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Reliability of deteriorated steel structures

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The paper reviews recent results on the reliability of deteriorated structures subjected to corrosion and fatigue. The models to quantify the effect of inspection and maintenance repair work on the structural reliability are discussed too. First the reliability assessment of corroded plate and pipelines is presented. Then a fatigue reliability assessment of maintained structures is presented. Finally numerical examples of failure behaviour and reliability analyses are also shown.

1. Introduction

The widespread use of new materials with higher strength capacity, and the utilisation of refined structural analyses in the design processes have made possible an optimisation of the weight of structures. However the production of more economical structures with less redundancy has made them more prone to the effect of the strength degradation phenomena such as fatigue and corrosion. Thus attention has moved from design considerations to monitor more closely the effect of maintenance actions.

The simplified procedures adopted in many cases are sufficient for fatigue screening of the details of the structure by pointing out the potentially fatigue critical ones, but they cannot be realistically applied for the design of new types of structures. For realistic fatigue life assessment these procedures need to be further calibrated and the database of fatigue behaviour of structural details should be further developed and integrated with the results of both tests and theoretical investigations.

Reliability based methods have been gaining acceptance as being proper tools for design decisions and for assessing the level of safety in the structures. They have also been shown appropriate tools for structural maintenance planning.

The scope of the present work is to review the application of reliability methods to assess the steel structures resulting from the degrading effect of corrosion and fatigue.

2. Deterioration mechanisms

Deterioration of structural materials evolves from a gradual loss of quality, corresponding to micro-structural changes, through a deterioration initiation, corresponding to the beginning of macro-structural effects, up to a real propagation until a threshold level of damage is exceeded. Several degradation mechanisms may also combine with each other.

Material scientists tend to study the microscopic aspects while from a structural point of view attention has been given mainly to macroscopic aspects. In many situations deterioration is difficult to describe and in this case Markov and semi-Markov models have been adopted for describing general degradation as a transition from one degraded state to another.

However, corrosion and fatigue are specific physical phenomena that can be described by appropriate mechanical laws, which can be extended to represent the uncertainty in the model and the basic variables.

2.1. Corrosion models

Two main corrosion mechanisms are generally present in steel plates. One is a general wastage that is reflected in a generalised decrease of plate thickness. Another mechanism is pitting, which consists of much localised corrosion with very deep holes appearing in the plate. This is an interesting problem from a probabilistic point of view, which is given by Scarf and Laycock (1994), but leads to a different failure mode. In fact, the pitting can lead to leakage but in general, because it is much localised, it does not affect the in-plane stress distribution in plates and thus the compressive strength of the plates is not affected.

The conventional models of corrosion assume a constant corrosion rate, leading to a linear relationship between the material lost and time. Experimental evidence of corrosion reported by various authors shows that a non-linear model is more appropriate.

Southwell et al. (1979) have observed that the wastage thickness increases non-linearly in a period of 2-5 years of exposure, but afterwards it becomes relatively constant. This means that after a period of initial non-linear corrosion, the oxidised material that is produced remains on the surface of the plate and does not allow the continued contact of the plate surface with the corrosive environment thereby stopping corrosions.

Melchers and Ahammed (1994) suggested a steady state model for corrosion wastage thickness and Yamamoto (1998) has presented results of the analysis of corrosion wastage, exhibiting the non-linear dependence of time and a tendency of levelling-off.

The reference to these earlier works shows that the non-linear time dependence of corrosion rate has been already identified. The model proposed by Guedes Soares and Garbatov (1998a) in addition to being a more flexible alternative to the previous ones also generalises the concept by including an early phase with corrosion protected surface. In fact the model proposed has free parameters to be adjusted to the data of specific situations.

The time dependent model of corrosion degradation may be separated into three phases as was suggested by Guedes Soares and Garbatov (1998a). In the first one there is in fact no corrosion because the protection of the metal surface works properly. The first stage depends of many factors and statistics show that in ships it varies in the range of 1.5-5.5 years (Emi et al., 1994). The second phase is initiated when the corrosion protection is damaged and corresponds really to the existence of corrosion, which decreases the thickness of the plate. This process was observed to last a period of around 4-5 years in typical ship plating. The third phase corresponds to a stop in the corrosion process and the corrosion rate becomes zero. Corroded material stays on the plate surface, protecting it from the contact with the corrosive environment and the corrosion process stops. Cleaning the surface or any involuntary action that removes that surface material originates the new start of the non-linear corrosion growth process.

Other authors have considered the problem of quantifying the corrosion rates (Tanker Structure Cooperative Forum, 1992) as well as of different types of ships (Huang et al., 1997).

It has to be noted that the effectiveness of the strength steels in reducing the weight of structures is generally limited by the present requirements of different design codes regarding minimum scantlings. These requirements assure adequate strength with respect to buckling to ensure elements subjected to compressive loads and sufficient stiffness to the elements with service deformability limits. On the other hand, adding account of a realistic corrosion margin to the value of the design thickness of the elements, assures that the hull girder and the single elements have enough strength for an adequate period of service, even considering severe corrosion conditions. Moreover, a given value of corrosion wastage has degraded effects, both to buckling stiffness and strength, larger for high strength steel elements, which have reduced scantlings, than on the mild steel ones.

2.2. Fracture Mechanics Approach based on LFM

The fracture mechanics approach is based on the criteria of initiation of fracture instability in the presence of a crack. It covers the prediction of the life of component in the crack propagation phase.

A crack in a body can be deformed in different ways. Irwin observed that there are three independent kinematic movements of the upper and lower crack surfaces with respect to each other, categorised as opening mode I, shearing mode II and tearing mode III. Here the we consider model I that describes the two crack surfaces pulled apart in the y-direction, but where the deformation are symmetric about x-z and x-y planes.

An expression, which is based on the analysis of experimental data of crack growth, was derived by Paris and Erdogan (1963), which takes into account the stress intensity factor:

$$\frac{da}{dN} = C\Delta K^m \tag{2.1}$$

where C and m are material properties and ΔK is the stress intensity factor.

For many studies the expression of Forman et al. (1967) is preferable since it describes the transition from a stable crack growth to the unstable damage and takes into account the asymmetry of the loading.

It has to be pointed out that for calculation of the stress intensity factor it is required to know the configuration of the component with a crack, the trajectory of the crack, the loading and material properties. For a welded joint the geometry function can be expressed as suggested by Yazdani and Albrecht (1990) by $Y(a) = F_e\, F_s\, F_w\, F_g$, where F_e, F_s, F_w and F_g are the crack shape, free surface, finite width and the stress gradient correction factor.

Gurney (1979) has presented an empirical stress intensity factor equation for a surface crack for a plate as a function of a parametric angle, the crack depth, crack length, plate thickness and the plate under condition of tension and bending load were presented by Yazdani and Albrecht (1990). Several analytical models have also been tested by Hou and Lawrence (1993) and Pang (1993) to simulate experimental results on crack propagation.

The Paris-Erdogan equation is recommended to use, where possible. It predicts the number of cycles required to propagate from an initial crack depth to a final crack size. Since the initial crack size must be known, this method is mainly of importance for the assessment imperfections together with consideration of shape imperfections as was demonstrated by Maddox (1993).

The complete fatigue life T_f is equal to the sum of the time of the crack initiation T_i and the time of crack propagation until its critical size T_p.

3. Reliability of corroded plates under compression

3.1. Compressive strength of plate

The most important parameter that governs the compressive strength of plate elements is the slenderness:

$$\lambda = \frac{b}{h}\sqrt{\frac{\sigma_y}{E}} \tag{3.1}$$

where b and h are the plate breadth and thickness respectively, σ_y is the yield stress and E is the Young's modulus of the material. This parameter is included in the classical formula due to Bryan for the critical elastic buckling stress σ_{cr} of infinitely long thin elastic plate with simply supported edges:

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{4\pi^2}{12\,(1-\nu^2)}\frac{1}{\lambda^2} \tag{3.2}$$

where ν is the Poisson ratio.

This expression was extended by Faulkner (1975) adding one extra term and fitting it to data of ultimate plate strength leading to:

$$\frac{\sigma_u}{\sigma_y} \equiv \phi_b = \frac{a_1}{\lambda} - \frac{a_2}{\lambda^2}, \qquad \lambda \geqslant 1.0, \tag{3.3}$$

where the constants a_1 and a_2 are given $a_1 = 2.0$ and $a_2 = 1.0$ for simple supports and $a_1 = 2.5$ and $a_2 = 1.56$ for clamped supports. This equation accounts implicitly for average levels of initial deflection and it can be complemented with others that dealt explicitly with the effect of residual stresses.

Guedes Soares (1988a) has extended that formulation by deriving a strength assessment expression for the compressive strength of plate elements under uniaxial load, which deals explicitly with initial defects as:

$$\phi = (\phi_b B_b)\,(R_r B_r)\,(R_\delta B_{r\delta}) \tag{3.4}$$

where ϕ_b is given by Eq. (3.3), B_b, B_r and $B_{r\delta}$ are the model uncertainties factors and R_r and R_δ are the strength reduction factors which are due to the presence of weld-induced residual stresses and initial distortions respectively.

