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An efficient method to minimise the preventive maintenance cost of series-parallel systems

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General preventive maintenance model for input components of a system, which improves the reliability to "as good as new", is used to optimize the maintenance cost. The cost function of a maintenance policy is minimized under given availability constraint. An algorithm for first inspection vector of times is described and used on selected system example. A special ratio-criterion, based on the time dependent Birnbaum importance factor, was used to generate the ordered sequence of first inspection times. Basic system availability calculations of the paper were done by using simulation approach with parallel simulation algorithm for availability analysis. These calculations are based on direct Monte Carlo technique and applied within the programming tool Matlab. A genetic algorithm optimization technique is used and briefly described to create the algorithm (in Matlab as well) to solve the problem of finding the best maintenance policy with a given restriction.

Key words: *preventive maintenance, cost, availability, optimization, Monte Carlo method.*

Notations

N – total number of components,
$\mathbf{T}_0 = (T_0(1), T_0(2), \ldots, T_0(N))$ – first inspection time vector,

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$\mathbf{T}_0^{\text{ord}} = (T_0^{(1)}, T_0^{(2)}, \dots, T_0^{(N)})$ – ordered first inspection time vector,
$T_0^{(1)} \leqslant T_0^{(2)} \leqslant \dots \leqslant T_0^{(N)}$,
$\mathbf{T}_P = (T_P(1), T_P(2), \dots, T_P(N))$ – vector of optimal periods of system component,
T_M – mission time,
$C(e(i,k))$ – cost of one inspection of i^{th} component in k^{th} parallel subsystem,
$A(t)$ – availability of system at the time t,
A_0 – availability constraint.

1. Introduction

The evolution of system reliability depends on its structure as well as on the evolution of the reliability of its elements. The latter is a function of the element age on a system's operating life. Element ageing is strongly affected by maintenance activities performed on the system. Preventive maintenance (PM) consists of actions, which improve the condition of system elements before they fail. PM actions such as the replacement of an element by a new one, cleaning, adjustment, etc. either return the element to its initial condition (the element becomes "as good as new") or reduce the age of the element. In some cases the PM activity does not affect the state of the element but ensures that the element is in operating condition. In this case the element remains "as bad as old".

Optimizing the policy of preliminary planned PM actions is the subject of many papers. In the past, the economic aspects of preventive and corrective maintenance have been extensively studied for monitored components in which failures are immediately detected and subsequently repaired. Far less attention has been paid to the economics of systems in which failures are dormant and detected only by periodic testing or inspections. Such systems are especially common in industrial safety and protection systems. Both the availability models and the cost factors differ considerably from those of monitored components (see [2]).

This paper develops availability and cost models for systems with periodically inspected and maintained components subjected to some maintenance strategy.

The aim of our research is to optimise for each component of a system the maintenance policy minimising the cost function, with respect of the availability constraint such as $A(t) \geqslant A_0$, for all t, $0 < t \leqslant T_M$, and a given mission time T_M.

A genetic algorithm (GA) is used as an optimisation technique. GA is used to solve the above mentioned problem to find the best maintenance policy using a simulation approach to assess the availability of the studied system. The solution comprises both the availability and the cost evaluation.

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Properties of the applied simulation code were intensively studied in [4]. The Matlab program was also successfully used in [5], for the reliability and availability optimisation based on design of a Distribution Area System under Maintenance. New improvements of the simulation program focused on enhancing of computational efficiency are implemented at the present time, including the implementation of a parallel computing procedures.

A similar optimisation problem applied to series parallel multi-state system was studied in [3] taking into account imperfect component preventive maintenance actions. This model uses universal z-transform for reliability calculations (universal moment generating function) but the duration of the PM activity is neglected. In [3], the optimisation procedure is also based on a heuristic genetic algorithm. We propose in this paper to study the example from [3] and others to prove the efficiency of our model.

2. Preventive maintenance model for general series-parallel system

2.1. Input component's model

In the paper we will assume for input component the PM model, which reduces the reliability to "as good as new". It means that the element's age is restored to zero (replacement). The model is demonstrated in Fig. 1, where:

T_F – Time to failure,

T_P – deterministic time-period of inspection,

T_0 – first inspection time (deterministic time),

T_M – mission time.

