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# Stochastic loads in the fatigue assessment prediction of sea-going ships

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The problem of ship hull dynamic loads generated by waves and the ship response constitutes an essential element in the ship design process. The paper deals with components of dynamic loads for direct and simplified calculations. Simplified calculation are based on Rules issued by PRS [8]. Direct calculations require numerical methods to determine transfer functions of ship motions, loads and long-term stresses projection. The paper presents verification of simplified formulae [8] for simultaneous coefficients of acting loads.

## 1. Introduction

Due to the fact that waves motion, which is an input function, is of stochastic nature, the ship wave dynamic system should be described by stochastic initial-value boundary problems. The theory of stochastic differential equations is not as yet so developed to allow practical applications and therefore simplified assumptions are made. It is assumed that the waves motion is a stationary stochastic process and the ship-wave dynamic system is linear. This allows one to apply the spectral analysis to solve the problems of the ship motion on waves and the ship loads.

Wave loads are sources of time changing, asymmetrical stresses in structural elements of a ship. They cause cracks in high grade steel when higher range stress is allowed. The old construction with section corroded in salt sea water which change the cross section of elements is also under investigation [1]. Examples of such constructions and guidelines for repair are published in [4]. Considerable progress has been made during the last ten years in the design procedure for checking the fatigue. In many papers attention is focused on the problems of uncertainty in such areas as [6, 7]:

- long-term probabilistic distribution,
- scatter diagram used for long-term model,
- prediction of fatigue damage,
- fitting of Weibull distribution for wave induced loads effects,
- other effects (influence of ship speed, wave short-crestedness, number of responses cycles in the structure lifetime, ship type, ship routes),
- high-frequency whipping loads,
- non-linear effects (slamming, sloshing, non-vertical ship sides).

There is still some uncertainty in this procedure and lack of summation of load components. Simple formula in the classification rules do not cover full range of construction work under complex loads. This problem is investigated in this paper.

## 2. Long-term prediction scheme

In short and long term calculations when suitably short period of time of the alternating wave condition in a given area $A_i$ $i = 1, \ldots, n$, can be defined by sea states and expressed by three parameters: $H_S$ (significant wave height), $T_1$ (characteristic period), $\mu_w$ (wave direction), with the assumption that:

- ship routes can be expressed by the sum of areas $A_i$ where the scatters diagrams are known (Table 1),
- the ship can be described by parameters concerning motions and load cases.

The wave characteristics $H_s$, $T_1$, $\mu_w$, become random variables over a long period of time in each area $A_i$ and the probability of occurrence of the sea condition in a given interval of the seaway characteristics:

$$\begin{aligned} H_s &\le H_s' \le H_s+\Delta H_s, \\ T_1 &\le T_1' \le T_1+\Delta T_1, \\ \mu_w &\le \mu_w' \le \mu_w+\Delta \mu_w, \end{aligned} \tag{2.1}$$

which is given by formula:

$$P_{wi}\big(H_s \le H_s' \le H_s+\Delta H_s,\ T_1 \le T_1' \le T_1+\Delta T_1,\ \mu_w \le \mu_w' \le \mu_w+\Delta\mu_w\big) = \int\limits_{H_s}^{H_s+\Delta H_s} \int\limits_{T_1}^{T_1+\Delta T_1} \int\limits_{\mu_w}^{\le\mu_w+\Delta\mu_w} f_{wi}(H_s', T_1, \mu_w')\, dH_s'\, dT_1'\, d\mu_w', \tag{2.2}$$

where $f_{wi}$ is the density function of sea states occurrence.

It is assumed that the random variable $\mu_w$ does not depend on the variables $H_s$ and $T_1$, and the probability density function of random variable $\mu_w$ does not depend on the area $A_i$. This implies that:

$$f_{wi} = f_{wi}^{(1)}(H_s, T_1)\, f_{wi}^{(2)}(\mu_w). \tag{2.3}$$

In calculation practice the probability of occurrence of sea state in a subarea is used $[H_{sj}, H_{sj+1} \times T_{1i}, T_{1i+1}]$ instead of the function of probabilistic distribution (Table 1).

According to the concept of the long-term prediction of ship response $y$ is the mathematical expectation of the random variable $y$ in a long period of time represented by formula:

$$\bar{y} = \sum_{i=1}^{n} \int_0^{2\pi} \int_0^{\infty} \int_0^{\infty} y f_{wi}^{(1)}(H_s, T_1)\, f_{wi}^{(2)}(\mu - \mu_c)\, dH_s dT_1\, d\mu_w\, p_i, \tag{2.4}$$

where:

$\mu_c$ – the ship course,

$p_i$ – the probability of the occurrence of the ship in the area $A_i$ over a long period of time.