3.2. Failure criteria

Applying the non-linear general corrosion wastage allows two-failure criteria for a plate element to be formulated (Guedes Soares and Garbatov, 1999a). Failure is considered to be caused by reaching a specified value of the thickness reduction or the plate ultimate strength. Having a reduction of the original thickness does not mean that the plate will reach the level of ultimate strength and the opposite is also true.

The ultimate strength does depend not only on the thickness but also on many other factors. It is assumed that these two failure modes are independent. Using the limiting thickness as an additional failure criteria accounts for the fact that in addition to ultimate strength there may exist other design or operational considerations that need to be taken care of. The relative ultimate strength $P_{U_r}(t)$ and the relative corrosion depth $C_r(t)$ may be described as follows:

$$P_{U_r}(t) = \frac{P_u(t) - P_T}{P_u(0) - P_T}, \quad P_{U_r}(t) \in [1,\, 0]\,, \tag{3.5}$$

$$C_r(t) = 1 - \frac{A(t)}{A(0)}, \quad C_r(t) \in [0,\, 1]\,, \tag{3.6}$$

where $P_u(t)$ is the ultimate collapse force as a function of time, P_T is the total axial loading, $A(t)$ is the plate cross sectional area as a function of time.

The cross section area is presented as $A(t) = b\,h(t)$ and after transformations, the cross section area and ultimate strength as a function of a relative corrosion depth are given by:

$$A(t) = A(0) - b\,d(t) \equiv A(0) - A(0)\,C_r(t), \tag{3.7}$$

$$P_u(t) = \phi(t)\,\sigma_y A(0) - \phi(t)\,\sigma_y A(0)\,C_r(t) \tag{3.8}$$

Taking into account Eqs. (3.7) and (3.8) for the safe domain may be written as follows:

$$\left|\begin{array}{l} \dfrac{P_{U_r}(t)}{\left(\dfrac{k_\phi\, k_l\, P_u(0) - (1-k_\phi)\, P_T}{P_u(t) - P_T}\right)} + C_r(t) = 1, \\ C_r(t) \leqslant k_c, \end{array}\right. \tag{3.9}$$

where k_c is the corrosion coefficient and k_l measures the relative initial reserve strength; moreover

$$k_\phi(t) = \frac{\phi(t)}{\phi(0)}, \qquad k_l = \frac{P_u(0) - P_T}{P_u(0)},$$

$$P_{U_r}(t) = k_u(t)\,[1 - C_r(t)]\,, \quad C_r(t) \leqslant k_c,$$

$$k_u = \frac{k_\phi\, k_l\, P_u(0) - (1-k_\phi)\, P_T}{P_u(t) - P_T}.$$

3.3. Reliability of a plate

Reliability assessment requires the modelling of random variables and the assessment of their statistical properties. A variable that governs the resistance of the plate to buckling collapse is its slenderness, which is dependent on the thickness. The thickness also influences other collapse mechanisms and may itself be used as a design variable. It is considered that general corrosion will occur, affecting plate thickness and localised pit corrosion will not be accounted for since its effect on plate collapse is not meaningful.

Guedes Soares (1988b) showed that the plate collapse is insensitive to the uncertainties in plate dimensions except for the thickness, which has some influence. In the presence of corrosion, the uncertainty in plate thickness will depend mainly on that effect and this can be modelled by a random variable that dominates the uncertainty in plate thickness resulting from the manufacturing process.

For a plate element, the cross section area A is given by the product of its breadth b by thickness h. The plate thickness starts from an initial value h_0 and decreases with time by a corrosion reduction $d(t)$:

$$h(t) = h_0 - d(t). \tag{3.10}$$

According to Eq. (3.3) the ultimate compressive force (P_u), which is a function of time, may be written as follows:

$$P_u\,(h_0,\, b,\, \sigma_y,\, E,\, d_\infty,\, t) = \phi\,(h_0,\, b, \sigma_y,\, E,\, d_\infty, t)\; \sigma_y\, A(t), \tag{3.11}$$

which is a product of the ultimate strength by the yield stress of the material and the net section area. For the sake of simplicity, this formulation is not considering the effect of initial distortion and residual stresses described in the previous section.

In many cases, plates are protected with anti-corrosion paints, which are effective during limited period of time. This implies that corrosion will only start at the random point in time in which the protection ceases to be effective. Therefore, the effect of corrosion just described is conditional on the initiation of the corrosion process.

The limit state for buckling collapse failure is defined as:

$$P_T > P_u(t) = \varsigma(t), \tag{3.12}$$

where P_T is the total axial loading, $P_u(t)$ is the ultimate collapse force, which has the threshold limit $\varsigma(t)$.

A failure will occur if Eq. (3.12) is fulfilled and the probability of the load exceeding $\varsigma(t)$ during the period of the time $[0, T]$ is:

$$P_f(T) = 1 - \exp\left[-\int_0^T \nu\left[\varsigma(t)\right] dt\right] \tag{3.13}$$

where $\nu\left[\varsigma(t)\right]$ is the mean upcrossing rate of the threshold $\varsigma(t)$.

If one assumes that the plate studied is an element of a deck of a ship, it will be loaded by wave-induced compressive loads which are often assumed to follow the Weibull distribution and in this case we have:

$$\nu\left[\varsigma(t)\right] = \nu_0 \exp\left[-\left(\frac{\varsigma(t) - \bar{P}_T}{\gamma_L}\right)^{\alpha_L}\right] \tag{3.14}$$

where α_L and γ_L are the Weibull parameters and $\bar{P}_T$ is mean value of total compressive force (Naess, 1984).

Considering that during the plate lifetime h_0, b, σ_y, E, d_∞ and P_T are random variables and also that the threshold limit can be described by Eq. (3.14), the probability of failure $P_f(t)$ is just a conditional probability and may be obtained from:

$$P_f(t) = 1 - \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty f_{\bar{P}_T}(\bar{P}_T)\, f_{d_\infty}(d_\infty)\, f_E(E)\, f_{\sigma_y}(\sigma_y)\, f_b(b)\, f_{h_0}(h_0) \cdot \exp\left[-\nu(t|h_0, b, \sigma_y, E, d_\infty, \bar{P}_T)\right] dh_0\, db\, d\sigma_y\, dE\, dd_\infty\, d\bar{P}_T. \tag{3.15}$$

The time dependent reliability after corrosion has started, implying that the corrosion rate is larger than zero, is denoted as reliability after loss of effectiveness of anticorrosion coating $R_a(t)$:

$$R_a(t) = 1 - P_f(t), \qquad t > \tau_c, \tag{3.16}$$

where τ_c is the time of failure of the coating protection and therefore, the reliability before $R_b(t)$ corrosion starts to decrease thickness is modelled in a similar manner for the time when $t \leqslant \tau_c$.

Since the time to loss of effectiveness of the anticorrosion coating is a random variable, the reliability is conditional on the probability of coating time failure, $R(t|\tau_c)$. Therefore, the unconditional reliability of a plate is given by:

$$R(t) = \int_0^t R(t|\tau_c)\, f_{\tau_c}(\tau_c)\, d\tau_c\,, \qquad t \in [0, T]. \tag{3.17}$$

The non-occurrence of failure due to corrosion may be expressed as a sum of two terms, which describe the contribution before corrosion action, and after if has started (Guedes Soares and Garbatov, 1999b):

$$P_{nf} = \{E_{1,1}(t < \tau_c) \cap E_{1,2}(t < \tau_c)\} \cup \{E_{2,1}(t \geqslant \tau_c) \cap E_{2,2}(t \geqslant \tau_c) \cap E_{2,3}(t \leqslant T - \tau_c)\}, \tag{3.18}$$

where T is the lifetime.

The first event is sub-divided into two sub-cases which represent probability of that corrosion will not occur during the time t, $(E_{1,1})$, where $t \in [0,\, T]$ with probability $[1 - F_{\tau_c}(t)]$ and the probability of non-failure $R_b(t)$ under the condition that corrosion does not appear before end of coating life $(E_{1,2})$.

The second case includes three conditions. The first one $(E_{2,1})$ is when corrosion occurs at time τ_c where $\tau_c \in [0,\, t]$ with probability $f_{\tau_c}(\tau_c)\, d\tau_c$. The second sub-case $(E_{2,2})$ represents the probability of non-failure before corrosion starts decreasing the thickness of element at time τ_c and $\tau_c \in [0,\, t]$, $R_b(\tau_c)$. The third condition $(E_{2,3})$ gives probability of non-failure under condition that the corrosion appears $R_a(t - \tau_c)$.

