Structural Monitoring

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Application of modal analysis for monitoring and diagnostics of mechanical structures

T. UHL

University of Mining and Metallurgy
Department of Mechanical Engineering and Robotics
al. Mickiewicza 30, 30-059 Kraków, Poland

In this paper author has presented innovative approach to diagnostics of mechanical structure. This approach is based on idea of model based diagnostics. In proposed diagnostic procedure the central role plays modal model identified during structure operation. The method is analytically formulated and software tools for performing in-operation modal analysis is shown. Based on in-operation modal analysis results, during the first step of diagnostics procedure the increasing vibration level due to structural resonances or forced vibrations are recognized. If the first case is detected the modal model can be used for damage localization but in the second one inverse identification problem should be formulated. Additional experiment is required to identify loads vector which loads structure during operation. This information is needed to localized damage in the structure. State of the arts of operating loads identification is shown. The application of genetic algorithms for this task solution is studied. Some case studies are presented for all presented procedures.

1. Introduction

The aim of diagnostic procedure of mechanical structures is to determine its technical state and if is not satisfactory to localise faults and assess its dimension. One of the most commonly use diagnostics of rotating machinery is vibrodiagnostics that consist of vibration measurements during operation and estimation of chosen state symptom. As a state symptom some signal estimates like mean value of amplitude, spectrum, RMS, p-p value can be used. The vibration can be measured in continuous way by monitoring system or can be measured periodically with given state dependent period. When during testing vibration level exceeds given standards or manufacturer recommendations reasons should be identify. Proposed diagnostic procedure diagrammatically is shown in Fig. 1.

There are two basic reasons of increasing vibration levels of operating machinery:

- too big excitation level caused by machinery faults,
- resonance of the structure.

Diagrammatically such problems can be shown as in Fig. 2.

The first steps of proposed procedure consist of vibration measurements at target location and comparison of overall vibration level (RMS) filtered in the frequency range from 10 Hz to 1000 Hz with standards. To identify too big vibration level reasons, at the first step of formulated procedure, the frequency decomposition of measured signals

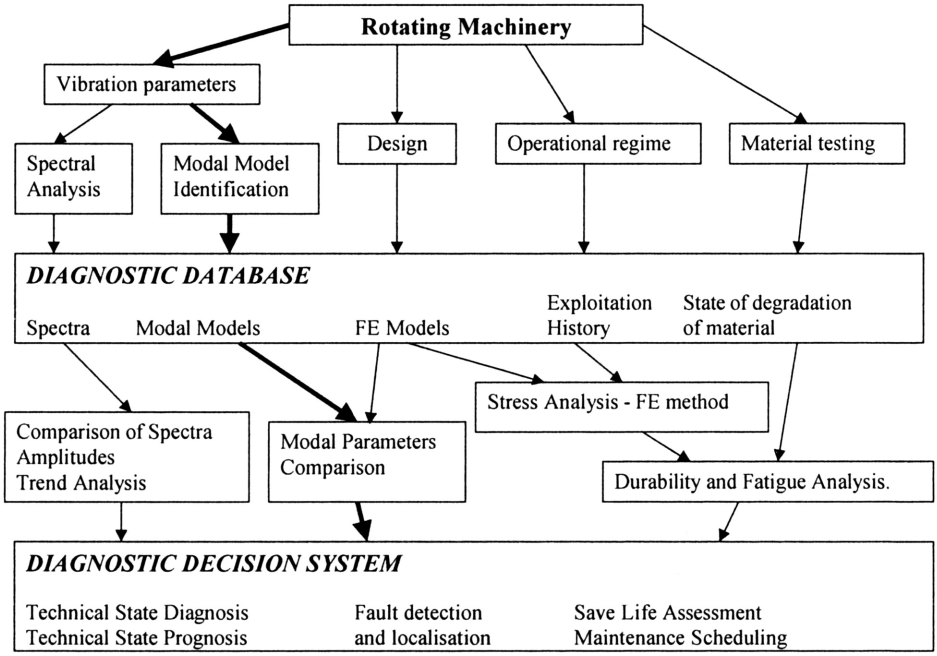

FIGURE 1. Proposed diagnostic procedure for rotating machinery.

is directed. The frequency analysis of signals gives information of the frequency range of problem occurred in structure. At this frequency range farther analysis should be execute. This is very important to limit frequency range to such wide range, which contains effects of dominating damages in the system because it decreases cost of modal experiment. For operating machinery, such a case is in practical diagnostics problem (there are a lot of difficulties to switch of machinery for special diagnostic experiment) in-operation modal analysis (INOP) procedure has to be applied. The results of INOP allow distinguishing structural modes and excited vibration (harmonics of excitation). The main difference is in damping coefficient for structural modes the damping is non zero but for excited vibration INOP gives estimation of damping very close to zero (if excited vibrations are stationary). In the first case the reason of increased vibration level has to be looking for at structural properties of the structure (or changes in structural properties of the structure), but in second case in big excitation level.

To localise the damage in investigated structure for the case of recognised dominating influence of structural properties of the structure the mass, stiffens and damping parameters have to be identified to find which structural elements are "responsible" for increasing vibration level. In a case when main reason of decreasing vibration level is big excitation level the source of this excitation should be identify by means of localisation and assessment. This requires special procedures for identification of loading forces for operating structures based on response measurements. If loads of the structure can be known an usage monitoring can be implemented.

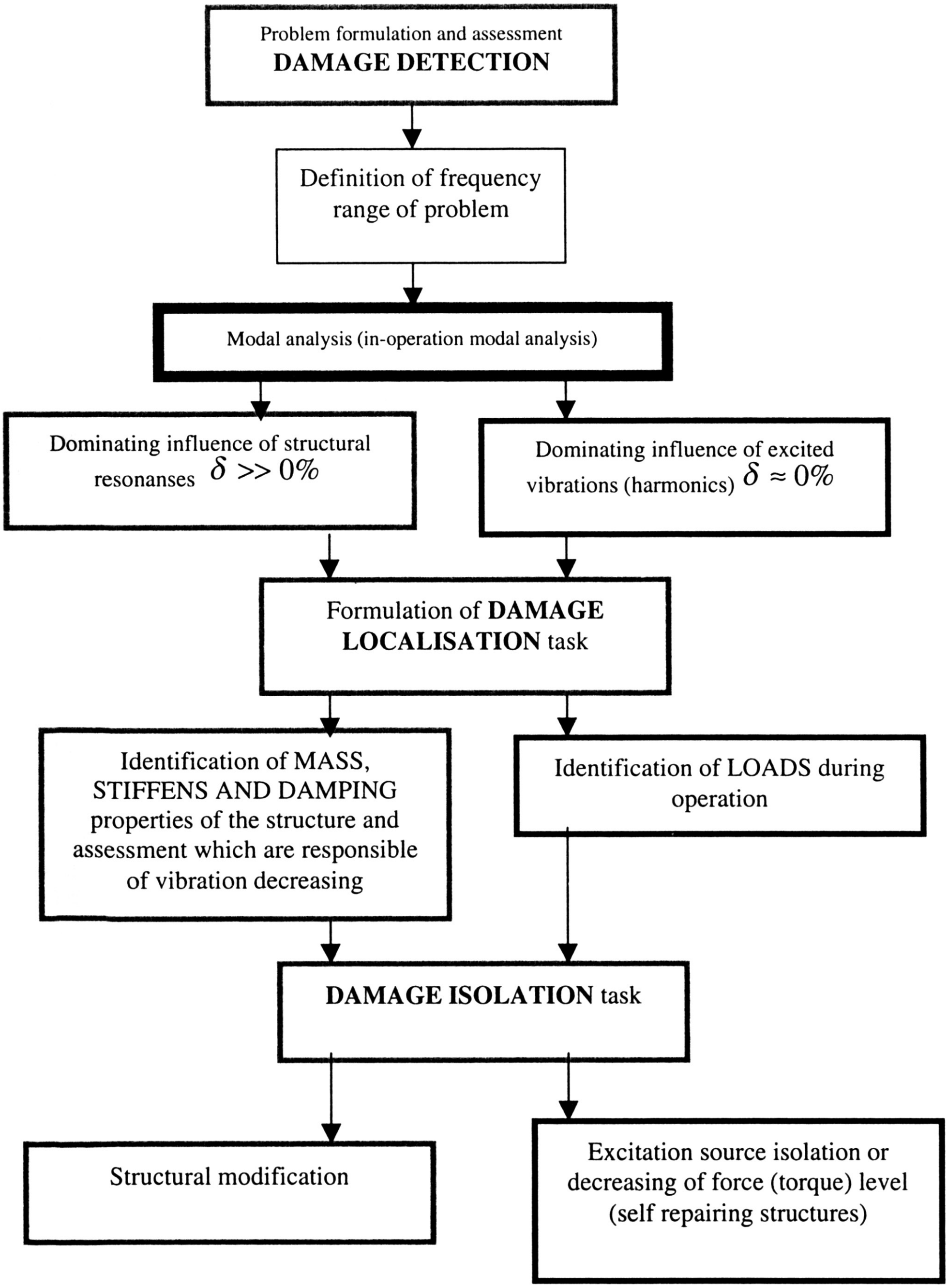

FIGURE 2. Scheme of presented diagnostic procedure based on modal analysis and load identification.

The last step in diagnostic procedure is to find solution to decrease of influence of detected damage on structure operation. This task can be solved with application of structural modification for a case of structural vibration problem or by changing control signal profiles (damage isolation) in a case of too big excitation level.

The main goal of this paper is to explain in-operation modal analysis and loads identification methods ideas and to show benefits of application of these methods. Several examples of practical diagnostic measurements of complex mechanical structures are considered.

2. In-operation modal analysis and its application for damage localisation

Classical experimental modal analysis procedure is based on experiment in which structure under a test is excited using controlled input [4, 5, 15]. Both, input and output in the form of force and acceleration are measured simultaneously in many points of the structure. Based on these experimental results modal parameters are estimated using time domain or frequency domain algorithms. During operation that is very difficult or even impossible to measure forces exciting structural vibrations. This indicates less applicability of classical modal analysis for condition monitoring. There is a need to formulate modal analysis algorithms in which only output measurements are necessary for modal parameter extraction. Such methodologies are described in literature [1, 3, 6, 8, 9, 16].

Nowadays, modal analysis of mechanical structure can be done using many different algorithms. These algorithms can be grouped in several main groups. From the point of view of application of modal analysis in monitoring systems the most important are two classes of methods: in operation modal analysis and methods based on regression models (AR, ARMA, etc.). Many papers have been published in the field of in-operation modal analysis [1, 3, 6, 8] and procedures are available in commercial software like CADA-X, SVS and VIOMA. But still open is a problem of model order selection in automated modal analysis. Two different approaches can be distinguish:

- modal model identification for fixed order model (model order selection before identification procedure),
- modal model identification with automatic order selection.

The first approach requires experimental investigation of monitored structure before starting monitoring process. This investigation is required to determine model order. During monitoring of structure only modal parameters are estimated.

The second approach is based on dedicated algorithms for model order selection, these algorithms are based on cluster analysis, ARMA model analysis, SVD analysis, artificial intelligence. But this problem is currently under investigation at (University of Mining and Metallurgy (UMM), LMS International, Catholic University of Leuven (KUL) within FLITE EUREKA project.

In-operation modal model identification algorithms have been developed in the frame of SINOPSYS EUREKA project which has been done from 1997-2000. The major partner of this project were LMS, INRIA Paris, KUL and UMM.

2.1. Identification of structural dynamics model based on in-operation measurements

Two structural dynamic models of mechanical structures, which can be identified during machinery operation, will be considered:

- modal model of structure,
- operational model of structure (ODS – Operational Deflection Shape).

These two models can be identified using in-operation measurements. The required measurements are the same in considered methodology but data processing procedures required for model parameter estimation are different.

2.1.1. Modal analysis identification method based on the operating excitations. In the classical modal analysis the identified modal parameters are determined using the measurements of frequency characteristics on the tested structure in an active identification test, involving controlled excitation of vibrations and measurements of the system response in the form of vibrations acceleration spectrum. Knowing the response spectrum and the excitation signals, we are able to identify the frequency characteristics of the structure. That procedure is applied in the frequency-domain methods [1, 8].

These methods allow to find the modal parameters in the neighbourhood of a single natural frequency (SDOF methods) or throughout the selected frequency band covering more than one natural frequency (MDOF methods). Unlike those methods, the time-domain methods require multiple channel measurements of time characteristics of response and excitations signals. The first step in the procedure applied in most well known methods is finding the system impulse response. Once it is known, the modal parameters can be duly estimated [8].