The probabilistic density function of the maximum values of the process is given by the formula:

$$f_\sigma(\sigma) = \frac{\sigma}{m_0} \exp\left(-\frac{1}{2}\frac{\sigma}{m_0}\right), \tag{2.5}$$

where $m_0$ is the spectral moment of order 0 of the response process. The conditional probability of exceeding a given level $\sigma_0$ by the process is determined by the Rayleigh distribution:

$$Pr\{\sigma \geq \sigma_0\} = \exp\left(-\frac{\sigma_0^2}{2m_0}\right). \tag{2.6}$$

The moment of the spectrum is given by the formula:

$$m_i = \int_0^{\infty} f_s\, \omega^i\, S_{\sigma\sigma}(\omega)\, d\omega, \quad i = 0, 1, 2, \ldots, \tag{2.7}$$

where $S_{\sigma\sigma}$ is the spectral density function which for North Atlantic is given by formula:

$$S(\omega) = \frac{H_s^2}{4\pi}\left(\frac{2\pi}{T_z}\right)^2 \omega^{-5} \exp\left(-\frac{1}{\pi}\left(\frac{2\pi}{T_z}\right)^4 \omega^{-4}\right), \tag{2.8}$$

TABLE 1. Probability of sea state in the North Atlantic described as occurrence per 100000 observation, after BMT's Global Wave Statistics [3].

| hs \ tz | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 | 11.5 | 12.5 | 13.5 | 14.5 | 15.5 | 16.5 | 17.5 | 18.5 | SUM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.0 | 0.0 | 1.3 | 133.7 | 865.6 | 1186.0 | 634.2 | 186.3 | 36.9 | 5.6 | 0.7 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3050 |
| 1.5 | 0.0 | 0.0 | 0.0 | 29.3 | 986.0 | 4976.0 | 7738.0 | 5569.7 | 2375.7 | 703.5 | 160.7 | 30.5 | 5.1 | 0.8 | 0.1 | 0.0 | 0.0 | 0.0 | 22675 |
| 2.5 | 0.0 | 0.0 | 0.0 | 2.2 | 197.5 | 2158.8 | 6230.0 | 7449.5 | 4860.4 | 2066.0 | 644.5 | 160.2 | 33.7 | 6.3 | 1.1 | 0.2 | 0.0 | 0.0 | 23810 |
| 3.5 | 0.0 | 0.0 | 0.0 | 0.2 | 34.9 | 695.5 | 3226.5 | 5675.0 | 5099.1 | 2838.0 | 1114.1 | 337.7 | 84.3 | 18.2 | 3.5 | 0.6 | 0.1 | 0.0 | 19128 |
| 4.5 | 0.0 | 0.0 | 0.0 | 0.0 | 6.0 | 196.1 | 1354.3 | 3288.5 | 3857.5 | 2685.5 | 1275.2 | 455.1 | 130.9 | 31.9 | 6.9 | 1.3 | 0.2 | 0.0 | 13289 |
| 5.5 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | 51.0 | 498.4 | 1602.9 | 2372.7 | 2008.3 | 1126.0 | 463.6 | 150.9 | 41.0 | 9.7 | 2.1 | 0.4 | 0.1 | 8328 |
| 6.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.2 | 12.6 | 167.0 | 690.3 | 1257.9 | 1268.6 | 825.9 | 386.8 | 140.8 | 42.2 | 10.9 | 2.5 | 0.5 | 0.1 | 4806 |
| 7.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3.0 | 52.1 | 270.1 | 594.4 | 703.2 | 524.9 | 276.7 | 111.7 | 36.7 | 10.2 | 2.5 | 0.6 | 0.1 | 2586 |
| 8.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.7 | 15.4 | 97.9 | 255.9 | 350.6 | 296.9 | 174.6 | 77.6 | 27.7 | 8.4 | 2.2 | 0.5 | 0.1 | 1309 |
| 9.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.2 | 4.3 | 33.2 | 101.9 | 159.9 | 152.2 | 99.2 | 48.3 | 18.7 | 6.1 | 1.7 | 0.4 | 0.1 | 626 |
| 10.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.2 | 10.7 | 37.9 | 67.5 | 71.7 | 51.5 | 27.3 | 11.4 | 4.0 | 1.2 | 0.3 | 0.1 | 285 |
| 11.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.3 | 3.3 | 13.3 | 26.6 | 31.4 | 24.7 | 14.2 | 6.4 | 2.4 | 0.7 | 0.2 | 0.1 | 124 |
| 12.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 1.0 | 4.4 | 9.9 | 12.8 | 11.0 | 6.8 | 3.3 | 1.3 | 0.4 | 0.1 | 0.0 | 51 |
| 13.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.3 | 1.4 | 3.5 | 6.0 | 4.6 | 3.1 | 1.6 | 0.7 | 0.2 | 0.1 | 0.0 | 21 |
| 14.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.4 | 1.2 | 1.8 | 1.8 | 1.3 | 0.7 | 0.3 | 0.1 | 0.0 | 0.0 | 8 |
| 15.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.4 | 0.6 | 0.7 | 0.5 | 0.3 | 0.1 | 0.1 | 0.0 | 0.0 | 3 |
| 16.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.2 | 0.2 | 0.1 | 0.1 | 0.0 | 0.0 | 0.0 | 1 |
| SUM: | 0 | 0 | 1 | 165 | 2091 | 9280 | 19922 | 24879 | 20870 | 12898 | 6245 | 2479 | 837 | 247 | 66 | 16 | 3 | 1 | 100000 |