The total reliability $R(t)$ is given by the reliability of the plate without corrosion plus the reliability of the plate with corrosion:

$$R(t) = [1 - F_{t\tau_c}(t)]\, R_b(t) + \int_0^t R_b(\tau_c)\, R_a(t - \tau_c)\, f_{\tau_c}(\tau_c)\, d\tau_c, \qquad t \in [0,\, T]. \tag{3.19}$$

The first term of this equation represents the probability that no corrosion appears and that failure does not occur in time $[0,\, t]$. The second term represents the probability of non-failure under the condition that the corrosion is initiated.

3.4. Modelling of corrosion inspection and repair

The state of general corrosion in a plate is assessed by measuring the plate thickness at several points. There are two sources of uncertainty in this procedure. One results from the precision of the measuring instrument and the other from sampling variability. Measurements are made at few points of a panel and they are considered to be representative of the thickness in the whole plate.

Inspections are routinely made for structures in service and they may result in detection or no detection of a plate that has a mean thickness smaller than the acceptable value that is a fraction k of the original mean value (Guedes Soares and Garbatov, 1996).

$$h(t) \leqslant k\,h(0), \quad k \leqslant 1.0, \quad k = 1 - k_c. \tag{3.20}$$

The uncertainty of the method of detection is considered small in this work, and it is assumed not to influence plate detection. It is assumed that an element will be inspected every four years. It is further assumed that the method of inspection is such that if a plate is smaller than a limit value then it will be replaced and after replacement, their thickness will be h, which is their original value.

The reliability is computed for each period between inspections by using Eq. (3.19) being a function of the repairing time t_r:

$$R(t) = [1 - F_{\tau_c}(t - t_r)]\, R_b(t - t_r) + \int_0^{t-t_r} R_b(\tau_c) R_a(t - t_r - \tau_c)\, f_{\tau_c}(\tau_c)\, d\tau_c$$
$$\text{for} \quad T_{j-1} \leqslant t < T_j. \tag{3.21}$$

At each repair operation, a value of t_r must be determined. This value is substituted in Eq. (3.21) and the reliability can be evaluated for the next interval between inspections. In Eq. (3.21) the first term denotes the probability of non-failure when corrosion is not initiated in the service interval before $[T_{j-1}, T_j]$ and the second term denotes the probability of non-failure when corrosion appears in the service interval $[T_{j-1}, T_j]$.

3.5. Numerical example

The proposed approach was applied to assess the reliability of a plate element with corrosion. The plate is simply supported on the edges. The breadth is 0.6 m, and the breadth of tension zone of welding residual stresses $\eta = 5$. Detailed information on the probabilistic models adopted to describe the random variables is presented in Guedes Soares and Garbatov (1996).

The reliability of a plate as given by Eq. 3.21 is shown in Fig. 1. The restoring action is provided when the thickness of the element is less than 75% of original thickness independently of the time between inspections.

The reliability of the plate element after repair will be equal to the initial value for the new plate. However, in the present example, the plate replacement was made at 13 and 26 years and this brought the reliability to its initial value.

The formulation presented here can be used to assess the effect of different parameters such as the repair criteria, the time interval between inspections,

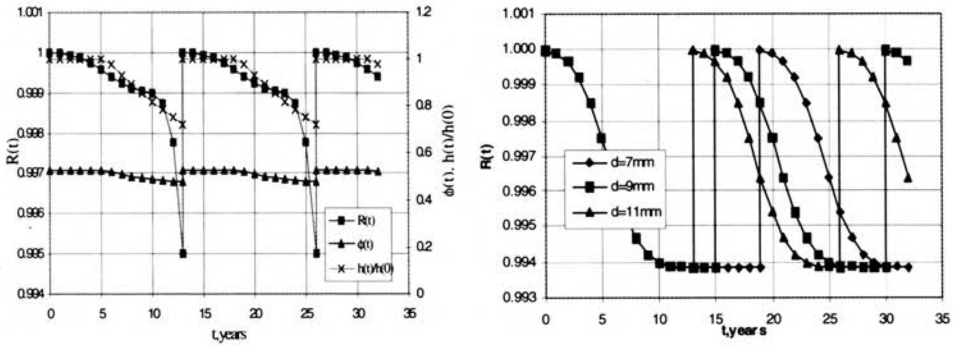

FIGURE 1. Reliability of a plate element (left) and a pipeline (right).

the corrosion rate, the coating life and the initial thickness as was done in Guedes Soares and Garbatov (1998b), for the case of a linear decrease of thickness as a result from a constant corrosion rate.

4. Reliability of corroded pipelines

4.1. Strength of corroded pipelines

Predicting the failure of damaged oil and gas pipeline is essential for the determination of design tolerances, remaining strength assessment and for the effective maintenance. The existing and widely accepted criterion used in the assessment of corrosion damage is ASME (1991).

The failure equation incorporated into the B32G code, were derived based upon a fracture mechanism calibrated by extensive testing. By limiting the maximum hoop stress to 10% higher than the specified minimum yield stress the following failure equations were defined, (Batte et al., 1997):

$$p_f = \frac{\sigma_f}{D\,l\,2h}\left[\frac{1-\dfrac{2}{3}\dfrac{d}{h}}{1-\dfrac{2}{3}\dfrac{d}{h}\dfrac{1}{M}}\right] \qquad \text{for} \quad \sqrt{0.8\left(\frac{l}{Dh}\right)^2} \leqslant 4.0, \tag{4.1}$$

$$p_f = \frac{\sigma_f}{D\,l\,2h}\left[1-\frac{d}{h}\right] \qquad \text{for} \quad \sqrt{0.8\left(\frac{l}{Dh}\right)^2} > 4.0, \tag{4.2}$$

where D is the outside diameter of the pipe, h is the nominal wall thickness, d is the corrosion wastage, M is a coefficient, σ_f is the yield flow stress and p_f is the failure pressure and $0.1 < d/h < 0.8$.

Recently a number of researchers have suggested modifications to the failure equations to reduce conservatism or scatter as for example Kiefner and Vieth (1989, 1990).

Comparing the current test data, finite element analyses and published test data, Batte et al. (1997) have proposed a method that describes a reserve strength factor RSF, which relates the defect behaviour to the failure of a pipe as:

$$RSF = \frac{p_f}{p_0} = \frac{1 - \dfrac{d}{h}}{1 - \dfrac{d}{h}\dfrac{1}{M_l}}, \qquad M_l = 1 + a\left(\frac{l}{\sqrt{Dh}}\right)^b, \tag{4.3}$$

where p_f is the failure pressure of the defective pipe and p_0 is the failure pressure of plain pipe whilst a and b are constant multiplier and constant exponent, respectively.

According to Eq. (4.3) the ultimate or failure pressure (P_u), that is a function of time, may be written (Guedes Soares and Garbatov, 1998a):

$$p_u\,(h_0,\, D,\, \sigma_y, l,\, d_\infty,\, t) = RSF\big(h_0,\, D, \sigma_y,\, l,\, d_\infty, h(t)\big)\, k_\sigma\, \sigma_y \tag{4.4}$$

is the product of the reserve strength factor by the yield stress of the material and the coefficient k_σ.

4.2. Reliability of pipelines

The limit state for failure is defined as follows:

$$P_T > P_u(t) = \zeta(t), \tag{4.5}$$

where P_T is the total loading, $P_u(t)$ is the ultimate collapse load, which has the threshold limit $\varsigma(t)$.

There will be a failure if the above equation is fulfilled and the probability of the load exceeding $\varsigma(t)$ during the period of the time $[0, T]$.

Considering that during the lifetime for a plate $x_1 = h_0$, $x_2 = b$, $x_3 = \sigma_y$, $x_4 = E$, $x_5 = d_\infty$ for a pipeline $x_1 = h_0$, $x_2 = D$, $x_3 = \sigma_y$, $x_4 = l$, $x_5 = d_\infty$ and P_T are random variables the probability of failure $P_f(t)$ is just an unconditional probability.

Since the time to the loss of effectiveness of the anticorrosion coating is a random variable, the reliability is conditional on the probability of coating failure, $R\,(t\,|\tau_c)$.

The total reliability $R(t)$ is given by the reliability of the pipeline without corrosion plus the reliability of the plate with corrosion.

The proposed approach was applied to assess the reliability of a pipeline with a non-linear corrosion wastage (see Fig. 1, right). Detailed information on the probabilistic models adopted to describe the random variables is presented in Guedes Soares and Garbatov (1998a).

5. Fatigue reliability

Fatigue design is one of the most complicated problems in engineering, especially for the structural components subjected to stochastic loading. A practical method of predicting component reliability under fatigue failure mode is generally difficult, not only because of the difficulty in describing the mechanics of fatigue failure, but also because of the complexity of the reliability model.

In fact both the environmental loads acting on a structural component and the corresponding stress vary with time and can be modelled as stochastic processes.

In general, structural components are subjected to a complicated pattern of randomly varying load amplitudes and frequencies. Also the strength degrades with time due to the development of fatigue cracks and of corrosion and discrete changes occur at repairs.