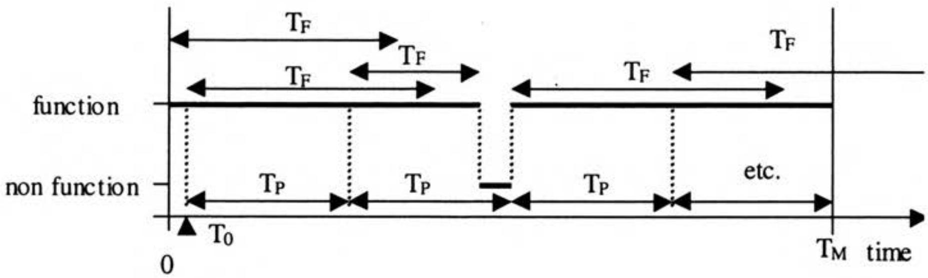

FIGURE 1. PM model for periodically tested elements.

The problem of finding the optimal T_P for each of input components is closely connected with the problem of determination of an optimal first inspection time T_0. Of course, it makes no sense to realise inspections at the

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beginning of the life of the component, when both the system and the input component are very reliable. Consequently the preliminary calculations must be performed to find the optimal T_0 for each of input components. At the same time, the finding of the T_0 must be in good agreement with both the cost and the reliability effect, as is explained bellow.

2.2. General series-parallel structure

Optimal PM plan is found for a general series-parallel structure, that is demonstrated in Fig. 2.

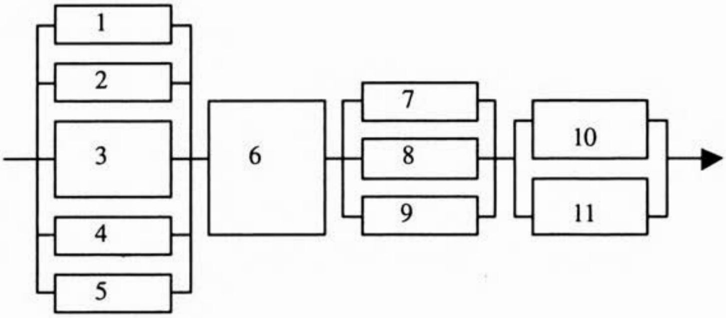

FIGURE 2. General series-parallel structure.

2.3. Cost model

Cost of the above mentioned preventive maintenance policy of a given system is simply given by summarizing each of the PM inspection done on the components that are under maintenance policy:

$$C_{PM} = \sum_{k=1}^{K} \sum_{i=1}^{E_k} \sum_{j=1}^{n_{e(i,k)}} C_j\big(e(i,k)\big),$$

where:

$n_{e(i,k)}$ represents the total number of inspections of the i^{th} component in the k^{th} parallel subsystem in the course of mission time,

$C_j\big(e(i,k)\big)$ is the cost of the j^{th} inspection of the i^{th} component in k^{th} parallel subsystem,

E_k is the number of components in given k^{th} parallel subsystem,

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K is the number of parallel subsystems,

$N = \sum_{k=1}^{K} \sum_{i=1}^{E_k} e(i,k)$ is the total number of components.

In most cases the cost of one inspection of a given component is constant in the course of the mission time, i.e.,

$$C_{PM}\big(e(i,k)\big) = \sum_{j=1}^{n_{e(i,k)}} C_j\big(e(i,k)\big) = n_{e(i,k)} \times C\big(e(i,k)\big),$$

where $C\big(e(i,k)\big)$ is the cost of one inspection of i^{th} component in k^{th} parallel subsystem, and

$$n_{e(i,k)} = \left\lfloor \frac{T_M\big(e(i,k)\big) - T_0\big(e(i,k)\big)}{T_P\big(e(i,k)\big)} \right\rfloor,$$

which means the integer part of the fraction, and T_M, T_0, T_P, are mission times, the first inspection time and period of the respective component.

3. Problem formulation

A system consisting of subsystems connected in series is considered. Each subsystem contains different elements connected in parallel (see, for example, Fig. 2). Each component is characterized by its failure rate function $h_j(t)$ and PM cost of one inspection $C\big(e(i,k)\big)$.

