

We take advantage of the forming limit diagrams and define LSF as the signed minimal distance from the FLC of the point corresponding to principal strains in the given finite element (Fig. 3). In the sign convention adopted the minus sign is for points above the curve.

Depending on the realization of the vector of random variables the 'cloud' of the points on the forming limit diagram can take a different shape which implies that different points in the sheet metal may be located close to the FLC. Adding to it the shape of the FLC (piecewise linear with vertices) and some 'numerical noise' introduced by using the explicit dynamic approach in the finite element analysis it appears unjustified to assume that the LSF is differentiable.

5. Numerical illustration

Deep drawing of a square cup (Fig. 4),which is the benchmark problem of the Numisheet'93 Conference [7], has been analysed. The material (aluminium) properties are taken as follows:, thickness 0.81 mm, Young modulus $E = 71$ GPa, Poisson's ratio $\nu = 0.33$, uniaxial true stress – true strain curve $\sigma = 576.79(0.01658 + \varepsilon^p)^{0.3593}$ MPa, friction coefficient $\mu = 0.162$. The blankholding force is 19.6 kN. Sensitivities of the vertical position of FLC with respect to n and t, defined by Eq. (4) have been assumed according to the available data [6] as: $a = 0.9$, $b = 130\,\mathrm{m}^{-1}$. With these data FLC is shifted vertically by the following distance expressed in terms of the logarithmic strain measure:

$$h(n,t) = 0.9(n - 0.3593) + 130(t - 0.81 \cdot 10^{-3}).$$

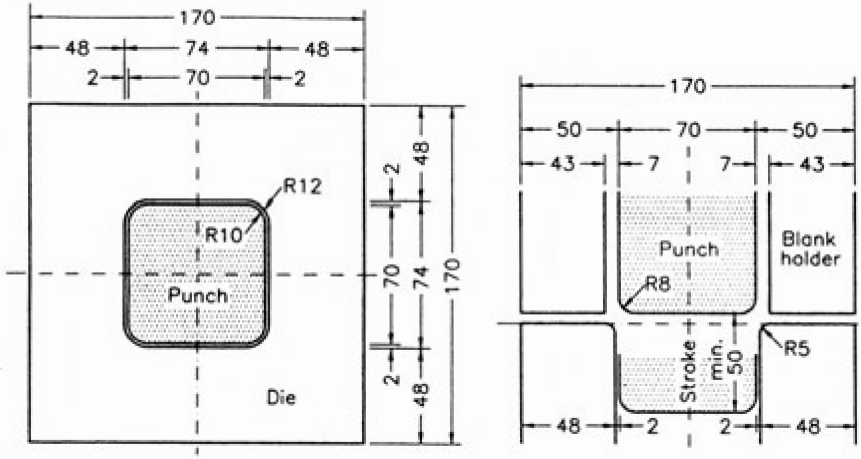

FIGURE 4. Deep drawing of a square cup – definition of the tool geometry.

Prior to a stochastic analysis the deterministic analysis of deep drawing of a square cup has been carried out. The results have been compared against available experimental results given in [7]. Experimental results have been reported for the punch travel of 15 mm. Good agreement between experimental and numerical results has been shown in [8].

The major and minor strains are plotted in the forming limit diagram in Fig. 5. Here the strains are still far from FLC and no danger of failure is expected. The failure conditions are first met for the punch travel of 20 mm. The strains at some point in Fig. 6 lie practically on FLC. This indicates the

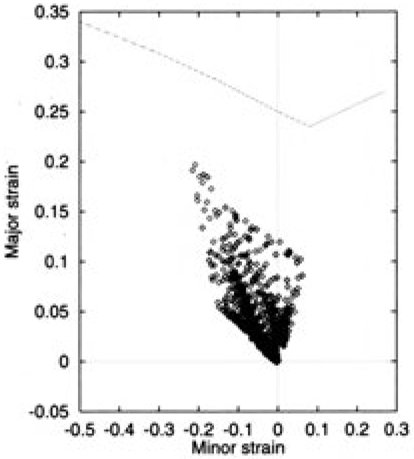

FIGURE 5. Forming limit diagram for 15 mm depth of drawing.

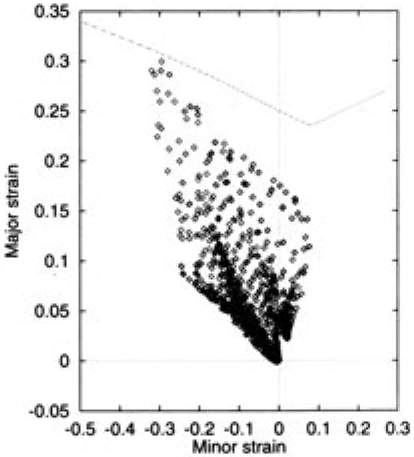

FIGURE 6. Forming limit diagram for 20 mm depth of drawing.

danger of failure. This is in agreement with experimental results – in [9] the failure in laboratory tests at punch stroke of 19 mm is reported.