To predict crack propagation, the fatigue life Paris-Erdogan equation has been adopted. The limit state for cracked component may be defined as follows:

$$a_{cr} - a(t) \leqslant 0, \tag{5.1}$$

where a_{cr} is the critical crack size, and $a(t)$ is the crack size, which depends on time.

Failure will take a place if the stress time history upcrosses the limit $\zeta(t)$, which can be written as follows:

$$\varsigma(t) = (\sigma_1 - b_1\,\nu_0\,\sigma_1\,t)^{\frac{1}{m-2}}, \tag{5.2}$$

where

$$\sigma_1 = \frac{K_{cr}}{Y\sqrt{\pi a_0}}, \qquad b_1 = \frac{m-2}{2} Y^m \pi^{\frac{m}{2}}\, a_0^{\frac{m}{2}-1}\, C\,\Delta\sigma^m.$$

The probability that the stress would exceed $\varsigma(t)$ during the period of the time $[0,\ T]$, i.e., the probability of failure, is (Corotis et al., 1972):

$$P_f(T) = 1 - \exp\left[-\int_0^T \nu\,[\varsigma(t)]\,dt\right] \tag{5.3}$$

where $\nu\,[\varsigma(t)]$ is the mean upcrossing rate of the threshold $\varsigma(t)$.

It has been shown that an exponential distribution is an adequate approximation for the long-term distribution of wave-induced stress ranges, (Guedes

Soares and Moan, 1991). Therefore this distribution was adopted and making $\alpha_L = 1$, finally, the probability of failure after crack initiation is written as follows:

$$P_a(T) = 1 - \exp\left\{-\frac{\gamma_L}{b_1\sigma_1}\left[\exp\left(\frac{\sigma_1 - b_1\sigma_1\nu_0 T}{\gamma_L}\right) - \exp\left(-\frac{\sigma_1}{\gamma_L}\right)\right]\right\}. \quad (5.4)$$

This approximation is correct for a stationary process but the long-term distribution considered here is non-stationary. Therefore, this result is only an approximation.

Making t=0 we define the zero upcrossing rate before cracks have initiated and the probability of failure before crack initiation is given:

$$P_b(T) = 1 - \exp\left\{-\exp\left[-\frac{\sigma_1}{\gamma_L}\right]\nu_0 T\right\} \quad (5.5)$$

and the probability of failure after crack initiation may be described in a similar manner.

Since the time to crack initiation is a random variable, the conditional reliability of the component with a crack may be expressed under the condition that the crack has initiated at time t and $f_{t_i}(t)$ is the probability density function of the time to crack initiation. If $R(T)$ is the reliability in the service life [0, T] the following equation can be written:

$$R(T) = [1 - F_{t_i}(T)]\, R_b(T) + \int_0^T f_{t_i}(t) R_b(t) R_a\,(T - t)\, dt. \quad (5.6)$$

The first term is the probability of non-failure under the condition that the crack is not initiated during the service time [0, T]. The second term is the probability of non-failure under condition that the crack is initiated during the service time [0, T].

6. Reliability of maintained structures

6.1. Modelling of crack inspections

Inspections are routinely made for structures in service and they may result in the detection or non-detection of the cracks. The size of a detected crack is measured by a non-destructive method. For welded structures, cracks are generally assumed to be present after fabrication. Fatigue damage is expressed with a fatigue crack size that increases with time.

A purpose of periodic inspections is to detect the fatigue cracks. It is assumed that if a fatigue crack is detected, it is repaired to its original condition (a_0), which increases the reliability of the ship's structure. However the formulation could be easily extended to account for the effectiveness of the repair actions described by Ma and Bea (1995).

The inspection quality depends on detecting the crack and quantifying its size. In principle each detection technique will have a limit size of detection $a_{d,0}$, under which cracks will not be detected, i.e., in general for detection it is necessary that $a(t) \geqslant a_{d,0}$.

The inspection procedure is not a deterministic one because of measuring inaccuracies and therefore the inspection capability may be described by the probability of detection (Packman et al., 1969):

$$P_d(t) = \begin{cases} 1 - \exp\left[-\frac{a(t)-a_{d,0}}{\lambda_d}\right], & \text{if } a(t) > a_{d,0}, \\ 0, & \text{if } a(t) \leqslant a_{d,0}. \end{cases} \tag{6.1}$$

The inspection quality is characterised by the parameter λ_d which has the values between 0 and ∞. The smallest limit corresponds to a perfect inspection and when $\lambda_d = \infty$ the plate has not been inspected.

Probability of non-detection of a crack size a using the ultrasonic inspection technique may be described by a log-normal distribution as is suggested by Harrison et al. (1967):

Based on the experimental results of Packman et al. (1969) a model, which gives a reasonable approximation when applying the liquid penetration method and is an upper bound for the ultrasonic method was proposed by Yang and Trapp (1974)

Another theoretical distribution was derived by Ichikawa (1985), based on the weakest link model and the linear elastic fracture mechanics.

It should be noted that the real measurements of a crack size usually involve a large scatter on the probability of detection, which reflects the different mathematical models describing $P_d(a)$. This uncertainty affects very much the reliability as is shown by Madsen (1985). Rudlin and Wolstenholme (1992) showed that the choice of an inspection method could have a noticeable effect on the probability of detection. Detailed analysis of the problem can be found in Delmar and Sorensen (1992).

After repair, the crack is assumed to start propagation after the time of crack initiation, which for simplicity is taken here as a percentage of the time to crack propagation to the critical size.

6.2. Reliability of cracked component with maintenance

The structural component can belong to one of the two groups. The first one is when the component is repaired at the time of the last inspection T_j.

The second one includes the situation when the component is not repaired at the time of the last inspection T_j.

The reliability in the service interval $[T_j, T_{j+1}]$ of the component with a crack that is repaired at the time of the last inspection $R^r_{l,j+1}(t)$ can be written as follows:

$$R^r_{l,j+1}(t) = \left[1 - F_{t_i}\left(T_{j+1} - T_j\right)\right] R_b\left(T_{j+1} - T_j\right) + \int_{T_j}^{T_{j+1}} f_{t_i}\left(t - T_j\right) R_b\left(t - T_j\right) R_a\left(T_{j+1} - t\right) dt. \quad (6.2)$$

This method was also applied for economical criteria of estimation of fatigue life of ship structure (Ivanov et al., 1991) and for jack-up platforms by Jensen and Pedersen (1992).

Using the third axiom of probability theory, (e.g. Lewis, 1987) the probability of non-failure in the time interval $[T_r, T_{j+1}]$ can be obtained as:

$$P\left\{C_{T_r,T_j} \cap C_{T_j,T_{j+1}}\right\} = P\left\{C_{T_r,T_j} \middle| C_{T_j,T_{j+1}}\right\} P\left\{C_{T_j,T_{j+1}}\right\}, \quad (6.3)$$

where $C_{T_r,t}$ is the probability of non-failure in the time interval $[T_r, t]$, where $t \in [T_j, T_{j+1}]$ and C_{T_r,T_j} are the probabilities of non-failure in the time interval $[T_r, T_j]$. The case $C_{T_r,t}$ implies the probability of non-failure in all intervals up to T_j, C_{T_r,T_j}. The probability of non-failure in the time interval $[T_j, t]$, where $t \in [T_j, T_{j+1}]$ is written as $C_{T_j,t}$.

Equation (6.3) may be rewritten as a definition of the conditional probability of non-failure in the service interval $[T_j, t]$, as follows:

$$R^{nr}_{l,j+1}(t) = P\left\{C_{T_j,t}\right\} = \frac{P\left\{C_{T_r,T_j} \cap C_{T_j,t}\right\}}{P\left\{C_{T_r,T_j} \middle| C_{T_j,t}\right\}}. \quad (6.4)$$

Equations (6.2) and (6.4) include all possible cases of the cracked component, including the states of the crack initiation, the crack propagation, the crack detection, and the crack repair. At the last state, the component is repaired to its original condition and the crack life starts again.

During the time of inspection if the crack is detected and the component is repaired the crack starts initiation and propagation until next detection and repairing.

The formulation presented here is applied for the determination of the fatigue reliability of cracked component based on fracture mechanics.

The stress range resulting from load is 244 MPa, which corresponds to the 10^{-8} probability level. The material constants are taken as $C = 1.7 \cdot 10^{-11}$

and $m = 3$. The initial crack size is $a_0 = 0.4\,\mathrm{mm}$ and the critical size (a_{cr}) is equal to $0.8\,\mathrm{m}$. It is assumed that the time to crack initiation is ten percent from the time of crack propagation until the critical size. The parameters of the Weibull distribution of time to crack initiation are taken as $\alpha_t = 2$, $\beta_t = 20$ and the geometry parameter $Y = 1.12$.