Our approach admits, in general, the maintenance actions with a non negligible time duration. Consequently, each maintenance action modifies temporary the system configuration (available components). Then, a dynamic model for periodic changes of the system structure (dynamic fault tree) is necessary to study to take into account this fact. Dynamic fault tree was intensively studied in [1], where house events matrix was considered as the representation of the fault tree time changes. In spite of the fact, in the first step of the research, the time in which the components is not available due to a PM activity is negligible if compared to the time elapsed between consecutive activities.

The basic assumptions of our approach are:

1. Testing actions(or inspections) are performed for the j^{th} component at time intervals $T_P(j)$. Inspections are ideal which means that given component is renewed (as good as new). The j^{th} component inspection begins at the time $T_0(j)$.
2. A system consisting of subsystems connected in series is considered. Each subsystem contains different components connected in parallel.

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Each component is characterized by its failure rate function $h_j(t)$, and PM cost of one inspection – $C(e(i,k))$ – is the cost of one inspection of i^{th} component in k^{th} parallel subsystem.

The aim of our research is to optimise for each component of a system the maintenance policy minimizing the cost function C_{PM}, with respect to the availability constraint such as $A(t) \geqslant A_0$, for all t, $0 < t \leqslant T_M$, and a given mission time T_M. In other words, it is necessary to find optimal vectors (cost minimizing) $\mathbf{T}_P = (T_P(1), T_P, (2), \ldots, T_P(N))$, and $\mathbf{T}_0 = (T_0(1), T_0(2), \ldots, T_0(N))$, under given availability constraint.

4. Availability estimation based on simulation technique

4.1. Availability estimation based on simulation approach

Basic availability calculations of the paper were done by using simulation approach. In fact, the simulation approach is employed when analytical techniques have failed to provide a satisfactory mathematical model or defy solution of the problem in closed form or the solution becomes unwieldy. The principle behind the simulation approach is relatively simple and easy to apply. However, the common real time simulation techniques are slow and take a lot of time to provide accurate results. Nevertheless, this technique is the only practical method of carrying out reliability studies, particularly when system is maintained and arbitrary failure and repair distributions are used or some special repair or maintenance strategy is prescribed.

The Monte Carlo method allows complex systems to be modelled without the need to make unrealistic simplifying assumptions, as is inevitably done when using analytical methods. With the increasing availability of fast computers, Monte Carlo methods become more and more powerful and feasible. As they have the potential of obtaining solutions to model which are very close to reality, they can yield relevant and useful results. In the past twenty years applications of Monte Carlo simulation methods to a variety of system engineering problems have indicated that such techniques can provide significant improvements in the realistic assessment of reliability and availability of complex systems, with relevant influence on the system life cycle cost and design.

Availability assessment method is based on the simulation program described in detail in the Ref. [6]. Parallel simulation algorithm for reliability and availability analysis, based on the direct Monte Carlo technique and applied within the programming tool Matlab, is demonstrated in the reference mentioned. The parallel simulation technique brings many improvements of the basic direct simulation technique, resulting in higher computational ef-

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ficiency first of all. The procedure is parallel in that it takes into account all the simulated transition times of each node in parallel. Using oriented acyclic graph as a system representation (composed from nodes and edges), the reliability characteristics of the system are obtained by the evaluation of the highest TOP node. Input and output characteristics of the program are presented in detail as well as the computational facilities including for example performance of a reliability analysis under dynamic structural changes of a system. Big diversity of maintenance strategy applied on input components is allowed using the code. Reliability calculations (both unreliability and unavailability dependent on time) of a complex technical system from practice (maintained and periodically tested) with graphically demonstrated outputs are possible to perform using the algorithm.