A slightly different approach is required when the system is subjected to immeasurable excitations due to processes taking place during machine operation [1, 2, 3, 4]. An obvious advantage of those identification methods is that the excitation conditions, boundary conditions and the distributions of operating loads are maintained. It is difficult or sometimes even impossible to meet these requirements during the active tests in laboratory conditions. Identification methods using in-operation measurements can be divided into three basic categories:

- methods using auto-correlation and cross-correlation of signals [1, 3],
- methods using autoregression function for the response signals [2],
- methods realised in the stochastic sub-space [4].

In modal model identification of operating structure the method involving auto-correlation function of the response signals and cross correlation of response and reference signals is preferred, because gives global estimates of modal parameters.

It can be proved that the correlation function can be expressed by means of damped harmonic functions for the MIMO systems subjected to random excitations. To determine the modal parameters the LSCE [Least Square Complex Exponential] method was applied [1, 8]. The correlation function is approximated with the sum of decaying exponential harmonic functions. This method applied to measure the impulse response is a well-known technique in classical experimental modal analysis, and it yields the global poles estimators. It can be proved that the cross-correlation function may be used in modal parameters identification in the same way as the impulse response. Accordingly, the equation of the system dynamic motion was considered:

$$M\ddot{x} + C\dot{x} + Kx = f(t), \tag{1}$$

where: M, C, K – mass, damping and stiffness matrixes; $\ddot{x}$, $\dot{x}$, x – acceleration, velocity and displacement vectors; f – vector of the excitation force.

Equation (1) can be transformed to the modal co-ordinates, using the transform given by the formula:

$$x(t) = \Psi q(t) = \sum_{r=1}^{n} \Psi_r q_r(t), \tag{2}$$

where: Ψ is the modal matrix; the columns being the modes of natural vibrations corresponding to the given natural frequency, q_r denotes the modal co-ordinate.

Assuming that damping is low or proportional, after substituting (2) into (1) and multiplying by Ψ^T we get an uncoupled system of equations of the form:

$$\ddot{q}_r(t) + 2\xi_r\omega_{nr}\dot{q}_r(t) + \omega_{nr}^2 q_r(t) = \frac{1}{m_r}\Psi_r^T f(t), \tag{3}$$

where: ω_{nr} is the frequency of natural vibrations; ξ is the modal damping ratio for r-th mode of vibrations, m_r is the modal mass.

Assuming the initial conditions are zero for any excitations, the solution to (3) may be written in the form of the following convolution:

$$q_r(t) = \int\limits_{-\infty}^{t} \Psi_r^T f(\tau) g_r(t-\tau)d\tau, \tag{4}$$

where: $g_r(t) = 0$ for $t < 0$, $g_r(t) = \frac{1}{m_r\omega_{rd}}\exp(-\xi_r\omega_{nr}t)\sin\omega_{nr}t$ for $t \geq 0$, $\omega_{rd} = \omega_{nr}(1-\xi_r^2)^{1/2}$ is the frequency of damped natural vibrations.

Applying the solution (4) that is valid for the modal coordinates to find the solution for generalised co-ordinates x(t), we get:

$$x(t) = \sum_{r=1}^{n} \Psi_r \int\limits_{-\infty}^{t} \Psi_r^T f(\tau) g_r(t-\tau)d\tau, \tag{5}$$

where: n is the number of vibration modes considered here.

For the single output and for single excitation signal at the point k the Eq. (1) has the form:

$$x_{ik}(t) = \sum_{r=1}^{n} \Psi_{ir}\Psi_{kr} \int\limits_{-\infty}^{t} f_k(\tau) g_r(t-\tau)d\tau, \tag{6}$$

where Ψ_{ir} is the i-th component of the r-th vibrations mode.

Impulse response induced by Dirac impulse at the point k measured as the response at the point i has the form:

$$x_{ik}(t) = \sum_{r=1}^{n} \frac{\Psi_{ir}\Psi_{kr}}{m_r\omega_{dr}} \exp(-\xi_r\omega_{nr}t)\sin(\omega_{rd}t). \tag{7}$$

The cross-correlation function determined for two response signals at the points i and j induced by white noise excitations at the point k has the form:

$$R_{ijk}(t) = E\big[x_{ik}(t+T)x_{jk}(t)\big], \tag{8}$$

where E is the expected value operator.

Substituting the solution given as (6) in the definition of the auto-correlation function (8) and assuming that the excitation comes in the form of white noise for which the correlation function is the constant α_k multiplied by the Dirac delta $\delta(t)$, we get:

$$R_{ijk}(t) = \sum_{r=1}^{n}\sum_{s=1}^{n} \alpha_k \Psi_{ir}\Psi_{kr}\Psi_{js}\Psi_{ks} \int\limits_0^\infty g_r(\lambda+T)g_s(\lambda)d\lambda, \tag{9}$$

where: $\lambda = t-\tau$; the integration limits being changed because of the form of the function g and the system causality.

Applying the definition of g given by (4) and distinguishing between the terms dependent on T and λ, we get:

$$g_r(\lambda+T) = \big[\exp(\xi_r\omega_{rn}T)\cos(\omega_{rd}T)\big]\frac{\exp(\xi_r\omega_{rn}\lambda)\sin(\omega_{rd}\lambda)}{m_r\omega_{rd}} + \big[\exp(\xi_r\omega_{rn}T)\sin(\omega_{rd}T)\big]\frac{\exp(\xi_r\omega_{rn}\lambda)\cos(\omega_{rd}\lambda)}{m_r\omega_{rd}}. \tag{10}$$

Substituting (10) and the cross-correlation function formula for $g_s(\lambda)$, analogous to (9), we get:

$$R_{ijk}(t) = \sum_{r=1}^{n}\big[A_{ijkr}\exp(-\xi_r\omega_{nr}T)\cos(\omega_{rd}T) + B_{ijkr}\exp(-\xi_r\omega_{nr}T)\sin(\omega_{rd}T)\big], \tag{11}$$

where: A_{ijkr}, B_{ijkr} are independent of T and are the functions of modal parameters:

$$\begin{Bmatrix} A_{ijkr} \\ B_{ijkr} \end{Bmatrix} = \sum_{s=1}^{n} \frac{\alpha_k\Psi_{ir}\Psi_{kr}\Psi_{js}\Psi_{ks}}{m_r\omega_{rd}m_s\omega_{sd}} \cdot \int\limits_0^\infty \exp(-\xi_r\omega_{nr} - \xi_s\omega_{ns})\lambda \sin(\omega_{sd}\lambda)\begin{Bmatrix} \sin(\omega_{sd}\lambda) \\ \cos(\omega_{sd}\lambda) \end{Bmatrix} d\lambda. \tag{12}$$

Equation (11) represents the relationship between the auto-correlation function as the sum of decaying exponential harmonic functions, and the impulse response function applied in classical modal analysis to modal parameters identification. To make a direct use of thus written correlation function, the formula (11) can be further transformed and written as:

$$R_{ij}(t) = \sum_{r=1}^{n} \frac{\Psi_{ir}G_{jr}}{m_r\omega_{rd}}\exp(-\xi_r\omega_{nr}T)\sin(\omega_{rd}T+\vartheta_r), \tag{13}$$

where the new phase shift angle ϑ_r and the constant G_{jr} are derived from the formula:

$$
\begin{aligned}
\tan(\vartheta_r) &= \frac{I_{rs}}{J_{rs}},\\
I_{rs} &= 2\omega_{rd}(\xi_r\omega_{nr} + \xi_s\omega_{ns}),\\
J_{rs} &= (\omega_{sd}^2 - \omega_{rd}^2) + (\xi_r\omega_{nr} + \xi_s\omega_{ns})^2,\\
\beta_{jkrs} &= \frac{\alpha_k\Psi_{kr}\Psi_{js}\Psi_{ks}}{m_s},\\
G_{jr} &= \frac{\Psi_{ir}}{m_r\omega_{rd}}\sum_{s=1}^{n}\sum_{k=1}^{n}\beta_{jkrs}(I_{rs}^2 + J_{rs}^2)^{-1/2}.
\end{aligned}
\tag{14}
$$

LSCE is one of the time-domain methods used in modal analysis. The method provides a global estimate of modal parameters in the form of natural frequencies and the damping ratios. This method was first published in the work [6]. It is a modification to the earlier CE (Complex Exponential) method. The basis for determining the modal models is the measured variability of the transfer function. In the identification methods using the measurements of system response to unknown excitations the transfer function is replaced with the cross correlation function. The cross-correlation function comes as the sum of decaying exponential harmonic functions (13). To better present the estimation methods, these functions were rewritten as:

$$
h_{jk}(t) = \sum_{r=1}^{2N} A_{rjk}e^{s_r t},
\tag{15}
$$

where $s_r = -\omega_{rn}\xi_r + i\omega_{rd}$.

By way of sampling action with the constant sampling time Δt, the function $h(t)$ can be transformed into the sample series h_0, h_1, h_2, ..., h_L. The value of each sample can be expressed by the formula:

$$
\begin{aligned}
h_0 &= \sum_{r=1}^{2N} A_r,\\
h_1 &= \sum_{r=1}^{2N} A_r V_r,\\
&\vdots\\
h_L &= \sum_{r=1}^{2N} A_r V_r^L,
\end{aligned}
\tag{16}
$$

where: A_r, V_r are the desired quantities, $V_r = e^{sr\Delta t}$.

These values can be found using the Prony's method [6]. According to that method there always exists a polynomial in V_r with real coefficients β such that the following relation is satisfied:

$$
\beta_0 + \beta_1 V_r + \beta_2 V_r^2 + \dots + \beta_L V_r^L = 0.
\tag{17}
$$

To determine the coefficients β it is necessary to solve the equation:

$$\begin{bmatrix} h_0 & h_1 & \cdots & h_{2N-1} \\ h_1 & h_2 & \cdots & h_{2N} \\ \vdots & \vdots & & \vdots \\ h_{2N-1} & h_{2N} & \cdots & h_{4N-2} \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{2N-1} \end{Bmatrix} = - \begin{Bmatrix} h_{2N} \\ h_{2N+1} \\ \vdots \\ h_{4N-1} \end{Bmatrix}. \tag{18}$$

The coefficients obtained from (18) enable to find the roots V_r of the polynomial. Using these values V_r and the corresponding complex conjugate value, we can obtain the natural frequencies and the damping factors. The values of V_r being known, we can determine the coefficients A_r, and consequently the modal constants and the phase angles, using (13). The coefficients A_r can be determined when the following equation is solved:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ V_1 & V_2 & \cdots & V_{2N} \\ V_1^2 & V_2^2 & \cdots & V_{2N}^2 \\ \vdots & \vdots & & \vdots \\ V_1^{2N-1} & V_2^{2N-1} & \cdots & V_{2N}^{2N-1} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{2N} \end{Bmatrix} = - \begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_{2N} \end{Bmatrix}. \tag{19}$$

Solving the Eq. (19) for A_r, we can find the modes of natural vibrations. As it can be easily seen, these relations are derived for SISO systems; which means that only one transfer impulse function or one cross correlation function – in the case of response measurements methods, should be analysed. LSCE is an extension of CE method for SIMO; it allows for simultaneous analysis of all measured cross-correlation functions, thus enabling us to determine global estimators of modal parameters of the tested structure. The relevant relationships will have the form:

$$\begin{bmatrix} [h]_1 \\ [h]_2 \\ \vdots \\ [h]_p \end{bmatrix} \{\beta\} = \begin{Bmatrix} \{h\}_1 \\ \{h\}_2 \\ \vdots \\ \{h\}_p \end{Bmatrix} \qquad \text{or} \qquad \underset{(2Np \times 2N)}{[h]} \; \underset{(2N \times 1)}{\{\beta\}} = \underset{(2Np \times 1)}{\{h_G\}} . \tag{20}$$

The solution to (19) for β can be found using pseudo-inverse:

$$\{\beta\} = \left([h]^T [h] \right)^{-1} [h]^T \{h_G\} . \tag{21}$$

The further procedures for finding the modal parameters are the same as in the CE method.

This algorithm for modal parameters identification was implemented in the CADA-X system, and VIOMA package as the part of the research project EUREKA "Sinopsys", in which author participate [9].