The reliability assessment considers that during the time for inspection the component is observed. If the crack is detected then it is perfectly repaired. The basic results are produced by detectable crack size $a_d = 0.05\,\mathrm{m}$ and the time between inspections $\Delta t_0 = 4\,\mathrm{years}$.

The results of calculation for the crack propagation and reliability as a function of time are presented in Fig. 2. It is clearly shown that after reaching the level of crack length of 0.05m the component is perfectly repaired and the fatigue crack life begins again and at the same time reliability is restored to one as in the case of new component.

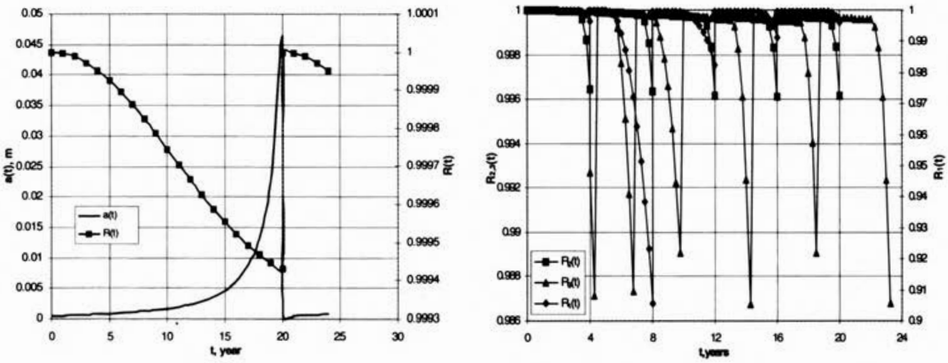

FIGURE 2. Crack length and reliability of component(left) and reliability of structural assemblies for different inspection polices (right).

Different assumptions were made about loading and material properties that are not essential to the method but are needed for the example.

This formulation was used for constant time intervals between inspections but it can also be used to determine the time of repair when adopting a criterion of repairing only at a specified reliability level. Alternatively, keeping the same inspection interval, the requirement of a minimum reliability level is related with the detection limit of the inspection method.

6.3. Reliability of cracked structural assemblies

The present approach deals with the application of reliability based techniques to system reliability of cracked ship structural assemblies subjected to the process of crack growth and repair. The fatigue reliability is predicted by a time variant formulation and the effects of maintenance actions in the reliability assessment are shown.

Consider a series system with $i = 1, 2, \dots, m$ series components. The safety margin of the i^{th} component is denoted as M_i, which is given by:

$$M_i = a(t) - a_{cr} > 0. \tag{6.5}$$

The system probability of failure can be defined by:

$$P_{sys} = P\left(\bigcup_{i=1}^{m} M_i \leqslant 0\right). \tag{6.6}$$

The additional information can be included for the events of no crack detection ND_j, the events of crack detection D_j and the events of repair, L_j which are presented by:

$$ND_j = a(t)_j - a_{d,j} > 0, \quad \text{where} \quad j \in [1,\, m_{nd}]\,, \tag{6.7}$$

$$D_j = a(t)_j - a_{d,j} \leqslant 0, \quad \text{where} \quad j \in [1,\, m_d]\,, \tag{6.8}$$

$$L_j = a(t)_j - a_{l,j} \leqslant 0, \quad \text{where} \quad j \in [1,\, m_l]\,, \tag{6.9}$$

where $a(t)_j$, $a_{d,j}$ and $a_{l,j}$ are respectively the crack size at certain point of time in the component j, the detectable crack size and the crack size which is repaired. The number of events related with the crack non-detection, crack detection and repair are m_{nd}, m_d and m_l, respectively.

The system probability of the repaired structure is expressed as:

$$P_{sys|ND,D,L} = P\left(M_{sys} \leqslant 0 \,\middle|\, \bigcap_{j=1}^{m_{nd}} ND_j \leqslant 0 \bigcap_{j=1}^{m_d} D_j = 0 \bigcap_{j=1}^{m_l} L_j = 0\right). \tag{6.10}$$

For the sake of simplicity, the following assumptions are made. All members that are considered to have a potential crack are checked by a visual inspection method and if crack damage is found in an element then it is replaced by a perfect one. The progressive fatigue failure of the structural system is considered. If n denotes the number of elements that have a repair at time T_j, and m is the total number of elements, then the reliability of the structure can be expressed as follows:

$$R_{j+1}(t) = \prod_{l=1}^{n} R^{r}_{l,j+1}(t) \prod_{k=n+1}^{m} R^{nr}_{k,j+1}(t), \tag{6.11}$$

where $t \in [T_j,\, T_{j+1}]$ and no correlation is considered.

6.4. Design for minimum repair

One of the main advantages of probabilistic structural design based on the first principles is that the reliability of the structure with respect to the various possible modes of failure may be examined and quantified. Fatigue is one of the main time dependent factors affecting structural condition, which is in fact the main object here.

Reliability requirements may vary greatly depending on different parameters, describing the loading condition and structure itself or they may sometimes be set by the designer. In other situations, the requirements may be imposed by the owner of the ship and in some instances, third parties such as government agencies may play a large role. For any structure there are likely to be trade-offs between minimum reliability level and repair works that have to be done, initial crack size which is related with quality of manufacturing of the structure, time between inspections, detectable crack size, etc.

The approach presented here may be used as a decision tool for different reliability based maintenance policies. One is if the interval between inspections is known to be $\Delta t_{j+1} = \Delta t_0$ and the detection limit of the method of inspection is $a_{d,j+1} = a_d$, for $j \in [0, n]$, and n is total number of inspections. Then the reliability can be calculated:

$$R_{j+1}(t) = g_1 (\Delta t_0, a_d), \quad \text{where} \quad t \in [T_j, T_{j+1}]. \tag{6.12}$$

The second case is when there is a minimum acceptable value of the reliability level $R_{\min}$ and the detectable crack size $a_{d,j+1} = a_d$, for $j \in [0, n]$, is fixed. Then the time interval between each inspection Δt_{j+1} can be calculated:

$$\begin{gathered} \Delta t_{j+1} = g_2 [R, a_d], \quad \text{if} \quad \{[R_{j+1}(t) \leqslant R_{\min}] \cap [a_{d,j+1} = a_d]\}, \\ \text{where} \quad \Delta t_{j+1} \in [\Delta t_{\min}, \Delta t_{\max}]. \end{gathered} \tag{6.13}$$

The third possible application is fixing the time interval between inspections, $\Delta t_{j+1} = \Delta t_0$ and the minimum level of the reliability $R_{\min}$, the calculated limit for the detectable crack size is:

$$\begin{gathered} a_{d,j+1} = g_3 (R, \Delta t), \quad \text{if} \quad \{[R \geqslant R_{\min}] \cap [\Delta t = \Delta t_0]\}, \\ \text{where} \quad a_{d,j+1} \in [a_{d,\min}, a_{d,\max}]. \end{gathered} \tag{6.14}$$

This implies the choice of the method of inspection that is able to accomplish it.

The formulation presented here is applied for the determination of the fatigue life of a structural assembly. The potential cracks are considered on the intersections of the longitudinal stiffener with the transverse frames. The material constants are taken as $C = 1.7 \cdot 10^{-11}$ and $m = 3$. The initial crack size is $a_0 = 0.5\,\mathrm{mm}$ and the critical size a_{cr} is equal of the height of the stiffeners. It is assumed, that the time to crack initiation is ten percent from the time of crack propagation until the critical size. The parameters of the Weibull distribution of time to crack initiation are taken as $\alpha_t = 2$, $\beta_t = 20$ and the geometry parameter is $Y = 1.12$ which implies the same manufacturing conditions for all elements.

The reliability assessment considers that during the time for inspection all elements are observed. If the crack is detected in an element, then it is perfectly repaired. The basic results are produced by $a_d = 0.02\,\mathrm{m}$, $\Delta t_0 = 4\,\mathrm{years}$.

The example (Fig. 3 (right)) shows the differences between the three cases, which were formulated here.

To examine how initial crack size, detectable crack size and time between inspections affects the reliability and the number of replaced elements, nine cases are studied. The variation of the parameters has been chosen because the intervals between inspection (and repair) are fixed for ships. This means that sometimes more critical situations may not result in a lower reliability. In fact, it may only indicate that the repair operations are made earlier.

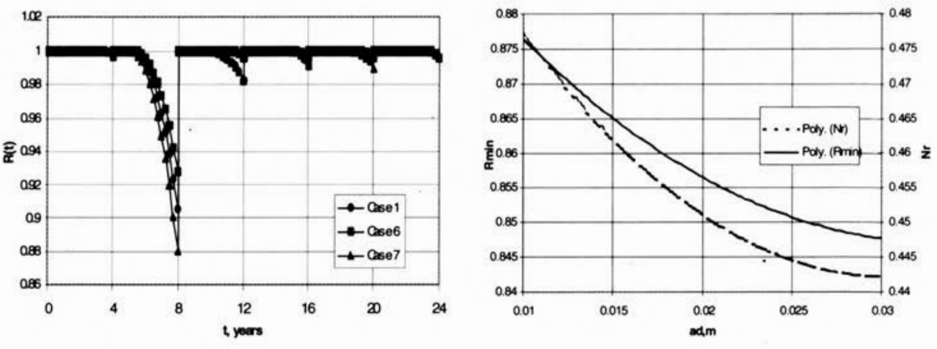

FIGURE 3. Reliability, relative number of replaced elements (Nr) and minimum reliability (Rmin) as a function of detectable crack size.