where: $T_z = 2\pi(\frac{m_0}{m_2})^{\frac{1}{2}}$. Then the long-term prediction of the probability of exceeding the given level $\sigma_0$ is obtained from the formula:

$$\overline{Pr}\{\sigma \geq \sigma_0\} = \sum_{i=1}^{n} p_i \int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} \exp\left(-\frac{\sigma_0^2}{2m_0}\right) f_{wi}\, dH_s\, dT_1\, d(\mu_w - \mu_c). \quad (2.9)$$

In numerical calculations scatter diagram (Table 1) of the probability is used and integrals change to the sequence:

$$\bar{P}\{\sigma > \sigma_0\} \approx \sum_l \sum_k \sum_j \sum_i \exp\Big(-\frac{\sigma_0^2}{2m_{0ijkl}}\Big) p_{ij} p_k p_l, \quad (2.10)$$

where:

$p_{ij}$ – the probability of occurrence of sea state (defined by $H_{si}$ and $T_{lj}$) according to Table 1,

$p_k$ – the probability of occurrence of angle $\mu_c$,

$p_l$ – the probability of occurrence that the ship is in subarea $A_l$.

Additionally, it is assumed that the transfer function $f_{wi}$ and spectral density function $S_{\sigma\sigma}$ are in the moving reference system on the forward speed of the ship. It means that the ship speed depends on the weather condition and varies due to:

- increased resistance and loss of propulsion efficiency,
- attempt to avoid an undesirable weather by the ship master.

The subjective loss of speed is generally caused by high values of acceleration at the bow, slamming, deck wetness, propeller emergence and roll. In the calculation model it is assumed that:

$a_{V\frac{1}{3}} \leq 0.4g$ – significant value of the vertical acceleration amplitude at the bow ($g = 9.81\frac{m}{s^2}$ – the gravity acceleration),

$P\{\text{S.A.} > \text{F}_{\text{BF}}\} = 0.05$ – the probability of the deck wetness at the bow (where S.A. – the vertical displacement, $F_{BF}$ – the high of freeboard),

$P\{\text{slamming}\} = 0.07$ – the probability of slamming occurrence at the bow,

roll $\leq 25$ degrees.

## 3. Load and stresses combination

The main purpose of the present study is the analysis of individual components of the ship hull dynamic loads from waves due to sea water and the carried cargo. The loads are non-stationary and occur with various intensity and time shift. This means that the loads components cannot be summed directly and for that reason the superposition method of the structure response (stresses) to particular components of loads should be so formulated as to obtain the approximate total response (stresses). The response is subsequently used in fatigue assessment of the ship structural elements.

It can be assumed that in each structural element the resulting total stress is a sum of stresses which arise:

- primary loads (shear forces and bending moments) act on hull beam,
- secondary loads as dynamic pressures acting on the element,
- zone strength of the structure modelled as 2 or 3-dimensional girder system.

This enables us to calculate each individual stress component according to the procedure showed in Sec. 2.