2.1.2. Measurements of the operating deflection of the structure. In practice of engineering structures testing it is often enough to measure the operating deflection of the structure; the method is called ODS (Operating Deflection Shape) or Running Mode. ODS is defined as the structure deflection for the selected vibration frequency or at the given time instant due to external excitation force; while the motion of at least two points must be analysed. In this way the structure deflection during its forced motion, understood as the relative motion of one reference point, may be determined. Since motion is a vector quantity (acceleration, velocity or deflection vectors), it has a point of origin, orientation and value, which determine the deflection shape during the structure motion. ODS in time domain may be determined basing on various types of time responses to random, impulse or harmonic excitations. Another techniques are applied to determine the frequency-domain ODS [7]. These involve mainly the measurements of the response spectrum, power spectral density and frequency characteristics or the transfer frequency characteristics for any reference point, specially defined to determine ODS. ODS differs from the modal parameters [modal vectors] discussed in earlier paragraphs in that it depends on the type of excitation signals. ODS will change with the change of the structure loading. On the other hand the modal vector is independent of the excitation mode. It characterises the dynamic properties of the tested structure; including the boundary conditions, its geometry and materials. The modes of vibrations [modal vectors] are dimensionless while ODS is expressed in the units of deflection, velocity or acceleration; depending on which units were adopted in the course of measurements. ODS may be determined analytically or experimentally. The first method involves solving the Eq. (1) for the accepted time characteristic of the excitations. That yields the time characteristics of the response signal in the form of the vector $x(t)$. Calculating the value of $x(t_0)$ for any time instant and for all the co-ordinates of the vector x we get the time-domain ODS(t). ODS can be also determined experimentally, by simultaneous measurements of vibrations parameters at several points. The vector obtained through selecting the amplitude for the given time instant is the time-domain ODS. A similar procedure is applied to determine ODS in frequency domain. System dynamics in frequency domain can be described with the formula:

$$X(j\omega) = H(j\omega)F(j\omega), \tag{22}$$

where $X(j\omega)$ is the vector of system response spectra, $F(j\omega)$ is the vector of excitation force spectra, $H(j\omega)$ is the frequency characteristics matrix. In the case of linear systems, the Eq. (22) is satisfied for all frequencies throughout the considered range. ODS in frequency domain is defined as the system response to the excitations $F(j\omega)$ for any frequency ω_0:

$$\mathrm{ODS}(j\omega_0) = H(j\omega_0)F(j\omega_0). \tag{23}$$

It follows from (23) that ODS depends on the type of excitation forces. ODS(t) can be also determined through applying the inverse Fourier transform to (23):

$$\mathrm{ODS}(t) = \mathrm{FFT}^{-1}\{H(j\omega)F(j\omega)\}. \tag{24}$$

In this way ODS can be determined for those time instants for which the value of the inverse Fourier transform is calculated. ODS in frequency domain is determined experimentally using multiple channel measurements of response spectra and the cross spectra between the measurements and reference points. Because of high costs of measurements, the number of channels for simultaneous response measurements is limited.

That is why some reference points on the structure are chosen, which do not change their position during measurements. The other measurement points may be moved during the tests. Such procedure is necessary as it is essential to know the phase shift angle between the system responses at the points for which ODS is determined. When the forced vibrations of the systems are dominated by the natural vibrations, the ODS vector will be similar to the modal vector. The degree of similarity depends on how strongly do natural vibrations dominate the measured responses.

ODS might be formulated for any frequency value and it describes form of vibration in this frequency for existing, unmeasured excitations. If assumption of little coupling of modes, no close modes and near white noise excitation forces properties might be justified then maximum of amplitude of measured response auto- and cross-spectra correspond to running (harmonics of excitation force) modes or structural modes.

When number of measurement channels exceeds number of signals to be measured transmissibility functions (response over reference response) might be used [4] if excitation is sufficiently stationary. ODS calculated for maximums of amplitude of measured spectral functions allow to approximate modal parameters of considered system. Such identification method is called Basic Frequency Decomposition (BFD) or Peak Picking [3]. The biggest advantage of ODS method application is its simplicity. ODS analysis results might be calculated only for one reference signal at a time in the result local estimate, corresponding to the selected reference direction, of natural frequency, modal damping values and mode shapes are evaluated.

2.2. Case study

The monitoring systems were used in many industrial structures. Bellow two examples of application are shown.

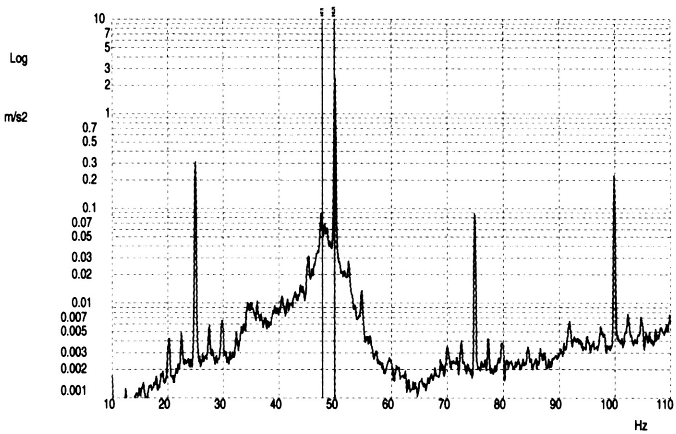

FIGURE 3. Amplitude of vibration acceleration spectrum measured on radial bearing housing in axial direction.

The first example deals with a fan used in a boiler part of a turboset. Vibration testing procedure performed according to ISO 10816 standard indicated unacceptably high amplitude of axial vibration of radial sliding bearing housing. Spectral analysis of the measured acceleration signals showed that the dominating effect on amplitude RMS value has the 2nd harmonic component of rotational speed (50 Hz). Additionally in the vicinity of value of 50 Hz the increase of amplitude spectrum might be also observed (see Fig. 3). To explain this phenomenon application of OMA technique was suggested, as it provided no need of the tested fan shutdown.

The analysis results showed that in the vicinity of 50 Hz there was identified a set of 5 similar structural modes. Relatively high modal density in the considered frequency sub-range causes that low values of dynamic stiffness are obtained in this frequency sub-range. Low dynamic stiffness is a reason for high vibration amplitude level at the frequency of considered harmonic component. Moreover amplitude of structural vibration of frequency 47.70 Hz, the closest to the 2nd harmonic component, proved to be also substantially high. Both the identified structural mode shape and running mode shape are presented (respectively) in Fig. 4 and 5. These mode shapes are dominated by bending vibration of radial bearing's pedestal concrete support. Correspondence of the considered mode shapes pair according to Modal Assurance Criterion (MAC) [15] is found to be very high (99%). Qualitative survey of the identified mode shapes explains why in the vicinity of the attachment of pedestal to the foundation base crack occurred for some fans of considered design. The presented application of OMA enabled to explain the increase of vibration amplitude level and indicated its structural cause. High amplitude bending of the support is caused by improper structural design of the fan foundation resulting in to low axial stiffness of the support.

Contrary to Structural Health Monitoring systems, symptom based diagnostic systems are usually focused on harmonic components of response arising due to polyharmonic excitation. The last example of application of OMA method deals with its use for modal decomposition of polyharmonic forced response. During performed experiment on high power gearbox, relative displacements of shaft journals with respect to their bushings were measured in 4 bearing cross-sections at 2 perpendicular directions for each bearing. Figure 6 shows location diagram of measuring cross-sections and measuring directions.

Application of OMA method to the measurement results allowed to identify all harmonic components of response caused by polyharmonic excitation. In Fig. 7, 8 and 9 results obtained for 2nd harmonic component of slow shaft rotational speed are presented.

Figure 7 shows trajectory of shafts journals on spatial plot (lines connecting pairs of points where measurement took place have only geometrical meaning and do not present shafts' motion). Figures 8 and 9 present 2nd harmonic component of shaft journal trajectories in measurement cross-section plane for input (faster) shaft and output (slower) shaft respectively.

Identified harmonic components might be used directly for diagnostic purpose. On the other hand the filtered trajectory of shaft journals might be composed of identified harmonic components for stationary conditions provided that dynamic stiffness of bearings is known for considered frequency range and rotational speed.

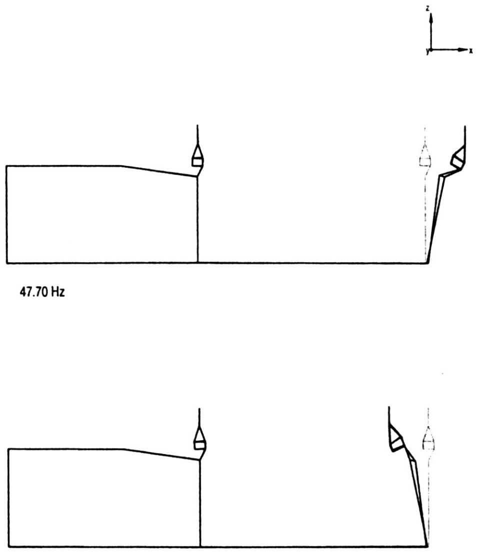

FIGURE 4. Example of structural mode shape.

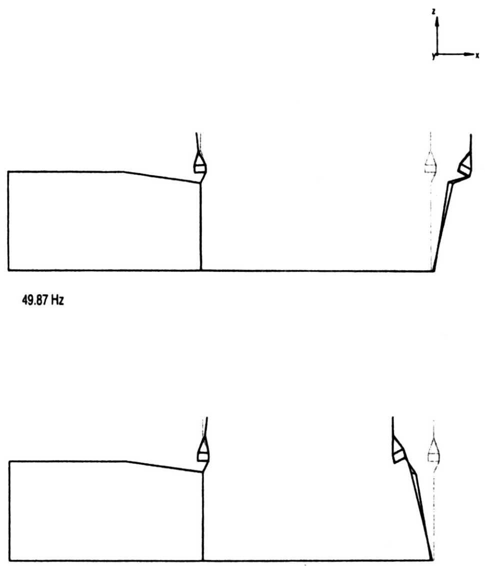

FIGURE 5. Example of running mode shape.

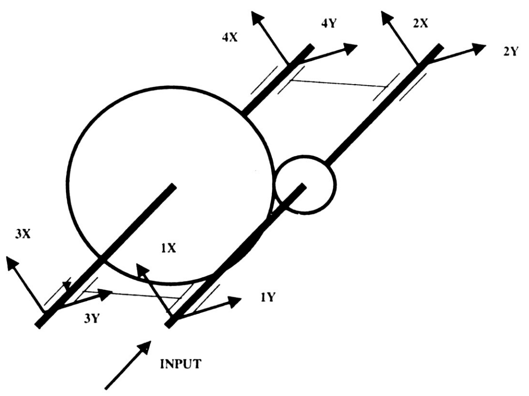

FIGURE 6. Diagram of relative vibration displacement measurement on the high power garbox.

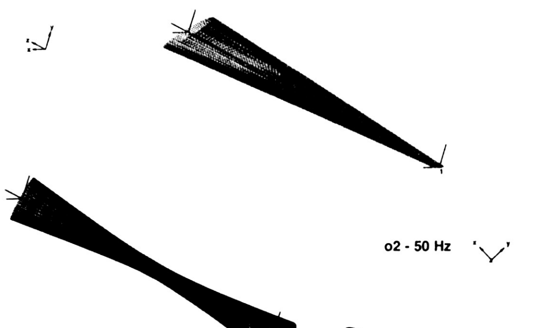

FIGURE 7. Example of estimated displacement trajectories of shafts in 4 bearing cross-sectons for 2^{nd} harmonic component of slower shaft rotation.

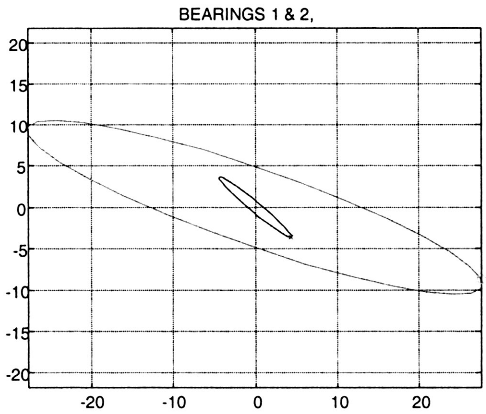

FIGURE 8. Example of 2nd harmonic component of orbit of output shaft for input shaft bearings cross-sections.

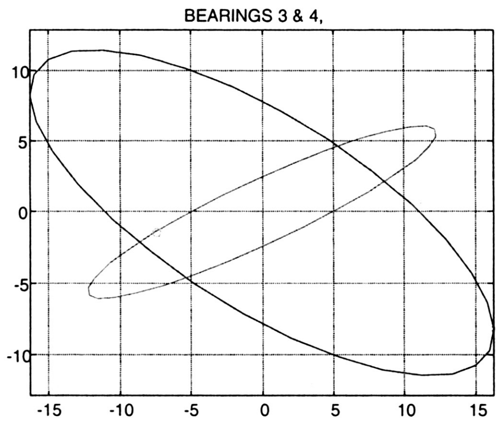

FIGURE 9. Example of 2nd harmonic component of orbit of output shaft for output shaft bearings cross-sections.

2.3. Conclusions

In operation modal analysis is a new and very effective tool for diagnosis of rotating and not rotating components of machinery. This method can be implemented in continuous monitoring system and can provide experts more deep knowledge about actual dynamic properties of a structure. As it was shown on genuine machinery element examples these information's give new possibilities in state assessment, fault localisation and detection.