Figure 3 shows the reliability as a function of time for the various values of detectable crack size. The right hand side of Fig. 3 presents the polynomial approximations to the minimum reliability and relative number of replaced elements. Having a small value of detectable crack size gives opportunity for keeping reliability on a relatively higher level. Having small a_d requires expensive techniques of inspection, which increases the repair work (number of detected and replaced elements) so as to increase the minimum reliability, as can be seen in Fig. 3.

Initial crack size is related to the quality of manufacturing and it has great importance for fatigue reliability. As can be seen from Fig. 4, increasing initial crack size leads to large reduction of minimum reliability, which is achieved during the ship life with a large number of repaired elements shown on the right side in the figure.

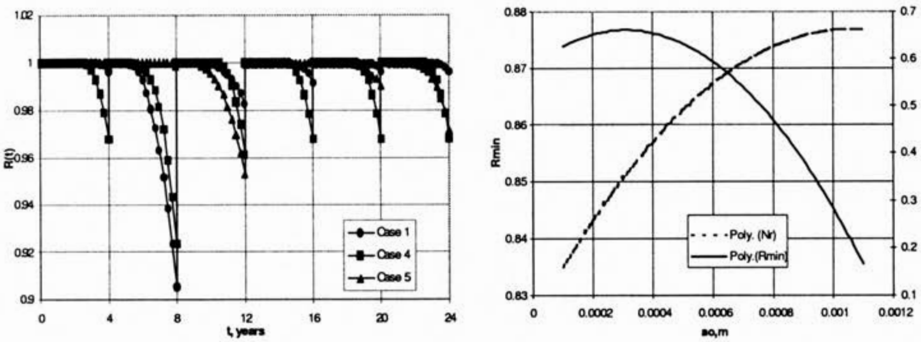

FIGURE 4. Reliability (R), relative number of replaced elements (Nr) and minimum reliability (Rmin) as a function of initial crack size.

Figure 5 shows the reliability as a function of time between inspections, which varies between 4 and 6 years. Decreasing the time interval will not allow the reliability to decrease as much as in the longer period. This can also be seen from Fig. 5, which shows that when having a smaller interval

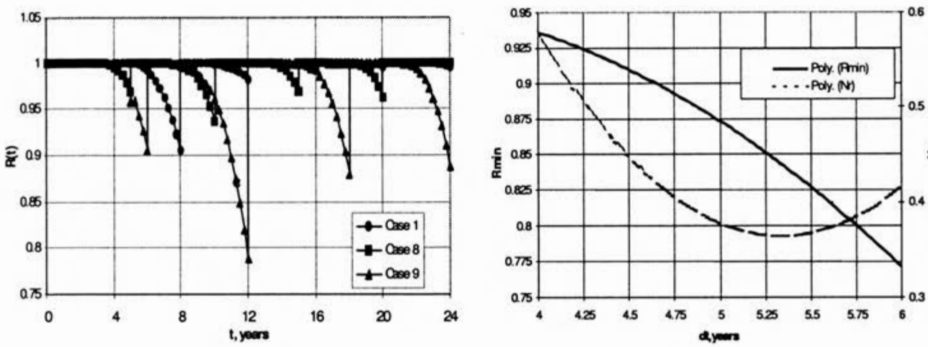

FIGURE 5. Reliability (R), relative number of replaced elements (Nr) and minimum reliability (Rmin) for various times between inspection.

between successive inspections a consistently higher minimum reliability level is achieved during ship operation. It is clear from the results that the time interval between inspections of approximately five years gives the minimum necessity of replaced elements.

Figure 6 shows that the minimum number of replaced elements may be achieved by keeping minimum reliability around 0.8- 0.85 which is also related with a certain time between inspections and with detected crack size. The mentioned value is valid only for the present example and the specific input data that have been used.

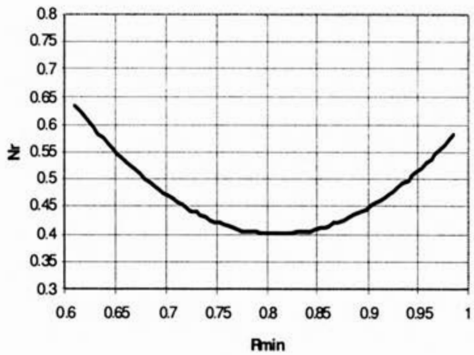

FIGURE 6. Relative number of replaced elements (Nr) as a function of minimum reliability (Rmin).

The formulation presented here can be used for reliability-based maintenance planning; in particular how to vary the inspection interval in order to vary the maximum of repaired elements keeping the same level of reliability. Alternatively, for fixed inspection intervals it is shown how the initial crack size, detectable crack size, time interval between inspection, average period of the sea state requirements of the minimum reliability and the number of repaired elements vary along the ship life. It is demonstrated that keeping a certain value of minimum reliability is reflected in the to minimum rate of replaced elements.

7. Cost and reliability based strategies

7.1. Cost analysis

Structures are designed in such a way that the appearance of fatigue failures cannot be avoided, implying the need for inspections during their life. Their maintenance has to be planned from an economic point of view so as to minimize maintenance costs but satisfying a minimum reliability level. A method presented to quantify the repair costs resulting of different reliability-based maintenance strategies. As an application of this approach a shell structure is analysed and the influence of different parameter with respect to the repair cost is also studied here.

In maintenance planning, optimisation can be achieved by appropriate selection of inspection interval, inspection methods, repair quality and so on. The interval between inspections depends on economical considerations, on expected losses due to maintenance downtime and on the requirements of classification societies. In general they require fixed intervals between inspections but the owners may decide on shorter intervals based on economical considerations.

The total expected cost for repair of the structure in the interval between succeeding inspections is classified into:

- inspection cost for each component of the structures (C_i);
- repair cost of the damages detected in the inspection (C_r);
- loss due to a member failure (C_f).

The above-mentioned cost items are grouped into two classes as losses due to damage (crack growth) ($\overline{C}_1$) and losses that are independent of the damage extent ($\overline{C}_2$), but are required for the maintenance process, leading to the cost function:

$$\overline{C}_{j+1}(t) = \sum_{i=1}^{n} \frac{\overline{C}_{1,i}(t|a)}{(1+r)^{t-T_{r,i}}} + \frac{\overline{C}_2}{(1+r)^{t-T_j}}, \qquad t \in [T_j,\, T_{j+1}], \tag{7.1}$$

where $\overline{C}_{j+1}(t)$ is the total repair cost, n is the number of repaired elements during the j^{th} inspection and $t \in [T_j,\, T_{j+1}]$, $T_{r,i}$ is the time for the last repair of i^{th} component and a is crack length and r is the interest rate.

Costs are defined in time and they are the result of the repairs that have been done along the life of the structures. The cost function is not continuous and has a local peak at the time of repair. For analysing the costs so as to look for an optimal planning of repair, the concept of intensity of repair cost $\overline{I}_{C,j}(t)$ is introduced as the cost per time unit:

$$\overline{I}_{C,j+1}(t) = \sum_{i=1}^{n} \frac{\overline{C}_{1,i}(t|a)}{(t-T_{r,i})\,(1+r)^{t-T_{r,i}}} + \frac{\overline{C}_2}{(t-T_j)\,(1+r)^{t-T_j}}, \qquad t \in [T_j,\, T_{j+1}], \tag{7.2}$$

where $T_{r,i}$ is the time of the last repair of the considered component.

The expected costs and intensity of costs are defined by:

$$E\left[\overline{C}_{j+1}(t)\right] = \sum_{i=1}^{n} \frac{\overline{C}_{1,i}\left(t - T_{r,i}\,|a\right)}{\left(1+r\right)^{t-T_{r,i}}} P_{d,i}\left[a\left(t - T_{r,i}\right)\right] + \frac{\overline{C}_2}{\left(1+r\right)^{t-T_j}}\left(1 - P_{sys,j+1}(t)\right), \qquad t \in \left[T_j,\, T_{j+1}\right], \tag{7.3}$$

$$E\left[\overline{I}_{C,j+1}(t)\right] = \sum_{i=1}^{n} \frac{\overline{C}_{1,i}\left(t - T_{r,i}|\, a\right)}{\left(t - T_{r,i}\right)\left(1+r\right)^{t-T_{r,i}}} P_{d,i}\left[a\left(t - T_{r,i}\right)\right] + \frac{\overline{C}_2}{\left(t - T_j\right)\left(1+r\right)^{t-T_j}}\left(1 - P_{sys,j+1}(t)\right), \quad t \in \left[T_j,\, T_{j+1}\right], \tag{7.4}$$

where $P_{d,i}$ is probability of crack detection and $P_{sys,j}$ is system probability of failure.