A structural element is existing in a special structure configuration and the formula for the sum of individual component should have a special form which takes into account the signs of stresses. For example, the stresses in the upper layer of longitudinal bottom girder can be expressed by (Fig. 1):

$$\sigma_j^{\Re} = \frac{M_{Vj}^{\Re}}{W_V} + \frac{M_{Hj}^{\Re}}{W_H} + \frac{M_{Tj}^{\Re}}{W_o} + \sigma_{sj}^{\Re} + \frac{p_j^{\Re} sl}{12W}, \quad j = 1, \ldots, n, \tag{3.1}$$

$$\sigma_j^{\Im} = \underbrace{\frac{M_{Vj}^{\Im}}{W_V} + \frac{M_{Hj}^{\Im}}{W_H} + \frac{M_{Tj}^{\Im}}{W_o}}_{\text{global}} + \underbrace{\sigma_{sj}^{\Im}}_{\text{zone}} + \underbrace{\frac{p_j^{\Im} sl}{12W}}_{\text{local}}, \quad j = 1, \ldots, n \tag{3.2}$$

where:

$j$ – the index referred to harmonic component of wave,
$M_V$ – the vertical wave bending moment in the hull,
$M_H$ – the horizontal wave bending moment in the hull,
$M_o$ – the torsion wave moment in the hull,
$\sigma_s$ – the stress in structural element due to zone strength,
$p$ – the non-stationary pressure (of wave) acting on structural element,
$W_V$ – the section modulus for vertical hull bending moment,
$W_H$ – the section modulus for horizontal hull bending moment,
$W_o$ – the section modulus for torsion moment,
$W$ – the section modulus of element,
$\Re$ and $\Im$ – the real and imaginary parts of calculated transfer function of $M_V$, $M_H$, $M_o$, $p$ and $\sigma_s$.

Other stress components can be neglected. This calculation procedure allows to take into account phase angles between all loading components acting on structural elements of the ship. The total transfer function of range stresses takes the form:

$$\Delta(\sigma_R)_j = \sqrt{(\Delta\sigma_j^{\Re})^2 + (\Delta\sigma_j^{\Im})^2}. \tag{3.3}$$

In the classification societies rules [8] the superposition of range stresses from global and local strength levels is used:

$$\Delta\sigma_R = \max(\Delta\sigma_g, \Delta\sigma_L) + K_{gl}\min(\Delta\sigma_g, \Delta\sigma_L), \tag{3.4}$$

where:

$\max(\Delta\sigma_g, \Delta\sigma_L)$ – the higher value of $\Delta\sigma_g, \Delta\sigma_L$,

$\min(\Delta\sigma_g, \Delta\sigma_L)$ – the lower value of $\Delta\sigma_g, \Delta\sigma_L$,

$K_{gl}$ – the coefficient of loads simultaneousness.

The local and global stress ranges take the form:

$$\Delta\sigma_l = \max(C\Delta\sigma_z, \Delta\sigma_w) + K_l\min(\Delta\sigma_z, \Delta\sigma_w), \tag{3.5}$$

$$\Delta\sigma_g = \max(C\Delta\sigma_V, \Delta\sigma_H) + K_g\min(\Delta\sigma_V, \Delta\sigma_H), \tag{3.6}$$

where:

$C$ – the coefficient of concentration loads,

$\Delta\sigma_z$ – the range stress for the local outer (sea) pressure,

$\Delta\sigma_w$ – the range stress for the local inner (cargo) pressure,

$\Delta\sigma_V$ – the range stress for the wave vertical bending moment,

$\Delta\sigma_H$ – the range stress for the wave horizontal bending moment.

## 4. Fatigue damage assessment of structural elements

The classification society rules [8] used the calculation method with Miner-Palmgren rule and $S$–$N$ curves. The accumulated damage parameter $D$ gives information about fatigue strength. If $D \le 1$, than fatigue damage of element is sufficient in a long period of time. On the other hand, when $D > 1$ we can expect a crack before the end of ship life at sea. Calculations of the fatigue damage are often based on the assumption that long-term distribution of probability of crack occurrence has a good approximation with the Weibull distribution:

$$p(\Delta\sigma) = \frac{\xi}{a}\left(\frac{\Delta\sigma}{a}\right)^{\xi-1}\exp\left[-\left(\frac{\Delta\sigma}{a}\right)^{\xi}\right] \tag{4.1}$$

where:

$p(\Delta\sigma)$ – the probability distribution,

$\xi$, $a$ – the Weibull distribution parameters.