3. Applications of neural networks for identifications of loads in mechanical structures

3.1. Introduction

We are facing important shift in a machinery management trough different types of mechanical structures. The majority of industrial and transport facilities know about condition-based maintenance (CBM) and many of those are using or planning to implement in the near future [24]. This subject is especially important in power plants, aviation systems, high-speed trains, etc were mechanical components are considered critical. In case of failure whole system is forced outage, which causes in very high economical losses and even danger for users. Up to now most of such systems operate with strictly followed overhauls program. This attitude helped to decrease number of failures but generated higher maintenance costs and did not guarantee failures avoidance. These goals can be better achieved with CBM. CBM requires that operator should know actual technical state of machinery and has possibility to predict its safe operation live. One of the most important factor that influences safe operation is fatigue usage.

To determine fatigue usage of mechanical components at given stage of their operations is one of main task of currently use monitoring and diagnostic system. The scheme of usage monitoring procedure is shown in Fig. 10.

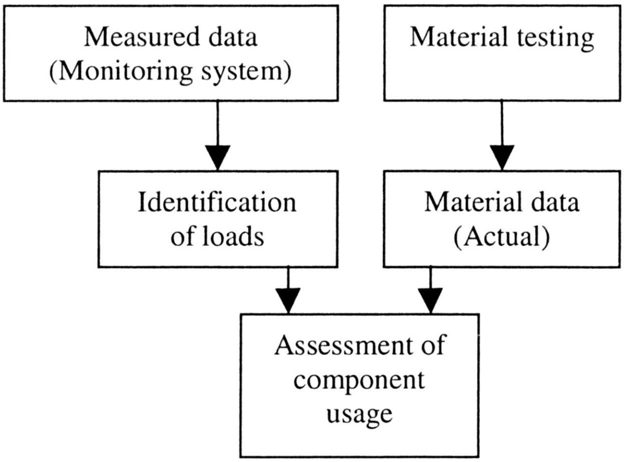

Figure 10. Scheme of usage monitoring system.

The process of predicting of safe retirement times for critical machinery components essentially involves three steps, which include:

- generation of operation conditions profile,
- acquisition of fatigue load from measurements for given operation conditions,
- establishment of S-N curve at statistically reduced endurance limit on full scale specimens in the laboratory test.

All of these steps are very important for correct usage determination. First two steps are related to machinery operation but the last one is related to materials data. Due to these facts a new identification problem in diagnostics of mechanical structure can be formulated. In the literature there are described two typical approaches to identification of loads of mechanical systems:

- based on measurements of process data (movement data),
- based on structure responses measurements due to loads under identification.

These two approaches can be applied in different operation conditions. The first can be applied if the process data for machinery (driving data for vehicle, flight data for airplanes) are measured, and have influence on load of operating structures. But the second in a case if response on a loads under identification can be measured directly. Relations between process data and load cycles of structures are commonly nonlinear and very difficult to analytical modeling. These reasons are main in choosing neural networks as a basic tool for identification of loads based on process parameters measurements.

The second problem of load identification, based on direct measurements of system responses is a classical inverse identification problem. Some deterministic and intelligent algorithms can be applied in this case. The overview of these methods is shown in a next section of this work.

In this section methodology of load identification based on process data measurements and neural networks is considered.

3.2. Formulation of load identification problem

The main idea of the method of load identification [17] is shown diagrammatically in the Fig. 11.

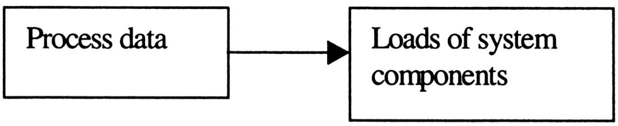

FIGURE 11. Scheme of presented load identification method.

The relation between process data and load vector is approximated using regression model or neural network. The regression model parameters are estimated based on measurement results [22]. In a case of application of neural network to approximate this relation, neural network is learned based on experimental data [22]. It will be shown bellow that for many real structures such approach gives not enough accurate results. In such a case new approach based on neural network algorithms is proposed. This method includes the following steps:

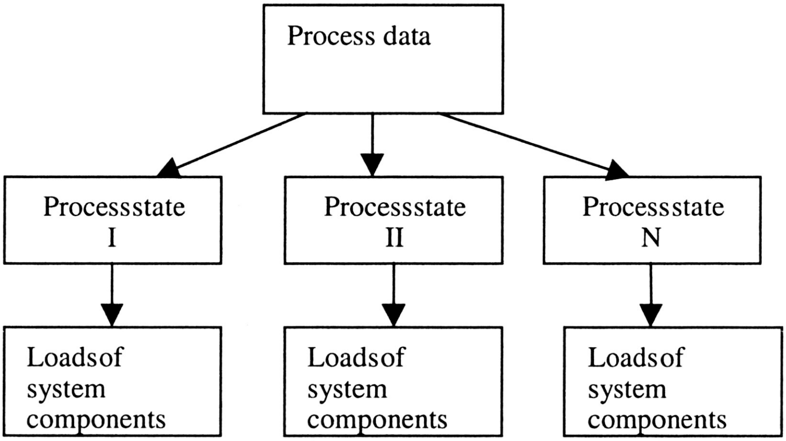

FIGURE 12. Proposed identification procedure.

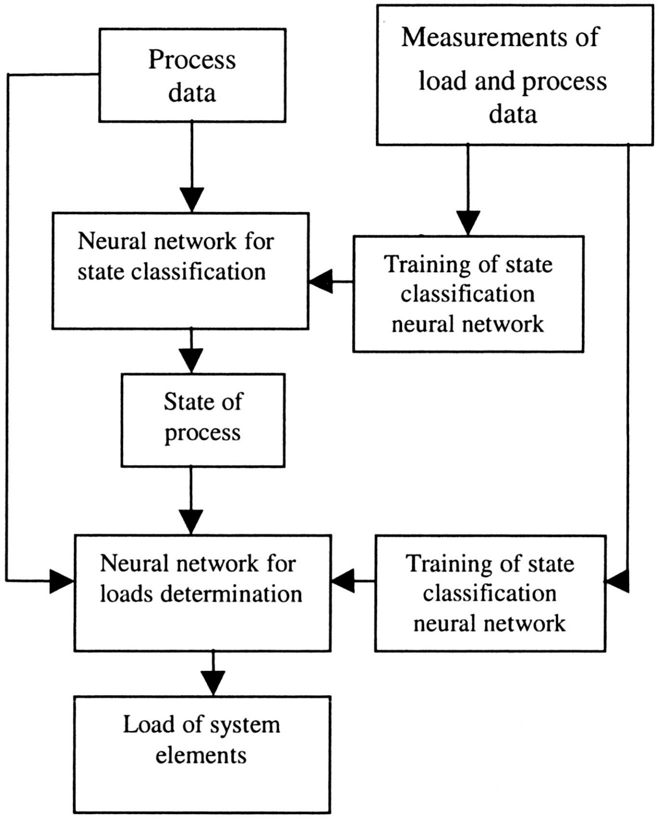

FIGURE 13. Scheme of proposed neural network based realisation of load identification task.

- identification of process state,
- identification of loads for particular state.

The approach is diagrammatically shown in Fig. 12.

Both steps of identification can be perform using neural networks. The first step is a classification task, but the second approximation. The task for neural network in the first step is to recognize process state based on measured process data. This classification problem can be solved using deterministic algorithms also. The second step of the approach is an approximation of relation between process data, state of the process and loads of the structure. This approximation has not so complex structures than has procedure shown in Fig. 10. Modified identification procedure implemented using neural networks is shown in Fig. 13.

Two applied neural networks have commonly different structures, sometimes for classification of process state simple ADALINE type of neural network can be applied. Type of networks depends on form of surface, which is a boundary between particular process states. For load identification task, which can be formulated as approximation task backpropagation neural networks is commonly use.

Realisation of loads identification procedure based on neural networks algorithms is beneficial for on line usage monitoring of complex structures that operate in various conditions. Identified loads can be an input data for procedures of fatigue life calculations and usage assessment.

Application of presented approach for identification of load of helicopter structures during flight is a subject of a next section.

3.3. Identification of helicopter loads during flight

A helicopter structure is subjected to severe loads due to the time varying flight conditions and mission profiles. These loads can result in fatigue damage of flight critical components that can accumulate and cause failure. If component loads can not be carefully monitored the classical maintenance procedures based on assumed operation time limitation has to be applied. But in many helicopter cases this time limit is overestimated due to safety reasons that cause of exploitation cost dramatically increase. Economical and safety reasons are motivations of usage monitoring in helicopter structures. The most critical helicopter components that are subjected to fatigue loads is rotor system.

Rotor system's component loads are not routinely measured during flight due to complexity of instrumenting in a rotating system. Several attempts have been made to predict rotor system loads from measurements in the fixed system using statistical approaches and intelligent algorithms. The approach based on regression model of relation between loads and flight data (data recorded using standard flight recorder) is presented in [22] and summarized in the section 5 of this text. Different approach, based on neural network algorithms is shown in [23]. There is shown a case study for SH-60B helicopter and three helicopter components have been under a test: the rotor blade pushroad, blade bending, main-rotor damper. Correlation coefficient between neural network approximation of loads of these component and measured loads has values from 84% to 95%. These values have been accepted by fatigue analyst. To improve quality of load prediction based on measured flight data the innovated approach is proposed and tested on SW-3 PZL Swidnik helicopter. The main rotor damper and bending of main rotor have been predicted using neural network algorithm. The measured loads in a form of strains time histories are shown in Fig. 14. The results of load measurements have been obtained with special helicopter instrumentation that is not installed as standard helicopter equipment. This data has been used to learn neural networks to predict helicopter load. As flight, data five parameters have been recorded using BUR-1-2 recorder: altitude, horizontal speed, yaw angle, pitch angle, slip angle. All measurement results are synchronised with main rotor rotation. Due to fact that recorder data set is not continuous and has different sampling frequency for different flight parameters to apply its for neural networks learning data processing has to be done.

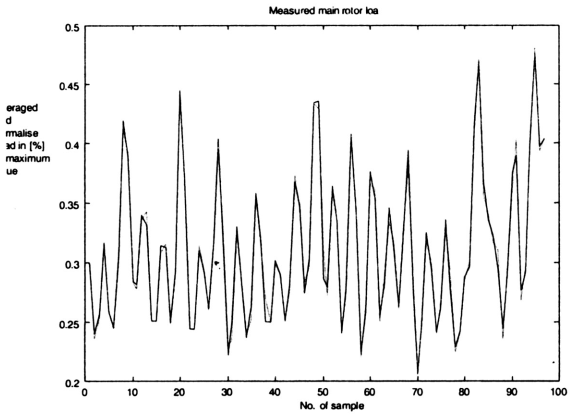

FIGURE 14. Example of measured load data after data processing and normalization.

Discontinuities of recorded signals have been cancelled by gluing plots at the point of discontinuity, author tried to omit samples in that period of shaft rotation in which discontinuities occurred, also. ut this approach gave bad results and was not recommended.

The biggest amplitudes of loads are in frequency domain up to 20 Hz that recorded signals (recorded with many different frequencies 400 Hz, 100 Hz) have been resample using sampling frequency of 50 Hz. To resample recorded signal averaging procedures are used.

To compress data for neural networks learning only a mean value and maximum amplitude of signal in one rotation of main rotor have been used.

Two main tasks are solved using neural networks:

- classification of flight state,
- determination of stress amplitude for particular flight states.