Since the repair cost depends very much of the shipyard where maintenance is being performed, the repair cost and the intensity of repair cost are normalized by the maximum expected cost at the time of repair during the total life of the structure:

$$\begin{aligned} E\left[C_{j+1}(t)\right] &= \frac{1}{\overline{C}_{\max}} E\left[\overline{C}_{j+1}(t)\right], \\ E\left[I_{C,j+1}(t)\right] &= \frac{1}{\overline{I}_{C,\max}} E\left[\overline{I}_{C,j+1}(t)\right]. \end{aligned} \tag{7.5}$$

The calculation of the repair costs is based on the standards for work content in the specific shipyard is considered. The repair costs include the cost of steel, electrodes, the work of shops and all yard expenditures that are defined on the basis of statistical analysis carried out for previous repairs in the shipyard. For the present analysis the ratio between the costs that dependent on repair work with respect to unit length damage to the additional costs independent of the damage is taken as 0.2. The ratio was calculated considering that for the steel cost, cost of electrodes, and the cost of remanufacturing of the replaced element, the work for dismounting and mounting and the additional cost in each maintenance operation. The ratio just defined varies from shipyard to shipyard and is important for maintenance planning.

At the beginning of the fatigue life of the structural component the intensity of repair cost is decreasing because no repair cost is normally incurred in the initial years. Expected repair costs are continuously increasing and the increase can be such that there will be a time in which the intensity of repair cost achieves its minimum and starts increasing. If at this time the reliability satisfies the minimum requirements then the economical criteria will determine the time for inspection.

If however, the intensity of repair work constantly decreases and after same time it becomes relatively constant as a function of time then the criteria that will determine the time for inspection will be the minimum acceptable reliability level.

In order to examine the applicability of life cycle cost minimization for decision making about the time of inspection numerical analyses are carried out by assuming the structure that was presented in previous section.

The repair ratio that was discussed in the above section was calculated as 0.2 considering that for a steel cost of 4000 units/ton, the cost of electrodes is 15 units/kg, the cost of remanufacturing of the replaced element is 3000 units/ton, the work for dismounting and mounting is 5 units/hour and the additional cost in each maintenance operation is 1000 units.

7.2. Pure economical criterion for inspection planning

This criterion defines the time interval between inspections based only on the consideration of optimal repair cost or minimal intensity of repair cost and constant detectable crack length without limitations about minimum reliability level and inspection interval:

$$R_{j+1}(t) = g_1\left(a_d, I_{C,\min}\right), \quad \text{where} \quad t \in [T_j, T_{j+1}]. \tag{7.6}$$

The results of calculations are presented in Figs. 7 and 8.

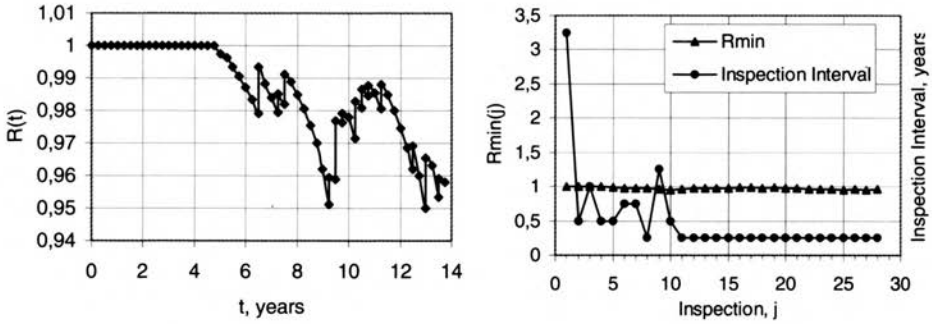

FIGURE 7. Reliability, minimum reliability and inspection interval for the pure economical strategy.

As can be seen from Fig. 7 the time interval of inspection varies between 0.25 and 3.25 years. After the 10^{th} inspection the time interval between inspections becomes a constant of 0.25 year, which makes such strategy non-realistic. Even with such a short interval and so frequent inspections and

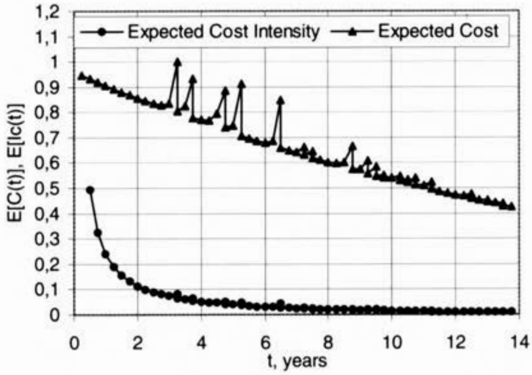

FIGURE 8. Normalized expected repair cost and expected intensity of repair cost for the pure economical strategy.

repairs the minimum reliability (reliability at the moment of inspection) is reduced. All repairs were done at the optimal condition for the intensity of repair cost. The economical criterion dominates.

7.3. Economic criterion with minimum time between inspection

To have more realistic strategy the example presented above were recalculated applying pure economical criterion with constraint of 1 year as the minimum time interval between repairs, ($\Delta t \geqslant 1$ year). The results for the reliability, normalized expected repair cost, expected intensity of repair cost, minimum reliability and inspection intervals are presented in Figs. 9 and 10.

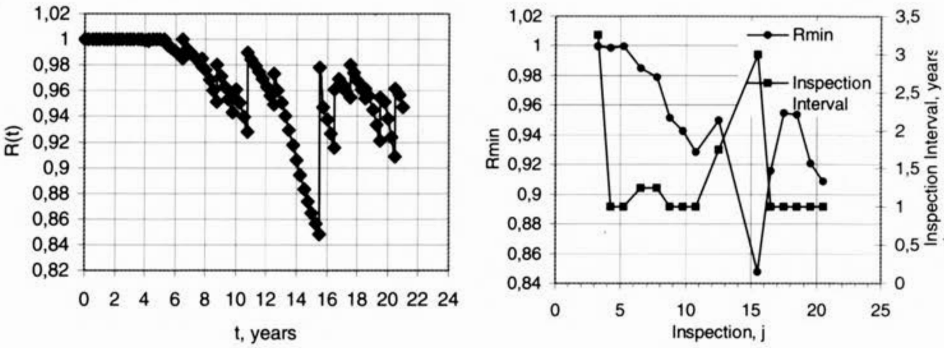

FIGURE 9. Reliability, minimum reliability and inspection interval for the pure economical strategy, ($\Delta t \geqslant 1$ year).

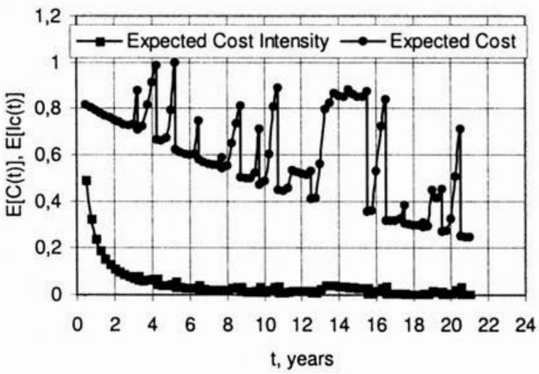

FIGURE 10. Normalized expected repair cost and expected intensity of repair cost for the pure economical strategy, ($\Delta t \geqslant 1$ year).

It can be noticed from the above figures that the expected cost is a decreasing time function as it was in the case without restriction of the time interval between inspections, but minimum reliability at the inspection is much below the one in the previous case. It is clear from the example that introducing some constrains about the time interval between inspections will increase the expected cost in the time.

7.4. Pure exploitation criterion – constant time interval of inspection

From an exploitation point of view it is preferable to have constant time intervals of inspection because in this case the ship operation can be planned with a large time horizon. The assumption adopted here is that the interval between inspections is known to be $\Delta t_{j+1} = \Delta t_0$ and the detection limit of the method of inspection is $a_{d,j+1} = a_d$, for $j \in [0, n]$ and n is the total number of inspections and the reliability can then be calculated by:

$$R_{j+1}(t) = g_2(\Delta t_0, a_d), \quad \text{where} \quad t \in [T_j, T_{j+1}]. \tag{7.7}$$

The results demonstrate the worst behaviour of the structures with respect to the minimum reliability level that varies between 0.905 and 0.9964. It can be seen from Fig. 12 that the intensity of repair cost does not satisfy the condition for optimal repair. During the repair more intensive works were performed reflecting in higher repair costs.