4.2. Finding the optimal first inspection time vector $\mathbf{T}_0$

Naturally, the problem of finding of optimal vector $\mathbf{T}_P$ is closely connected with another problem, namely the finding of a vector $\mathbf{T}_0$ which represents the beginning of inspection of each input component, i.e. the vector of first inspection times. Of course we will not realise inspections at the beginning of the life of an element, when the element is very reliable. Consequently the preliminary calculations must be performed to find the optimal T_0 for each of input components. At the same time, the finding of the optimal vector $\mathbf{T}_0$ must be in good accordance with both the cost and the reliability effect. The starting point for finding the optimal $\mathbf{T}_0$ is based on the idea that only such interventions into the system must be made, that are maximally effective both from reliability and from cost point of view as well. The measure of efficiency is a more or less subjective question and in many situations in practice may be dependent on a concrete reliability data files. For our research we decided to use the time dependent ratio-criterion of efficiency that is defined as follows:

$$\min\left\{R_j(t) \mid j = 1,\dots,N\right\}, \qquad R_j(t) = \frac{C(j)}{IF_j^B(t)},$$

where:

$C(j)$ is the cost of one inspection of the j^{th} component,

$IF_j^B(t)$ is Birnbaum's measure of importance of the j^{th} component at time t, cf. [7] for appropriate definitions.

Actually, Birnbaum's importance measure provides the probability that the system is in a state in which the functioning of component j is critical to system failure. The system fails when the j^{th} component fails.

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For a given time point, we obtain the component number, inspection of which is optimal, for which the ratio-criterion defined above is minimal.

The following procedure determines the vector $\mathbf{T}_0 = \big(T_0(1), T_0(2), \ldots, T_0(N)\big)$:

1. Calculate the dependence of reliability (availability) of analysed system on time for the given mission time T_M, supposing no maintenance; $i = 1$.
2. Obtain the time point t_i in which the system availability value A_0 is reached.
3. If $t_i < T_M$, then t_i is the i^{th} component of ordered first inspection time vector $\mathbf{T}_0^{\text{ord}}$; $T_0^{(i)} = t_i$; $\mathbf{T}_0^{\text{ord}} = \big(T_0^{(1)}, T_0^{(2)}, \ldots, T_0^{(N)}\big)$.
4. Determine component N° of j, using the above mentioned ratio-criterion applied at the time t_i; $1 \leqslant j \leqslant N$. Then $T_0(j) = T_0^{(i)} = t_i$.
5. Recalculate the dependence of availability of the system on time with the first inspection times of all respective components in all time points $T_0^{(k)} = t_k$; $k = 1, \ldots, i$.
6. $i = i + 1$, $i \leqslant N$, return to step 2.

Using the outlined procedure we obtain the complete vector $\mathbf{T}_0$. However, in some cases it is not necessary to use all of the components of the vector. That is just in the case when repeating inspections of one or more system elements is a more effective way to satisfy the given availability limit A_0. Consequently in such situations, it is necessary to select those elements that will be maintained. Final decision about system interventions depends particularly on the cost matrix according to which it must be made.

5. Cost optimization technique

The genetic algorithms were developed by John Holland in 1967 at the Michigan University (see details in [8]). They take as a starting point the principle of species reproduction, which consists in selecting the best adapted individuals among a population and in procreating by a crossing process. To implement a genetic algorithm, one first creates an initial population with given size (number of individuals). Then by a process of selection similar to that of the natural selection, which is defined by an adaptation function, one selects the individuals who will be crossed. These individuals are represented by a chromosome in the Genetic Algorithm. Then a current population is created by crossing the individuals. We start again the same step with the current population. The passage from a current population to another is called generation. For each generation, one retains the individual for which

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the optimization criterion is best satisfied. It is important to build well the chromosome representing the individual of the population.

In this work, the adopted coding method is a *direct coding* where each chromosome is composed of sub-chromosomes. The genes of these chromosomes are the durations for each component between two maintenance interventions in each subsystem. It is a real number randomly selected in the interval $[LB, UB]$ according to an uniform distribution. A chromosome comprises as many sub-chromosomes as there are subsystems in the studied system.

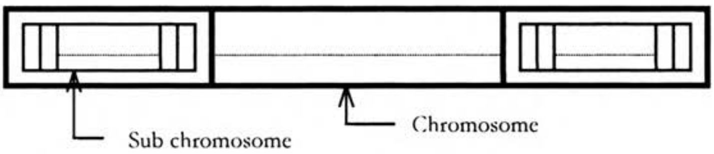

FIGURE 3. Solution Direct Coding.