The concept of neural network that solves such formulated tasks is shown diagrammatically in Fig. 15. For a state classification based on flight data backpropagation neural networks is applied. Input layer has five neurones but hidden layer has 6 neurones (it is the best configuration with minimal dimension to solve formulated classification task). The neural network has 44 outputs each for particular flight state. The architecture of this network has been chosen based on numerical experiment. Learning process

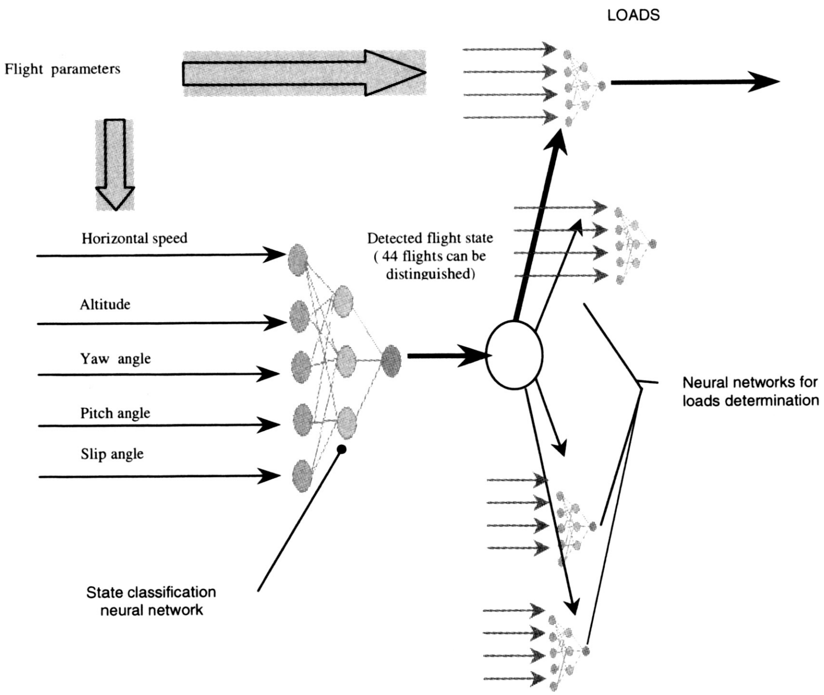

FIGURE 15. Scheme of applied load identification procedure using neural networks.

of such neural network is based on backpropagation mechanism. Particular output of the classification neural network is activated for particular flight state, Based on this decision the appropriate neural network for given flight state is chosen. This network is dedicated for load identification. Each neural network for load determination has five neurones in input layer and different number of neurones in hidden layer. This number depends on recognised flight state. As an output the main rotor damper load and bending loads of main rotor are obtained. The mean value and amplitude of these loads are given. These two load characteristics are necessary to determine fatigue of helicopter critical components.

The architecture of classification neural networks is shown in Fig. 16. This neural network has complex structure, contains two types of networks: two state decision element which classify provisionally flight state for four groups and backpropagation type of neural network for classification of state within these groups. At a first step speed is a feature which has different value for different state, generally we can distinguish states with velocity 0 and velocity different then zero. Helicopter manufacturer as possible flight states finds this number applied neural network distinguishes 44 different flight states, for tested helicopter.

The output signal form state classification neural networks is used in proposed procedure to select neural network for loads identification. These neural networks are chosen as typical backpropagation neural networks with sigmoid type of activation function mean value and amplitude of damper load and bending load of main rotor are deter-

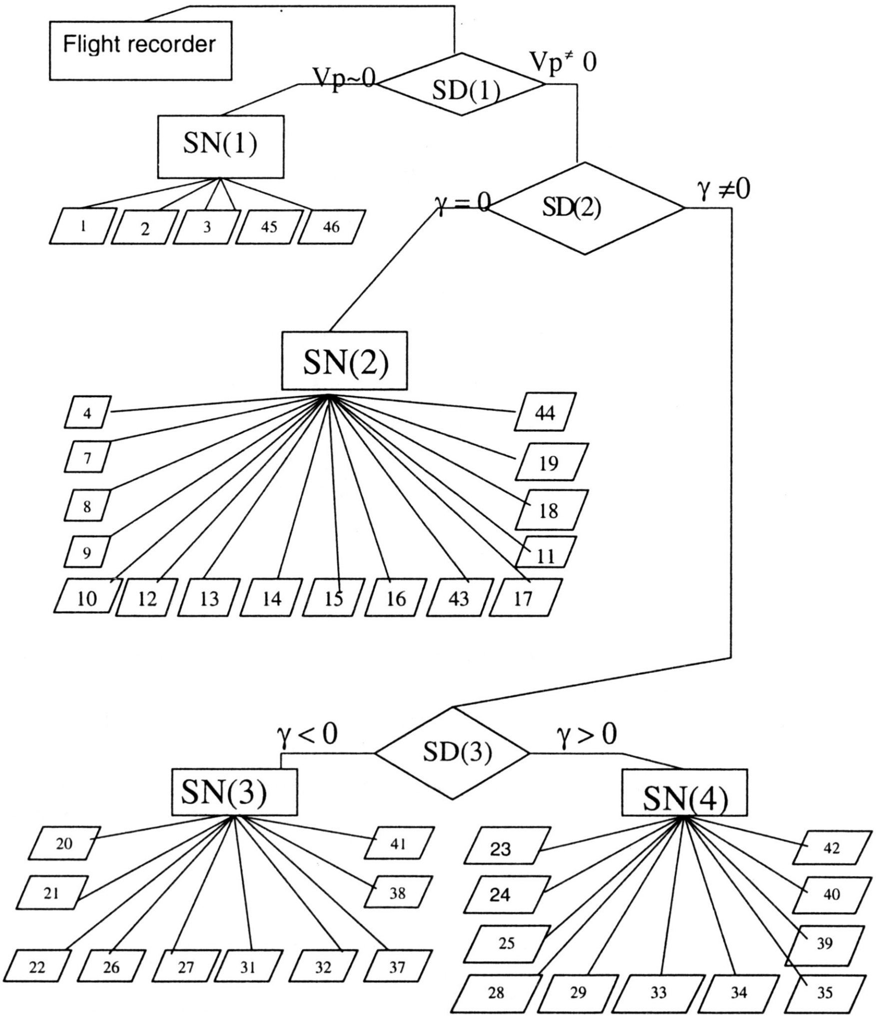

FIGURE 16. Architecture of classification neural network applied for flight state classification.

mined as an output of neural network. These data can be used for fatigue rest life calculation. The learning process is performed using backpropagation algorithm and SSE error as a quality criterion. The value of the error to stop learning procedure is set as 0.001. Very important step in neural networks learning is a choice of training data base. The training data set is the set of known input/output pairs that is used to train neural network. The training data set should be chose that captures all of the information required but not is biased to any given condition. If certain conditions are over emphasised in the training data set, they will dominate the solution process and bias the model to perform well for that condition but poorly for others. A testing data set is also extracted from the flight measurements data base. This data set is not used during training of the network but rather data from different flight test should be chosen. It is possible for poorly trained network to predict training set well but will be probably unable to generalise results for a new data set. A history of learning of neural network for state classification is shown in Fig. 17. In this case, 2338 epoch was necessary to achieve limit value of SSE error.

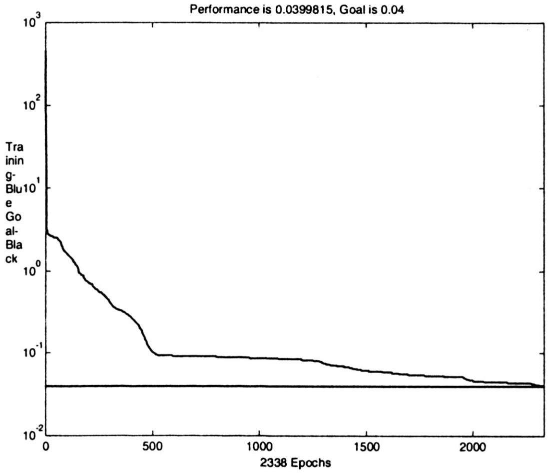

FIGURE 17. History of learning process for state classification.

Performance of neural network not only depends on the training data set but also on its physical design. Currently, the methodology to determine the optimum network design for given problem does not exist. A design sensitivity study was conducted in order to ascertain the appropriate number of hidden–layer nodes necessary for accurate performance. A single hidden layer design was chosen and the number of neurones in the hidden layer was varied from 5 to 20 in increment of 1. The neural network for load identification has been learned based on measurements load during flight for different flight conditions. The experiment was done for all 44 flight conditions and in data base used for learning all flight conditions have been represented. During the experiment, a strains at chosen points of helicopter structure have been measured. The backpropagation algorithm was applied for learning. The measured and predicted load are shown in Fig. 19 and in form of mean value of amplitude and maximum of amplitude

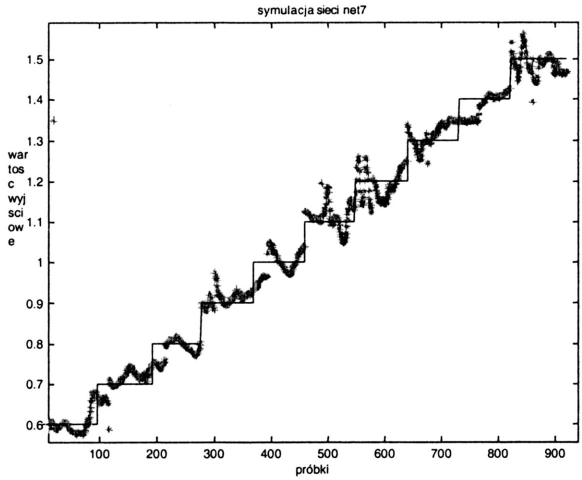

FIGURE 18. Performance of state classification neural network.

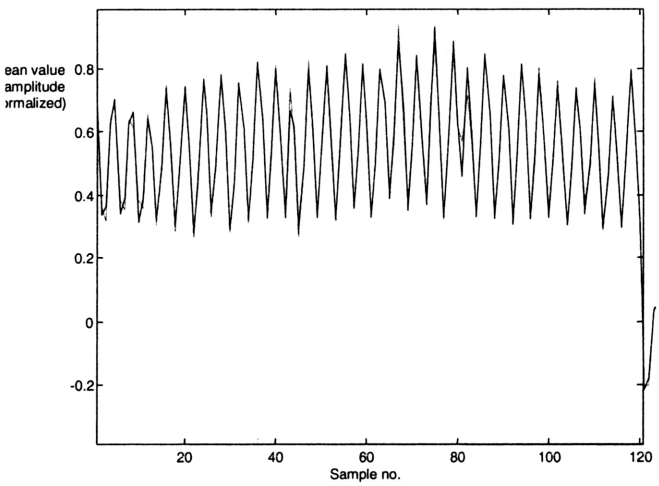

FIGURE 19. Measured and predicted mean value of amplitude (normalized) of bending load of helicopter rotor.

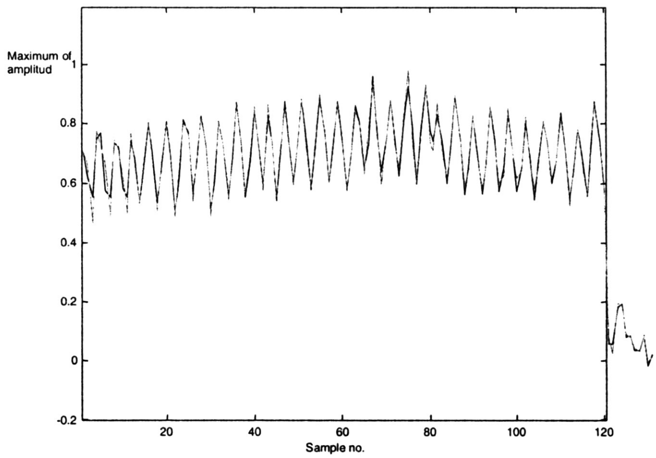

FIGURE 20. Maximum of amplitude of bending load of helicopter rotor.

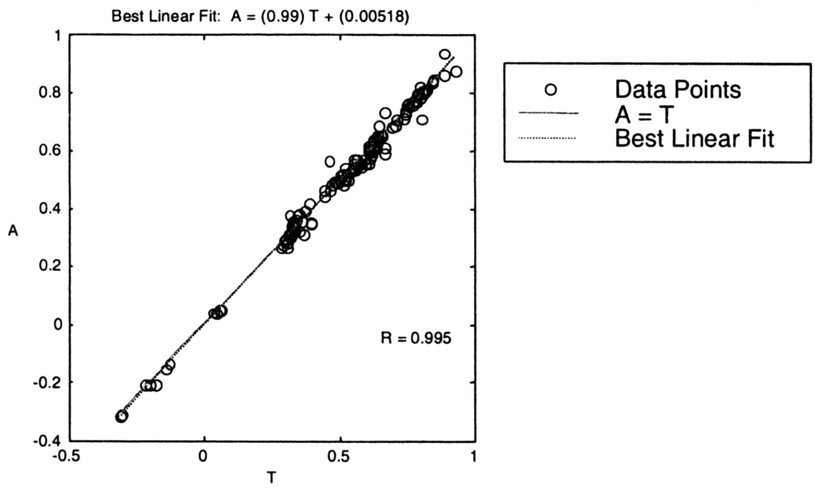

FIGURE 21. Correlation test results for maximum amplitude of bending load.

in Fig. 20.To show convergence between predicted and measured load the correlation has been tested. Correlation test results are shown in Fig. 21. As we can notice from above plots designed neural networks gives a good approximation of helicopter component loads.

3.4. Conclusions and final remarks

Neural networks seem to be a very effective tool in flight state classification and load identification of helicopter structures, but learning process is very time consuming process which strongly depends on used training data set. But load prediction process using neural networks approach is very effective and can be done on-line. Further research that is continuation of this study should go in direction of hardware realisation of trained neural network and direct application on helicopter board.