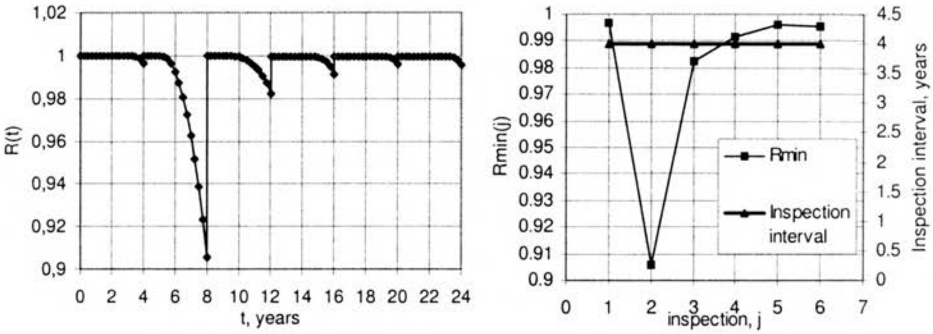

FIGURE 11. Reliability, minimum reliability and inspection interval for the pure exploitation strategy.

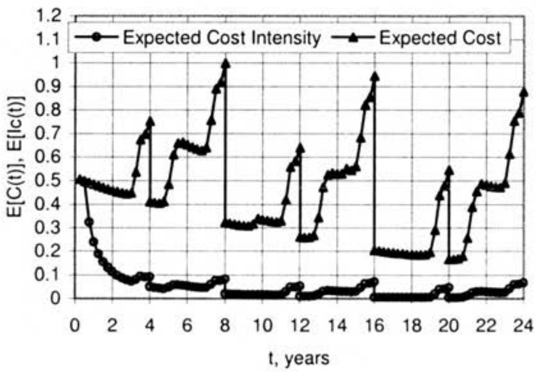

FIGURE 12. Normalized expected repair cost and expected intensity of repair cost for the pure exploitation strategy.

7.5. Pure reliability criterion achieved by variation of time interval of inspection

In this case the time interval of inspection is planned based on the fact that a minimum acceptable value of the reliability level $R_{\min}$ and the detectable crack size $a_{d,j+1} = a_d$, for $j \in [0, n]$, are fixed. The time interval between each inspection Δt_{j+1} can be calculated from:

$$\begin{gathered}\Delta t_{j+1} = g_3\left[R,\, a_d\right], \quad \text{if} \quad \left\{\left[R_{j+1}(t) \leqslant R_{\min}\right] \cap \left[a_{d,j+1} = a_d\right]\right\}, \\ \text{where} \quad \Delta t_{j+1} \in \left[\Delta t_{\min},\, \Delta t_{\max}\right]\end{gathered} \tag{7.8}$$

The results show better solution with respect to minimum reliability. The satisfied reliability condition is compensated by irregular inspection interval and systematical increase of the repair cost. The inspection interval varies between 1.75 and 4.5 years.

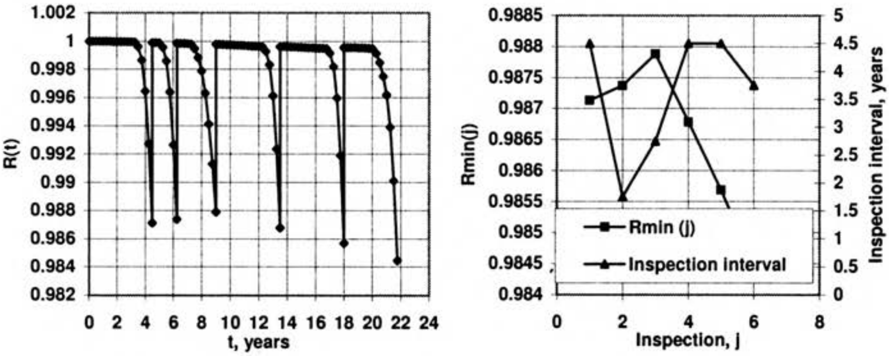

FIGURE 13. Reliability, minimum reliability and inspection interval for the reliability criterion, varying the interval of inspections.

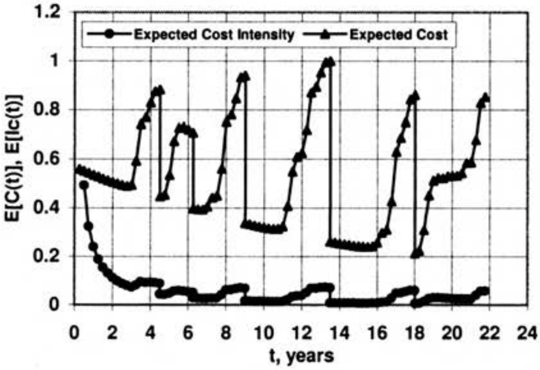

FIGURE 14. Normalized expected repair cost and expected intensity of repair cost for the reliability criterion, varying the interval of inspections.

7.6. Reliability criterion achieved by variation of inspection quality

The time interval between inspections, $\Delta t_{j+1} = \Delta t_0$ and the minimum level of the reliability $R_{\min}$ are fixed; the detectable crack size is the parameter that varies for satisfying the reliability conditions:

$$a_{d,j+1} = g_4\left(R, \Delta t\right), \quad \text{if} \quad \left\{ \left[R \geqslant R_{\min}\right] \cap \left[\Delta t = \Delta t_0\right] \right\}, \qquad \text{where} \quad a_{d,j+1} \in \left[a_{d,\min},\, a_{d,\max}\right]. \tag{7.9}$$

The inspection intervals are performed regularly keeping reliability level over the permissible level. The results show that repair cost is systematically constant at the time of inspection and at the same time the intensity of repair cost is not in optimal condition.

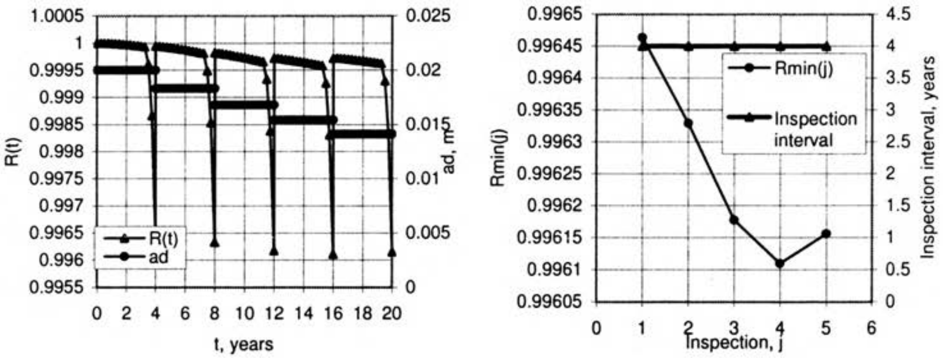

FIGURE 15. Reliability, minimum reliability and inspection interval for the reliability criterion varying inspection quality and detectable crack size as a function of time.

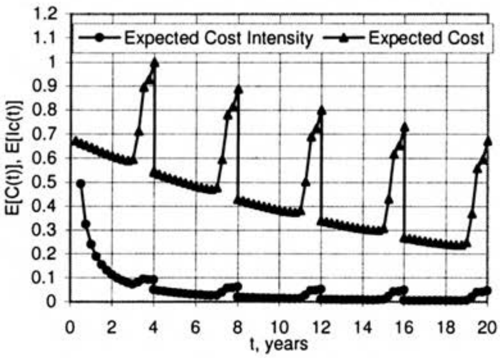

FIGURE 16. Normalized expected repair cost and expected intensity of repair cost for the reliability criterion varying inspection quality.

8. Conclusion

The appearance of deterioration failures cannot be avoided, and inspections and maintenance need to be planned at the design stage. This has to be based on economic criteria to minimize lifecycle maintenance costs but satisfying a minimum reliability level and as a result of that cost and reliability must be considered in the strategies for maintenance planning of floating structures.

Different approaches were proposed here to quantify the repair costs resulting in different reliability-based maintenance strategies. As an application of those approaches a shell structure was analysed and the influence of different strategies with respect to the repair cost were studied here.

It was recognized that for some strategies the dominating factors for the decision of inspection is the restriction of the time interval between inspection or the minimum reliability level and in those cases the minimum repair cost is not the governing factor. To achieve minimum repair cost a reliability criterion based on the variation of inspection quality has to be applied.

The simulated strategies for inspection planning pointed out that the application of repair cost optimisation for floating structures involves many uncertainties related to the costs of the shipyard that would perform the repair.

Evaluation of alternative criteria for maintenance planning in terms of the intensity of repair cost and availability of the platform to perform its intended functions is difficult to be achieved. The minimum required intensity or repair cost could be related with requirements of Classification Societies. However, this does not mean that the maintenance effort is optimised. When maintenance is intensified the costs associated with inspection and repairs increase. The search for a maintenance effort that will optimise the use of available resources should consider the lifetime cost of the solution.

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