Finally, the formula for the fatigue damage parameter of the element without correction for the sea water and cargo influence can be written in form:

$$D = \frac{N_L}{K} \frac{(\Delta\sigma_R)^m}{(lnN_R)^{m/\xi}} \Gamma\Big(1 + \frac{m}{\xi}\Big), \tag{4.2}$$

where:

$N_L$ – the number of loads cycles in the ship life at sea, for example 20 years,

$\Gamma$ – the gamma function,

$\Delta\sigma_R$ – the exceeded stress range according to the probability level of $\frac{1}{N_R}$,

$N_R$ – the number of total loads cycles in the whole life of the ship at sea.

The expression for the fatigue damage $D$ indicates that from the loading point of view it is necessary to define the total number of cycles $N_L$ and $N_R$, the characteristic value of the stress $\Delta\sigma_R$ which is associated with the value chosen for $N_R$ and the shape parameter $\xi$ of the Weibull distribution that describes the load spectrum [6]. It can be done with the use of the direct method. The results are shown in the next section.

## 5. Numerical results

The calculations were performed for a typical bulk carrier with the main dimension given by: $L = 225\,\text{m}$, $B = 32.2\,\text{m}$, $T = 12.2\,\text{m}$ and $C_B = 0.794$. The probability of occurrence of a crack was analysed in the bottom longitudinal at passage in a double bottom tank floor, at the midship in the middle part of cargo hold (Fig. 1).

For simplified calculations according to [8] the obtained range stress is equal to $\Delta\sigma_R = 152\,\text{MPa}$ (for $p = 10^{-4}$), and the fatigue damage parameter $D = 1.7$. It means that the structural element will be destroyed af-

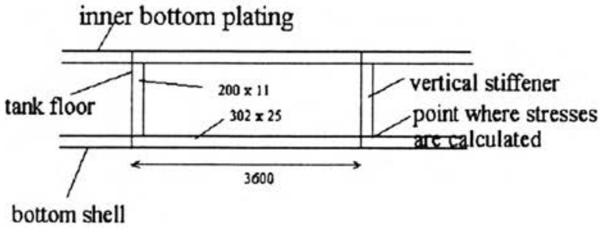


FIGURE 1. Bottom longitudinal

ter 11.6 years. Analogically, direct calculations gave $D = 1.27$, which means that the element will be destroyed after 15.7 years. This calculation was made without correction factors for element and welded joint and compressive stresses and influence of the salt sea water on $S$–$N$ curves. It was assumed that the ship sailed in one load case which gives a lower value of $D$. All these assumptions were made to avoid differences in the calculation methods. The results of the direct calculation of the long-term prediction stress range are showed in Table 2.

TABLE 2. Range stresses for the hull bending as a beam and local bending.

| $-\log(p)$ [–] | $\Delta\sigma_H$ [MPa] | $\Delta\sigma_V$ [MPa] | $\Delta\sigma_g$ [MPa] | $\Delta\sigma_R$ [MPa] | $K_{gl}$ [–] | $K_g$ [–] |
|---|---|---|---|---|---|---|
| 9 | 66.4 | 311.6 | 311.8 | 420.0 | 0.53 | 0.00 |
| 8 | 58.6 | 276.0 | 276.4 | 372.0 | 0.53 | 0.01 |
| 7 | 50.4 | 239.6 | 240.0 | 323.0 | 0.53 | 0.01 |
| 6 | 42.4 | 202.6 | 203.2 | 273.0 | 0.54 | 0.01 |
| 5 | 34.4 | 165.0 | 165.6 | 227.0 | 0.55 | 0.02 |
| 4 | 26.6 | 127.8 | 128.4 | 172.0 | 0.57 | 0.02 |
| 3 | 19.4 | 92.4 | 92.8 | 125.0 | 0.59 | 0.02 |

## 6. Concluding remarks

The long-term loads predictions may be determined using the parameters of the Weibull distribution, being the best approximation of these predictions. The results of calculation indicate the suitability of the Weibull distribution for the purpose of fatigue assessment. It will enable the elimination of the time-consuming method, consisting of:

1. solution of equations of the ship motion on regular waves,
2. determining the transfer function of loads,
3. superposition of the transfer function,
4. determining the long-term response on irregular waves,
5. fatigue life calculations.

Particular emphasis has been put on practical application of the present results of study so that the algorithm of stress calculations could be formulated and the formula approximating the long-term predictions of the ship behaviour on waves could be effectively used by designers and builders.

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