4. Deterministic methods of load identification – state of the art

Deterministic methods of load identification can be grouped in two categories:

- frequency domain methods [26, 28, 29],
- time domain methods [25, 30, 31, 32].

This classification is based on signal processing methodology applied for experimental data which are necessary to perform estimation of loads parameters process.

Basic methods of identification of excitation forces has been formulated for linear systems in which assumption about small damping and stationarity of parameters are valid. Methods in frequency domain require information about FRF (Frequency Response Functions) for investigated structures (inverse of FRF matrix) and spectrum of system responses measured during operation. Based on these information spectrum of excitation forces can be estimated.

Similar methods are formulated in time domain, using relation between excitation and system responses in a form of convolution. An iterative formula for calculation of excitation forces in mechanical structures is proofed based on properties of Toeplitz matrix. Nowadays identification of excitation forces can be perform using mutual energy theorem formulated by Heaviside at 1892. This method allows to identify spectrum of loads based on response (in a form of vibration velocity) measurements.

Given above identification methods can be used for systems which have linear properties. But for linear and nonlinear systems methods based on minimization of given objective function. Mainly a least square error between simulated and measured system response is used as objective function in this identification method. The dynamic programming optimization method formulated by Bellman [27] is commonly use for minimization of objective function to estimate excitation forces. Some examples of application of these methods are shown in [27].

4.1. Frequency domain method for load identification

In many practical problems knowledge of loads spectrum is required for diagnostics purpose. In such a case different method for load identification should be applied. This method requires measurements of spectrum of system responses at several points of a structure during operation, but frequency response function (FRF) as a characteristics of a system are also necessary to apply this method. FRF function can be identify experimentally or can be predicted using finite element model of a structure. Using finite element model of the system for predicting of the FRF the method can be apply to modify structure dynamics to achieve better performance e.g. lower level of loading forces.

4.1.1. Formulation of the method. The method is essentially based on inversion of FRF matrix and is commonly used in energy transfer path analysis in mechanical systems [25]. This method can be used under following assumption:

- system is linear,
- Maxwell theorem is preserved,
- system is stationary.

Relation between system excitation (loading force in discrete points) and system responses at discrete points of a structure under consideration can be written in the following form:

$$X_i(\omega) = \sum_{j=1}^{m} H_{ij}(\omega) f_j(\omega), \tag{25}$$

where:
$X_i(\omega)$ – spectrum of system response at location i,
$H_{ij}(\omega)$ – the frequency transfer function between response at location i and loading force at location j $(n \times m)$,
$f_j(\omega)$ – the operational loading forces at point j.

FRF function can be identified using impulse (hammer) or sine (shaker) excitation or can be computed from modal model of a structure. Usually only several first modes can be identified and used for computation of FRF functions. Matrix $H_{ij}(\omega)$ can be also obtained from finite element model simulation results. The last option is very helpful if there is no possibilities to make appropriate experiment.

The first method to identify operational forces is application of FRF matrix pseudo inversion. If the number of inputs and outputs is the same, results in square FRF matrix, direct inversion of FRF matrix can be applied. The formula to compute these forces has the following form (in a case of $n \neq m$) [26]:

$$F(\omega) = H(\omega)^{-1*} X(\omega), \tag{26}$$

where:
$F(\omega)$ – loading operational force vector,
$H(\omega)^{-1*}$ – pseudoinverse of matrix $H(\omega)$ given by equation:

$$H(\omega)^{-1*} = \left(H(\omega) H^T(\omega)\right)^{-1} H^T(\omega), \tag{27}$$

$X(\omega)$ – is response vector measured during the structure operation.

If the number of responses m is bigger then number of loading forces n $(m > n)$ allows to overdetermined the set of equations. By this, more then just the necessary information is used for the calculations of the operational forces. This allows you to obtain better and more accurate least square estimates results for the operational forces.

As response accelerations at n – points can be used, but for flexible structure with very low natural frequency and big displacement, displacement can be used as a system response. In these cases accelerance matrix should be used as the FRF for a first case and compliance matrix in the second one. To avoid numerical problems with in the matrix inversion singular value decomposition methods are commonly used.

The FRF matrix should be identify when the load source is disconnected from the structure. It means that laboratory experiment with not operated system is required, sometimes such system state is different from FRF function point of view, e.g. sliding bearings.

For the system in which loading force sources are connected to the structure via flexible mounts, the operational forces can be determined based on knowledge of complex dynamic stiffness of the mounts $K(\omega)$ and the difference of operational displacement over the mount during operation condition are applied. This difference can be measured using accelerometers and integration amplifiers.

The formula for operational loading force in j-th mount has the following form:

$$f_j(\omega) = K(\omega) \cdot \big(X_{j1}(\omega) - X_{j2}(\omega)\big), \tag{28}$$

where:
$f_j(\omega)$ – is operational force in month j-th,
$K(\omega)$ – is the complex dynamic stiffness,
$X_{j1}(\omega)$ – is the operational displacement on mount at the location of structure side,
$X_{j2}(\omega)$ – is the operational displacement on mount at the location of loading force side.

Described above method has been verified on experimental setup. During and experiment loading forces are measured and identify.

4.1.2. Experimental investigation. Experimental setup used for testing of presented approach is shown in Fig. 22.

During an experiment the frame under investigation has been hang on elastic suspension to avoid interaction with environment. The measuring points net (80 points on a structure) is shown diagrammatically in the Fig. 23. At measuring points during an experiment acceleration in three perpendicular directions have been measured.

As an excitations two electrodynamics shakers have been attached to the frame. The scheme of applied measuring setup is shown in Fig. 24.

Elements of matrix $H(\omega)$ have been estimated using classical procedure based on power spectral density measurements of outputs and inputs. In this case as input, forces generated by shakers and as outputs, accelerations at all points and directions are employed. Examples of measured $H_{ij}(\omega)$ are shown in Fig. 25. The main results of modal analysis of investigated frame are summarized in Table 1.

These modal parameters are used as the structure characteristics for operational loading force identification. To verify formulated identification method operational loading force has been measured. These measured forces have been compared with identified one. The comparison is shown in Fig. 26.

Comparison results shown in Fig. 26 indicate for a good agreement between measured and identified forces spectrum except frequency range about structural resonance's. At this frequency range the FRF inversion is not accurate and system nonlinearities which disturb FRF much more then out of resonance's.

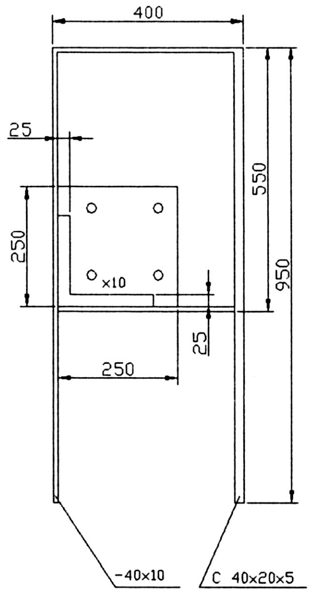

Figure 22. Structure under test.

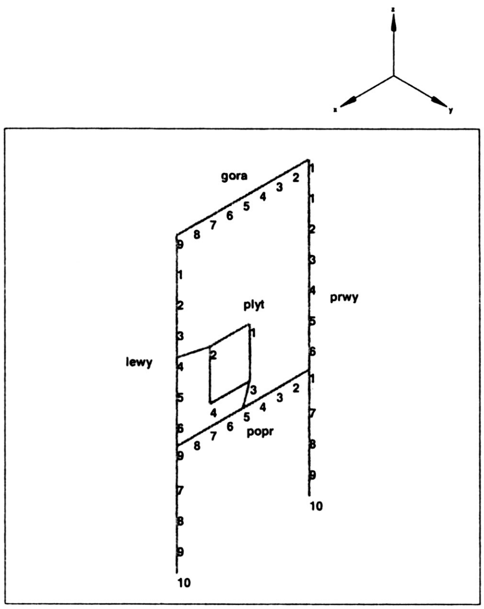

Figure 23. Measurement points located during a test.

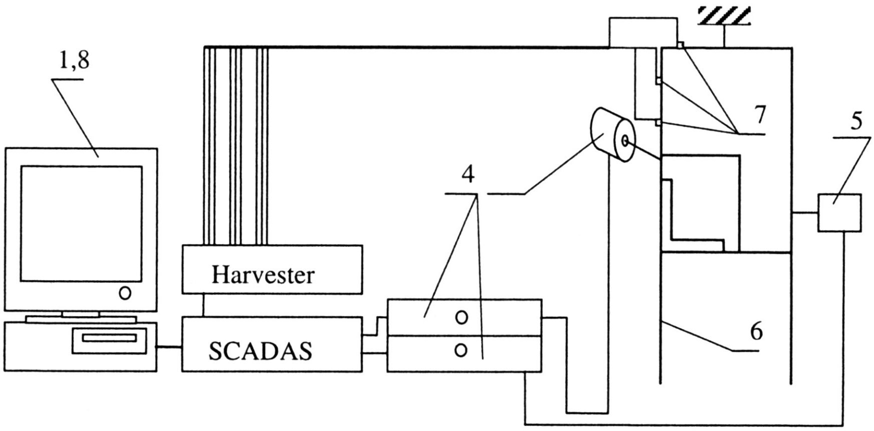

FIGURE 24. Applied measuring setup.

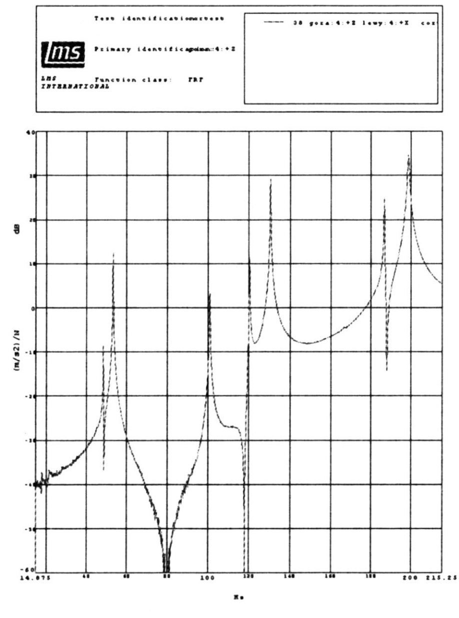

FIGURE 25. Example of measured FRF.

TABLE 1. Results of modal test of frame.

No. PDW	Frequency [Hz]	Modal dumping [%]
1	47,98	0,37
2	53,05	0,10
3	101,15	0,21
4	119,73	0,08
5	130,31	0,27
6	187,69	0,18
7	198,74	0,20

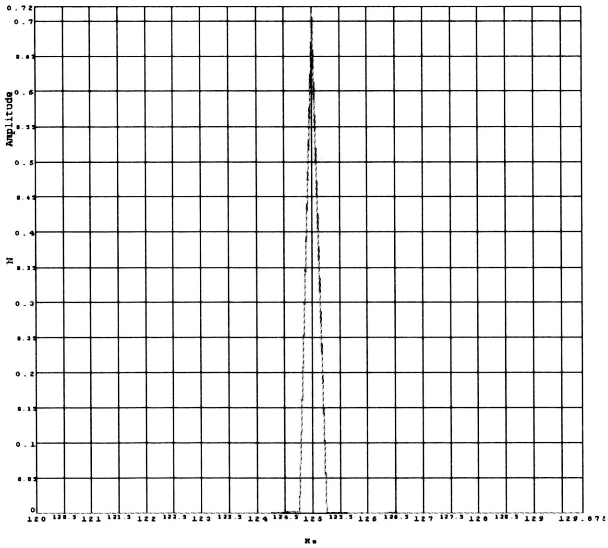

FIGURE 26. Comparison of measured and identified excitation force spectrum.

4.2. Time domain methods for loads identification

The methods in time domain can be grouped into three categories [30]:

- Methods based on minimization of the quality function defined on differences between model output and measured system response,
- Method based on inverse of convolution,
- Method based on mutual energy theorem.

The method based on minimization of the quality function is based on idea of inverse identification problem. This method is based on assumption that model simulation results and measurements on real existing structure for given excitation are the same. This requires model identification as a separate task to get high quality model, which gives simulation results that are convergent with measurements.

The main idea of this method involves minimization of the difference between measured response of the simulation results. The measure of this difference can be a sum of squared differences for given time samples of measured and simulated system responses.