The stochastic description of the system involves 3 random variables: the initial thickness of the sheet metal, the hardening exponent n and the Coulomb friction coefficient – between sheet metal and punch, die and blank-holder, respectively. Full correlation among friction coefficients describing these three states is assumed which appears close enough to reality. The random variables are assumed to be lognormally distributed with mean values μ and standard deviations σ shown in Table 1.

TABLE 1. Random variables.

Var.	Distribution type	Mean Value	Std. dev.	Description
t	lognormal	0.81 mm	0.04 mm	Initial thickness
μ	lognormal	0.162	0.015	Friction coefficients
n_1	lognormal	0.3593	0.015	Hardening coefficient
n_2	lognormal	0.3593	0.020	Hardening coefficient

To investigate the influence of a hardening exponent randomness on the sheet metal forming operation probability of failure, the reliability analysis employing RSM was performed. Two values of standard deviation n_1 and n_2 were examined.

It was clear that all gradient based algorithms would encounter difficulties while locating a design point $\mathbf{u}^*$ by FORM. Thus it was reasonable to employ the RSM approach by computing the gradients of limit state function by the plan of simulations (Fig. 1) to locate the central point instead of $\mathbf{u}^*$. The number of points used in the LSF approximation increases during optimization procedure (Eq. (3.4)), making the process stable and insensitive to local discontinuities of LSF. Therefore, the final hyperplane is in the least square sense the average description of LSF.

The objective of the reliability analysis was to study a change of probability of failure in terms of the safety margin. This allows us to verify the need of the marginal zone with a width of 10% which is used in practice. Change of the safety margin in the example studied has been obtained by the change of the depth of drawing. The results are presented in Table 2.

TABLE 2. Results of metal forming reliability analysis.

S [mm]	$d_{\min}$ [%]	St. dev. of n	RSM(FORM)			RSM(SORM)		
			P_f	β	N	P_f	β	N
16	7.44	0.015	$8.23 \cdot 10^{-7}$	4.793	46	$1.15 \cdot 10^{-6}$	4.725	61
		0.020	0.000100	3.718	39	$9.85 \cdot 10^{-5}$	3.723	54
17	5.50	0.015	0.000370	3.374	46	0.000314	3.419	61
		0.020	0.00437	2.622	29	0.00401	2.651	44
18	3.77	0.015	0.0165	2.132	29	0.0148	2.176	44
		0.020	0.0485	1.660	29	0.0476	1.669	44
19	2.14	0.015	0.132	1.119	39	0.108	1.238	54
		0.020	0.192	0.869	40	0.182	0.907	55
20	0.77	0.015	0.337	0.422	59	0.280	0.584	74
		0.020	0.373	0.324	23	0.395	0.267	38

Punch strokes between 16 and 20 mm were analysed, which corresponds to the safety margin ($d_{\min}$, see Fig. 3) variation from 7.44% to 0.77%, the values being obtained in the deterministic analysis based on the mean values of the random variables. The values of P_f estimated by these two methods are almost equal. The calculations performed with the more accurate RSM analysis based on the quadratic approximation prove that LSF relationship is almost linear in that region. Relatively small numbers of LSF calls for the RSM analysis are in contrast with large number (about 1000) of simulations necessary for the AMC method. The time consuming AMC method and, in one case, crude Monte Carlo techniques were used to check the accuracy of the RSM, Table 3.

TABLE 3. Results of verification by AMC.

S [mm]	$d_{\min}$ [%]	St. dev. of n	AMC		
			β	$\nu_{\bar{P}_f}$ [%]	N
16	7.44	0.020	3.721	7.10	10^3
18	3.77	0.020	1.670	5.32	10^3
20	0.77	0.020	0.342*	5.89	500

*Crude Monte Carlo method

It can be seen that probability of failure decreases fast with the increase of safety margin. For the safety margin of 7.44% these values are small enough so that the stamping process could be considered safe (reliable). Further increase of safety margin would reduce the probability of failure to the levels not required by manufacturers. The analysis shows that the width of marginal zone (10%) could be reduced in some cases. Of course we must remember that the results obtained by the reliability analysis are based on assumed distributions of random variables. It should be noticed that RSM does not give in the $|\beta| \leq 1$ case exact solution but only identifies an unsatisfactory level of P_f.

6. Conclusions

Methodology developed for reliability calculations of structures was successfully applied to estimate reliability of sheet metal forming operations. Response surface technique turned out the right tool to obtain assessment of probability of failure in the noise effect case of LSF, when it is impossible to use the standard gradient methods. For low number of variables computational cost of this method is much lower than in simulation techniques.

Adaptive Monte Carlo techniques proved useful for reliability assessment of sheet metal forming operations but for reasons of efficiency they appear suitable for medium size problems only.

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