The reproduction process consists in selecting the population elements ready to reproduce by evaluating their force using an adaptation function (G) which is the objective function in the case of maximization without constraint. Thus one selects N individuals which will be two by two crossed to give birth, the two individuals who will be elements of the population of the current generation. For the minimization problem, the adaptation function used is G', $G' = Cst - G$, G being the adaptation function for maximization problem (constant Cst is selected so that the quantity G' remains always positive).

The crossing is the genetic operator which allows, starting from two individuals of a given generation, to create one or more other individuals of the following generation. The purpose of the crossing is thus to create new individuals, i.e. to brew the old population. One crosses two by two the candidates with the reproduction probability of $p_c \geqslant 0.7$. Figure 4 below shows how two sub-chromosomes are crossed with the operator at a point. The circles are put on genes to make the difference between the two parents who will be crossed.

The purpose of the mutation is to bring a diversity among genes. It avoids falling in a local optimum. The mutation, contrary to the crossing should not be too often applied, because good genes in the individuals might be lost. The mutation probability adopted is $P_m \leqslant 0.07$.

For each type of coding, an operator of change was defined. It consists in modifying a part of gene in a random way. This modification consists in permuting between two genes chosen randomly for each selected chromosome.

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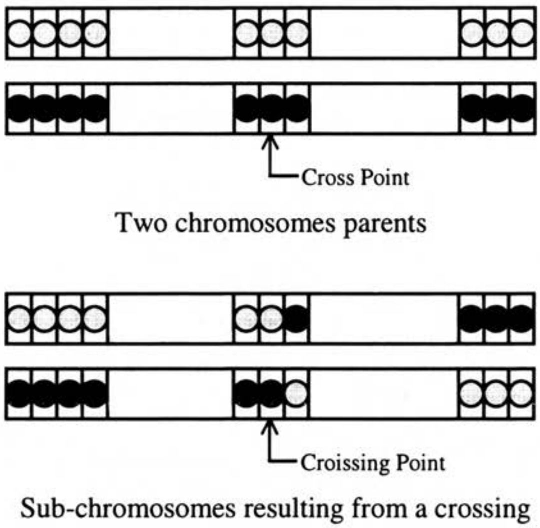

FIGURE 4. Principle of two elementary chains crossing.

The general structure of the genetic algorithm according to Davis [9] is as follows:

1. Initialization of the chromosomes population.
2. Evaluation of each chromosome of the population.
3. Creation of new chromosomes using crossing and mutation operators.
4. Evaluation of the new chromosomes.
5. Removing of the not selected chromosomes.

The last step is the final stop test (one considers for example the iteration count, or the no improvement of the solution value on a certain iteration count...). If the test is not verified, go to 3.

6. Results and illustrative data

Consider a series-parallel system consisting of four parallel subsystems connected in series (Fig. 2). The system contains 11 components with different reliability and PM cost data. The reliability of each component is defined by an exponential distribution with the failure rate $\lambda_0 = 1/\mathrm{MTTF}$ presented in Table 1. This table also contains the PM cost $C(e(i,k))$ of each component. The basic data are exponential modification of those of Weibull data presented in Ref. [3].

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TABLE 1. Parameters of system components.

No. of Component	Probability distribution	MTTF $= 1/\lambda_0$ [years]	$C(e(i,k))$
1	EXP	12.059	4.1
2	EXP	12.059	4.1
3	EXP	12.2062	4.1
4	EXP	2.014	5.5
5	EXP	66.6667	14.2
6	EXP	191.5197	19.0
7	EXP	63.5146	6.5
8	EXP	438.5965	6.2
9	EXP	176.0426	5.4
10	EXP	13.9802	14
11	EXP	167.484	14

6.1. Calculations for the mission time $T_M = 25$ years

6.1.1. Availability constraint $A(t) \geqslant A_0$; $A_0 = 0.9$

Optimal solution:

$$T_P(3) = 8.79, \quad T_P(5) = 12.64, \quad T_P(6) = 10.83, \quad T_P(11) = 10.82,$$

$$T_0(3) = 18, \quad T_0(5) = 14, \quad T_0(6) = 9.5, \quad T_0(11) = 12,$$

$$C_{PM} = 84.3.$$

Components N° 1, 2, 4, 7, 8, 9, 10 are not maintained.