This difference is consequence of differences between actual excitation and simulated one, only. Since not all coordinates used as model variables can be measured, used set of model variables which are under comparison has to be reduced to that can be measured. During the minimization of quality function, dynamic programming can be used. The decision variables for this minimization are time samples of excitation forces under estimation. According to dynamic programming methodology of minimization and Bellman optimality principle, the minimum value of quality function can be sought by minimization of this function in each step of computation (at each time sample). The basic algorithm of the method is shown in Fig. 27. The method is more detail described in [29, 30]. Author implemented this procedure in software and applied for real structures [28].

The second groups of methods are based on classical convolution theorem, which permits to predict dynamic system response based on knowledge of impulse response function for the structure and time history of excitation. To explain the idea a system with one input and one output will be considered.

This theorem formulated for response in a form of vibration velocity can be written as:

$$v(t) = \int_0^t P(\tau)g(t-\tau)dt, \tag{29}$$

where:
$v(t)$ – vibration velocity (response of the system),
$P(\tau)$ – excitation (input) force (to be identify),
$g(t-\tau)$ – impulse response of structure.

Formula (29) can be rewritten in the matrix form:

$$v = \Delta T[g]P, \tag{30}$$

where:
$v = [v(1), v(2), \cdots, v(N)]^T$ – is a velocity vector (vector coordinates are time samples of velocity),
$P = [P(1), P(2), \cdots, P(N)]^T$ – is a force vector (vector coordinates are time samples of force),

$$[g] = \begin{bmatrix} g(1) & 0 & \cdots & 0 \\ g(2) & g(1) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ g(N) & g(N-1) & \cdots & g(1) \end{bmatrix}$$ – is a Toeplitz matrix,

ΔT – sampling period.

Time samples of force (coordinates of force vector) can be computed from iterative formula:

$$P(i) = \frac{1}{g(1)}\left[\frac{1}{\Delta T}v(i+1) - \sum_{j=1}^{N} g(j)P(i-j)\right]. \tag{31}$$

Formula (31) can be used for estimation of excitation forces based on velocity of response measurements and impulse response functions characteristics of the system.

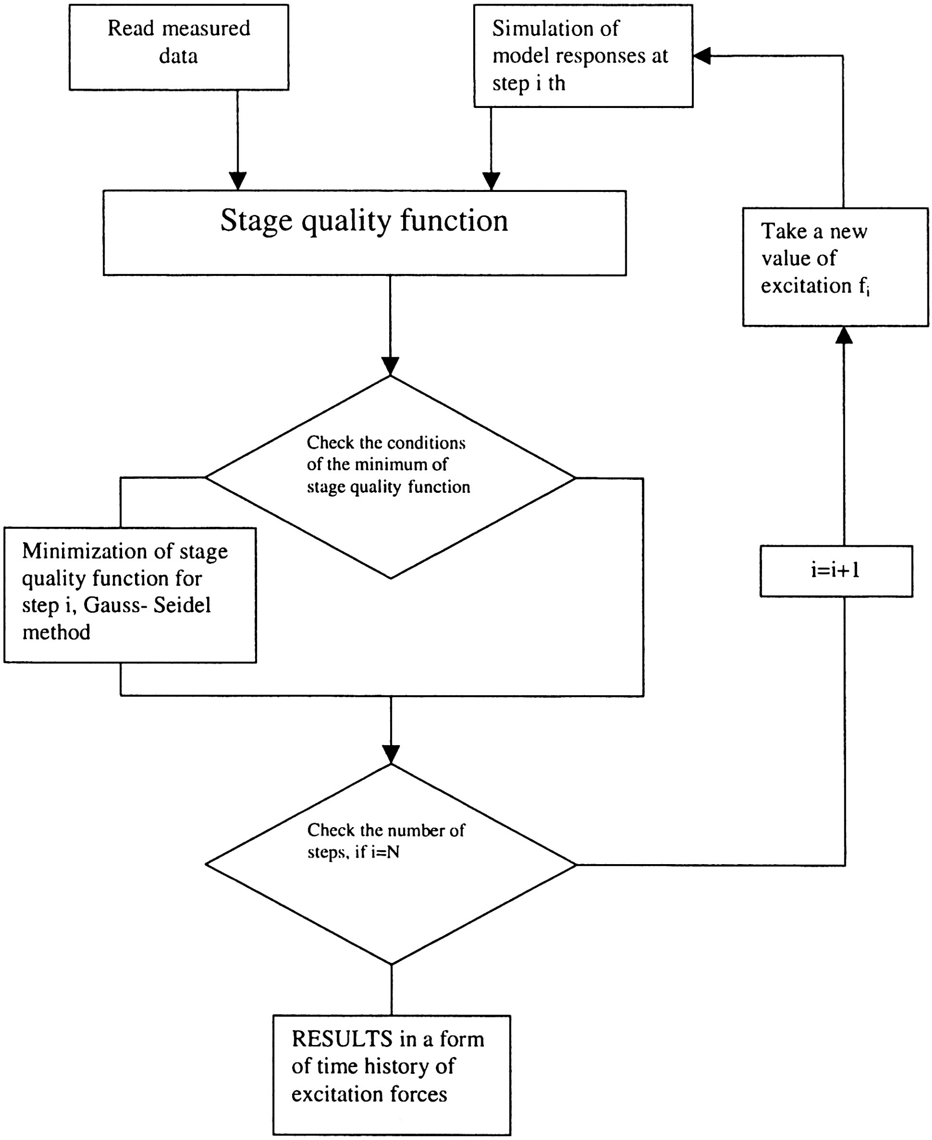

FIGURE 27. Scheme of loads identification algorithm based on objective function minimization.

The method can be extended for multi input and multi output system. Author for practical identification has applied the method and formulated a software tool for method realization [30].

The last group of methods involves methods based on mutual energy theorem formulated by Heaviside [33]. This method has been applied to identification of spectrum of excitation forces in mechanical systems in multi input and multi output case [31, 32]. The basic formula for estimation of excitation forces has a form:

$$F_A = V_B^{-1} V_A F_B, \tag{32}$$

where:

$$V_B = \begin{bmatrix} \begin{bmatrix} & 0 \\ V_{B_1} & \end{bmatrix} & \begin{bmatrix} & 0 \\ V_{B_2} & \end{bmatrix} & \vdots \\ \begin{bmatrix} & 0 \\ V_{C_1} & \end{bmatrix} & \begin{bmatrix} & 0 \\ V_{C_2} & \end{bmatrix} & \vdots \\ \cdots & \cdots & \cdots \end{bmatrix}, \qquad V_A = \begin{bmatrix} \begin{bmatrix} \ddots & 0 \\ V_{A_1} & \ddots \end{bmatrix} & & \\ & \begin{bmatrix} \ddots & 0 \\ V_{A_2} & \ddots \end{bmatrix} & \\ & & \ddots \end{bmatrix},$$

V_B – is a $(nN \times nN)$ matrix of measurements at points located on a structure due to force at point B and C,
V_A – is a $(nN \times nN)$ matrix of measurements at points located on a structure due to force at point A,
N – number of samples in time series,
n – number of excitation forces,
V_{A_i} – Toeplitz matrix formulated from measurements at point i due to force at point A,

$$F_B = \begin{bmatrix} F_{B_1}(1) \\ \vdots \\ F_{B_1}(N) \\ F_{B_2}(1) \\ \vdots \\ F_{B_2}(N) \end{bmatrix}_{(nN\times 1)}, \qquad F_A = \begin{bmatrix} F_{A_1}(1) \\ \vdots \\ F_{A_1}(N) \\ F_{A_1}(1) \\ \vdots \\ F_{A_2}(N) \end{bmatrix}_{(nN\times 1)}.$$

From the results of verification of shown that methods is enough accurate for practical application and the most error generating process is inversion of Toeplitz matrix [31].

4.3. Conclusions and final remarks

Presented frequency and time domain methods can be applied for multi-input systems if sufficient data are available. These methods required very efficient computer system due to complex algorithms. The best results are achieved if excitations are deterministic functions but not stochastic process. These methods require measurements of system response in the form of velocity, displacement of acceleration (time waveform or spectrum) but strains also can be applied.

5. Statistical methods – application of regression analysis for identification of loads based on process parameter measurements

The approach based on regression model of relation between loads and flight data (data recorded using standard flight recorder) is presented for helicopter in [40]. The quantitative measure of quality of regression model is coefficient of determination R^2 that is, for many components of investigated helicopter type H-53, relatively low 78% [40]. It was a motivation to find better methodology of loads prediction and shows that linear model is not sufficient to find loads of particular helicopter components based on flight data. Proposal of application of neural networks for such data is shown in section 3 of this text. But, conducting regression analysis during the evolution of flight loads survey can be a valuable tool in providing the fatigue analyst with understanding of the process parameters that influence loads. Such information is not available from neural networks based approach.

5.1. Formulation of the method

Regression analysis is a widely use mathematical tool for modeling of many different physical phenomena because of well known theory and parameters estimation procedures. Mainly linear multiple regression models are in use for modeling of mechanical system properties [37] in the form:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon, \tag{33}$$

where: β_i $(i = 0, 1, 2, \ldots, k)$ are regression coefficients, x_i are independent variables, y is dependent variable.

The model given by (33) is linear because output is a linear function of parameters β_i.

There are two main problems in identification of regression model for mechanical systems:

1. Choice of model order.
2. Estimation of model parameters based on experimental results.

Regression analysis has one big advantage, models that are more complex (which contain terms $x_i x_k$ or x_i^n) may often still be analyzed by multiple linear regression technique. The method of least squares is typically used to estimate the regression coefficients in multiple linear regression model. Suppose that $n > k$ observations on the response variables are measured $(y_1, y_2, \ldots, y_n)$ the regression parameters can be found from the formula:

$$b = (X^T X)^{-1} X^T y, \tag{34}$$

where:
$b = [\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k]^T$ is a regression vector contains estimators of regression parameters,
$y = [y_1, y_2, \ldots, y_n]^T$ is a vector of observations $(n \times 1)$,

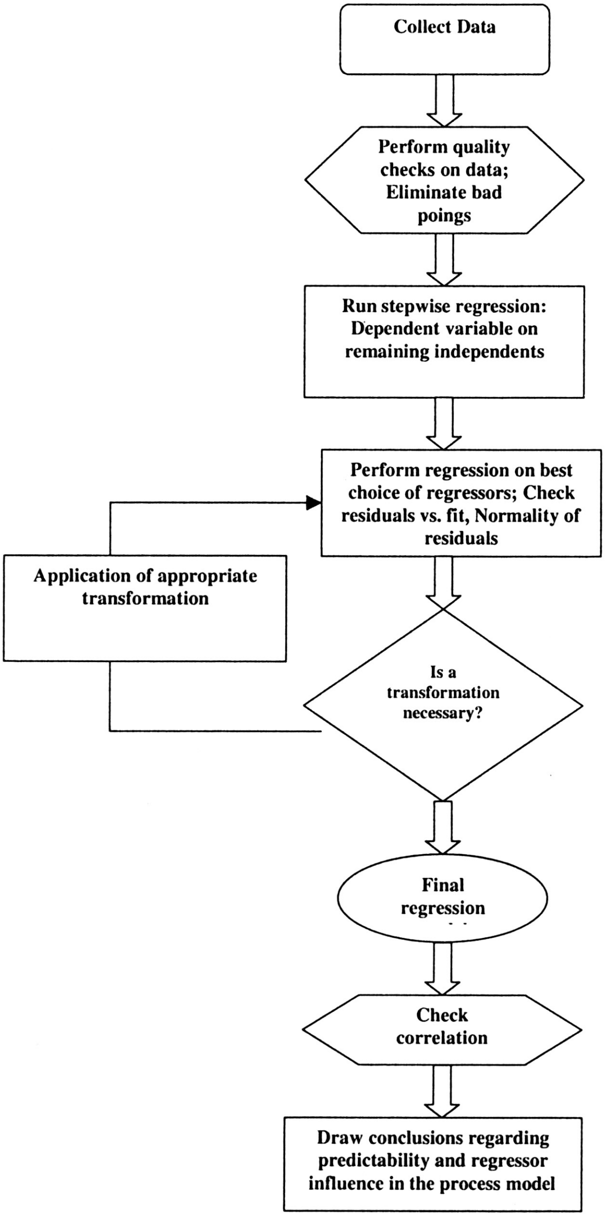

FIGURE 28. Algorithm of loads identification using regression analysis.

X is a $(n \times k)$ matrix of measurements (independent variables):

$$X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix}.$$

The estimator of regression parameters given by formula (2) is an unbiased estimator and residual sum of squares (estimation error) can be found from the formula:

$$SS_E = y^T y - bX^T y \tag{35}$$

The error given by formula (35) is a measure of quality of identified model. To find the best model structure certain tests of hypothesis about model significance and model parameters significance are very helpful. The most useful are:

- test for significance of regression,
- test on individual regression coefficient significance.