Dependence of availability on time is demonstrated in Fig. 5.

6.2. Calculations for the long term mission time $T_M = 50$ years

6.2.1. Availability constraint $A(t) \geqslant A_0$; $A_0 = 0.9$

Optimal solution:

$$T_P(3) = 9.466, \quad T_P(5) = 8.554, \quad T_P(6) = 10.301, \quad T_P(8) = 12.767, \quad T_P(11) = 10.573,$$

$$T_0(3) = 18, \quad T_0(5) = 14, \quad T_0(6) = 9.5, \quad T_0(8) = 20, \quad T_0(11) = 12,$$

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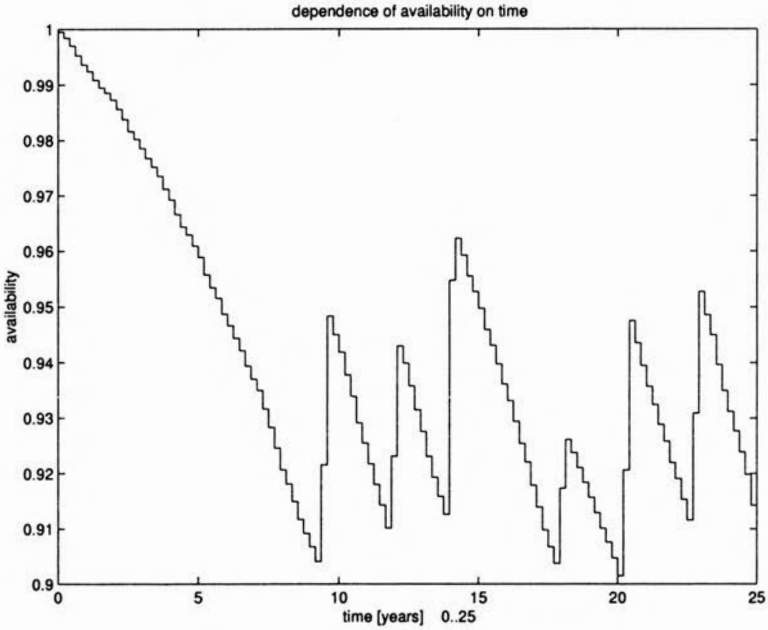

FIGURE 5. Dependence of availability on time under availability constraint $A(t) \geqslant 0.9$.

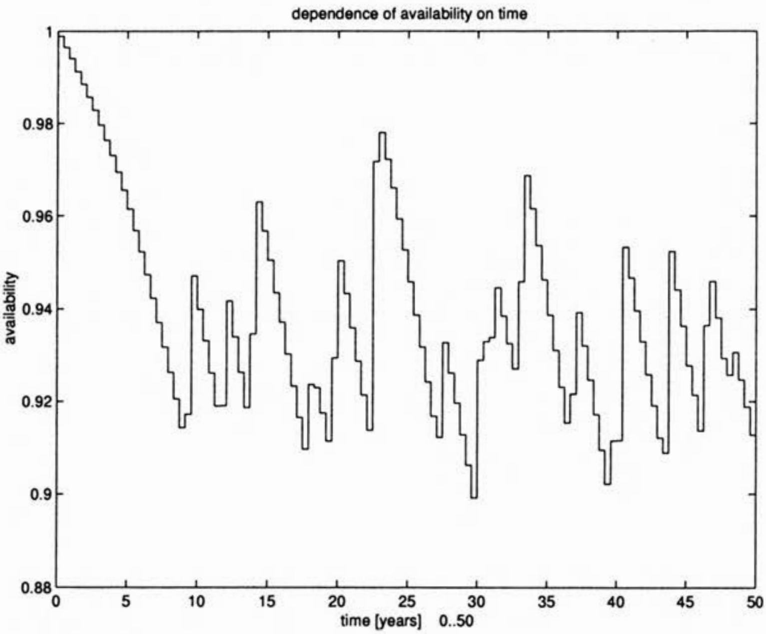

FIGURE 6. Dependence of availability on time under availability constraint $A(t) \geqslant 0.9$.

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$$C_{PM} = 238.0.$$

Components N° 1,2,4,7,9,10 are not maintained.

Dependence of availability on time is demonstrated in Fig. 6.

6.2.2. Availability constraint $A(t) \geqslant A_0$; $A_0 = 0.8$

Optimal solution:

$$T_P(1) = 24.344,\quad T_P(2) = 16.507,\quad T_P(3) = 23.136,\quad T_P(5) = 19.773,$$
$$T_P(6) = 20.504,\quad T_P(8) \doteq 22.107,\quad T_P(11) = 20.508,$$

$$T_0(1) = 26.5,\quad T_0(2) = 26.5,\quad T_0(3) = 21,\quad T_0(5) = 32.5,$$
$$T_0(6) = 15,\quad T_0(8) = 32.5,\quad T_0(11) = 18,$$

$$C_{PM} = 238.0.$$

Components N° 4,7,9,10 are not maintained.

Dependence of availability on time is demonstrated in Fig. 7.

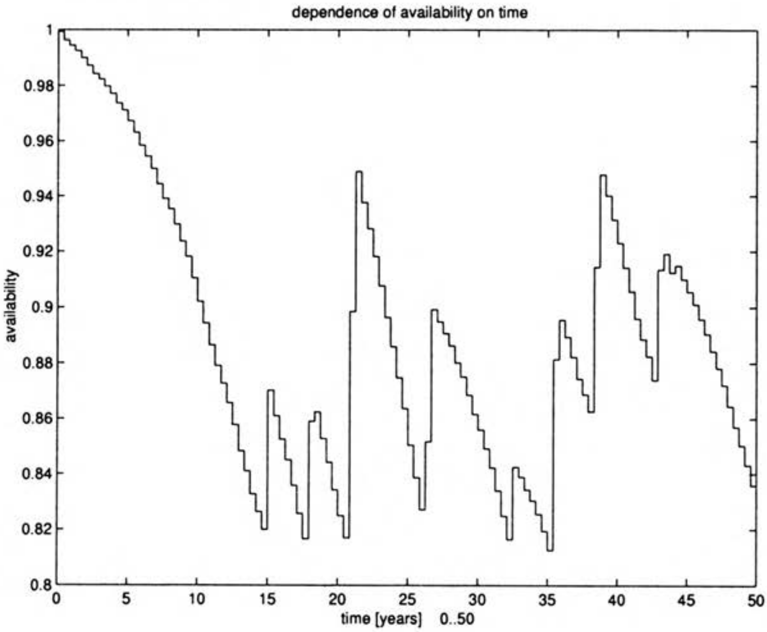

FIGURE 7. Dependence of availability on time under availability constraint $A(t) \geqslant 0.8$.

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7. Conclusions

We completed the results for two levels of reliability constraint, i.e. 0.9 and 0.8 and two levels of mission time, 25 and 50 years.

In Fig. 5 we can see that even if our GA program found the four optimal periods $T_P(3)$, $T_P(5)$, $T_P(6)$ and $T_P(11)$, in fact only the last two will be realised. The first two periods are so long that in its first application exceed the required mission time 25 years.

Highly reliable calculations (with the constraint $A_0 = 0.9$) are characterized by big sensitivity of obtained minimal cost results C_{PM} to small changes of the limit A_0. For example, if we change the limit to the value of $A_0 = 0.87$ (50 years calculations), we obtain new solution of the vector $\mathbf{T}_P$ with the optimal cost $C_{PM} = 203.4$ (compare with the value 238) , and for the value of $A_0 = 0.85$ we obtain $C_{PM} = 185.3$.

All availability calculations (dependencies of $A(t)$) are computed with the relative error of 5%, and by the confidence level of 90%. The use of the newly developed GA program required the need of an automatic ending of the simulation program. Ending on accuracy is, to our opinion, an optimal way to stop the program. Consequently, the possibility of "ending on accuracy" was built in into the simulation program. In fact that means that simulations are ended just in the case when a minimal number of successful trials is reached at the time point of worst availability value (worst during the mission time). The minimal number depends of course on the given accuracy and can be obtained according to the method presented in [4].

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