If given regression parameter is not significant for set of experimental data, this parameter should be canceled from the model and estimation process has to be repeated.

The idea of application of linear multiple regression for identification of loads in mechanical systems is based on approximation using regression model relation between process parameters and load vector. This method allows defining which process states have influence on load vector. For many different applications described above technique gave a promising results [35, 36, 37, 46].

To apply described above method for load identification the special MATLAB based procedure in a form of m.file has been formulated. The procedure executes the algorithm shown diagrammatically in Fig. 28.

The algorithm has been applied for identification of load of helicopter components during flight. The choice of model order has been done employing stepwise regression.

Stepwise regression is a technique for choosing the variables to include in a multiple regression mode [36, 43]. Forward stepwise regression starts with no model terms. At each step it adds the most statistically significant term (the one with the highest F statistic or lowest p-value) until there are none left. Backward stepwise regression starts with all the terms in the model and removes the least significant terms until all the remaining terms are statistically significant. It is also possible to start with a subset of all the terms and then add significant terms or remove insignificant terms. An important assumption behind the method is that some input variables in a multiple regression model do not have an important explanatory effect on the response. If this assumption is true, then it is a convenient simplification to keep only the statistically significant terms in the model.

5.2. Applications of the regression analysis method for loads identification of helicopter structures

The experiment has been done on SW-3 helicopter at PZL Świdnik. Collected data includes 46 different flight states. The general location and method of sensor placement has been outlined above. Table 2 lists the measurements that will be used in the process

TABLE 2. Recorded standard flight data.

No.	Symbol	Parameter	Sampling [Hz]
1.	LoadY	Loading in the vertical direction	8
2.	RollTs	Bank	4
3.	PitchTs	Pitch	4
4.	Psi	Geometric Heading	2
5.	OutTemp	Ambient temperature	2
6.	Px	Loading in the horizontal direction	2
7	Hg	Geometric Altitude	2
8.	Hmr	Swashplate input	4
9.	Slip Angle	Slip	2
10.	Htr	Deflection in tail section	4
11.	Kappa	Swashplate displacement vertical	4
12.	Eta	Swashplate displacement horizontal	4
13	MRrpm	Main rotor Rpm	1
14.	BFlapAng	Collective position	2
15.	BPitchAng	Longitudinal displacement of steering mech.	2
16.	LEmom	Torque on drive shaft of left engine	2
17.	REmom	Torque on drive shaft of right engine	2
18.	RollHel	Horizontal displacement of collective control	2
19.	Velocity	Indicated Air Speed	1
20.	PresAlt	Barometric Altitude	1
21.	RpmLeng	Left engine turbine RPM	1
22.	RpmReng	Right engine turbine RPM	1
23.	PitchHel	Pitch of Helicopter	
24.	AngVelX	Angular Velocity X axis	
25.	AngVelY	Angular Velocity Y axis	

modeling procedure. These measurements are registered during the flight by the flight recorder (BUR1-2). They are considered the "preferred" inputs, because they do not involve any extra instrumentation of the helicopter. Consequently these inputs will be used when a correlation with the loads is sought after.

In addition of flight parameters many different structural parameters listed in Table 3 have been measured using thirty-two-channel ESAM measurement unit.

The stepwise regression done for measured data shows that the model of helicopter load by means of damper moment and bending moment of a hube could be fit with a fairly high accuracy using only five explanatory variables namely, "Load Y, Pitch-Hel, Slip Ang, MR rpm and BFlapAng". After choosing these variables accordingly the regression has been performed which yielded the statistics and plots that follow.

To perform described above regression analysis the MATLAB m.file is prepared which includes the following steps:

- Read data set and load it into the MATLAB workspace environment.
- Eliminate erroneous and faulty readings.
- Identify and eliminate discontinuities.
- Resample portions of the data.
- Compress the data into usable form.

During the measurement process, because of faulty measuring instruments and techniques, imperfection of the observer and the influence of outside factors, the value ob-

Table 3. Recorded additional data during test flight.

PCM	CHAN	POINT	DESCRIPTION	Sampling [Hz]
1	1	400.0	Vibration under pilots seat X	400
2	2	401.0	Vibration under pilots seat Y	400
3	3	402.0	Vibration under pilots seat Z	400
4	4	420.7	Vibrations main transmission X	400
5	5	421.7	Vibrations main transmission Y	400
6	6	422.7	Vibrations main transmission Z	400
7	7	205.4	Damper moment MR around vertical joint 5-5'/I	400
8	8	215.4	Damper moment MR around vertical joint 5-5'/II	400
9	9	225.4	Damper moment MR around vertical joint 5-5'/III	400
10	10	235.4	Damper moment MR around vertical joint 5-5'/IV	400
11	11	207.4	Moment on the hub of the MR 7-7'/I	400
12	12	208.4	Bending moment on the hub of MR 8-8/I	400
13	13	209.4	Bending moment on the hub of MR 9-9/II	400
14	14	291.4	Bending moment of the link hub of damper 5-6-7-8/I	400
15	15	2.4	Blade bending moment of the MR LM (2-2')/I	400
16	16	102.4	Blade bending moment of the MR LD (2-2')/I	400
17	17	504.4	Blade oscillation angle MR around horizontal hub pos. beta/I	400
18	18	505.4	Blade oscillation angle MR around vertical hub pos. xi/I	400
19	19	515.4	Blade oscillation angle MR around vertical hub pos. xi/II	400
20	20	525.4	Blade oscillation angle MR around vertical hub pos. xi/III	400
21	21	535.4	Blade oscillation angle MR around vertical hub pos. xi/IV	400
22	22	1.3	Longitudinal Control 51-51'	400
23	23	10.3	Lateral Control 50-50'	400
24	24	2.3	Force on collective 52-52'	400
28	28	821.0	Gravity Loading Y axis	400
29	29	501.3	Pitch Ts	400
30	30	500.3	Bank Ts	400
31	31	502.3	Collective angle MR	400
32	32	503.3	Cyclic Control Angle	400
33	33	802.0	Slip Angle	400
34	91	981.0	ON/OFF Channel	100
35	34	600.0	Ambient Temperature	100
36	35	800.1	Barometric Altitude	100
37	36	800.8	Velocity of flight	100
38	37	852.8	Left Engine moment	100
39	38	851.8	Right engine moment	100
40	39	801.0	Pitch helicopter	100
41	40	800.0	Bank of helicopter	100

TABLE 3 [cont.]. Recorded additional data during test flight.

PCM	CHAN	POINT	DESCRIPTION	Sampling [Hz]
82	81	810.0	Angular velocity X	100
83	82	811.0	Angular velocity Y	100
95	984	984.0	Fuel consumption RE	100
96	985	985.0	Fuel consumption LE	100
97	986	986.0	Right engine rpm	50
98	987	987.0	Left engine rpm	50
99	988	988.0	Main Rotor rpm	50
100	989	989.0	Marker	50
101	990	990.0	frame counter – unused	50
102	991	991.0	frame counter – average	50
103	992	992.0	frame counter – fast	50
104	993	993.0	PCM format number	50
105	994	994.0	Operational number	50
106	995	995.0	IRIG Time	50
107	996	996.0	IRIG Time	50
108	997	997.0	Synchro A|B11101101	50
109	998	998.0	Synchro 1011010010	50
110	999	999.0	Synchro 0001010000	50

served often differs from the actual value. The difference between the value measured and the actual value is termed absolute error. Because one does not know the actual value nor the absolute error the task of anyone performing measurements is to determine the interval $x \pm \Delta x$ in which their is a large probability of finding the actual value. This is addressed in literature [41] along with three major types of errors that may be encountered either simultaneously or separately during the measurement process:

- systematic error,
- random or stochastic error,
- gross error.

This third error was encountered with the data sets obtained from PZL. Gross error most often occurs due to the experimenter's negligence. This negligence can manifest itself in the form of improper measurement readout or miscalculation. The gross error in the case of the datasets from PZL however, was caused by external noise and momentary lapses (periods of inactivity) of the measuring devices. Such lapses appear as "NaN" (not a number), in the output vector for a given time instant. The normal procedure would be to perform the measurements once again using more diligence in the acquisition process. This is not economically feasible with this particular data. Missing values are calculated as a weighted sum of linear interpolations from nearest available points. Altogether 5 estimates from column-wise and 5 for row-wise 1-d linear interpolation are calculated. Weights are such that for the best case (isolated missing points away from the boundary) the interpolation is equivalent to average of 4-point Lagrangian polynomial interpolations from nearest points in a row and a column.

This sampling problem had to be resolved before further analysis of the data could be considered. The reason for this is that for a specific output at certain time instants data that is sampled at lower frequencies will simply not exist. For example if one were to try to determine the ambient temperature (sampled at 100 Hz) and moment acting on the damper (sampled at 400 Hz) at time interval $(0 + 2T)$, where T is the highest sampling rate, 400 Hz in this case, the result would yield an eight hundred long vector for the damper moment but only a two hundred long vector for the temperature would be returned. This essentially means that there would not be a value for temperature at certain values of the moment. Consequently the approach to solve this synchronization problem was to interpolate the intermediate values.

The loading characteristic is one of a periodic nature, where the period is related to one revolution of the rotor. During this cycle the loading achieves its maximum and minimum value As a consequence of this, compression was performed by taking the average value of the loading for every revolution of the main rotor.

This procedure derived from reference [38] normalizes the values of each vector into the interval $[-1 < 0 < 1]$. It is a procedure that often greatly increases the speed of calculation. It is necessary when the compared data differ in orders of magnitude. When a matrix is formed and one column vector is of the magnitude 10^3 while the value of another vector are of 10^{-3} the smaller values would be listed by MATLAB as zeroes (insignificant digits). The normalization procedure helps to eliminate this problem. An important point of notice is that sometimes the data was not normalized. This was done so that one could "get a feel" for the orders of magnitude of the parameters that were worked with. Such an approach was specifically adopted for the purposes of graphing the predicted against the measured moments.

The parameters that were identified of central importance pertaining to loads in a helicopter were the blade bending moment as well as the moment acting on the damper

TABLE 4. Regression model for helicopter data in flight no. 6245.

Regression Analysis: Blade Bending Mom versus LoadY; PitchHel; ...

```
Weighted analysis

The regression equation is
Blade Bending Mom = 0,0633 + 13,4 LoadY - 0,832 PitchHel
                 + 0,277 Slip Angle + 2,44 MRrpm + 30,1 BFlapAng
```

Predictor	Coef	SE Coef	T	P	VIF
Constant	0,06326	0,01047	6,04	0,000	
LoadY	13,4318	0,2903	46,27	0,000	1,2
PitchHel	-0,83224	0,09280	-8,97	0,000	1,8
Slip Ang	0,27679	0,02010	13,77	0,000	1,1
MRrpm	2,4395	0,1282	19,03	0,000	1,2
BFlapAng	30,0983	0,1492	201,71	0,000	2,1

S = 0,01451   R-Sq = 96,2%   R-Sq(adj) = 96,2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	5	22,3089	4,4618	21188,85	0,000
Residual Error	4194	0,8831	0,0002		
Total	4199	23,1921			

of the main rotor. Such is the case presented in this particular load identification problem. Simply stated the problem lies in the fact that the loads occur in the rotating frame. This causes direct measurement to be an expensive and challenging procedure. In fact it is a procedure that is not applied in general aviation unless for experimental purposes such as the ones brought forth in this thesis. An alternative exists, which eliminates direct instrumentation measurement, to obtain the loading spectrum. It has been found that some of the parameters measured within the fixed frame are highly correlated with those of the rotating one. There are certain statistical analysis techniques, which identify these parameters as well as enable the formulation of the correct mathematical equation of this relationship. This type of approach to the problem is termed "process modeling".

The use of these specific analysis techniques enables the engineer to construct a statistical model that describes a particular scientific or engineering process. These types of models can be used for:

- prediction of process outputs,
- calibration,
- process optimization.

The prediction of process outputs serves as a basis for the work that follows. The goal of prediction is to determine either the value of a new observation of the loading or

Table 5. Regression model for helicopter data in flight no. 6421

Regression Analysis: Blade Bending Mom versus LoadY; PitchHel;...

```
Weighted analysis

The regression equation is
Blade Bending Mom = 0,506 + 19,7 LoadY + 2,25 PitchHel
                  + 2,09 Slip Angle + 0,701 MRrpm + 19,5 BFlapAng
```

Predictor	Coef	SE Coef	T	P	VIF
Constant	0,505851	0,005135	98,51	0,000	
LoadY	19,7306	0,4176	47,25	0,000	1,1
PitchHel	2,2483	0,1100	20,44	0,000	1,2
Slip Ang	2,09178	0,03981	52,55	0,000	1,4
MRrpm	0,70146	0,05883	11,92	0,000	1,1
BFlapAng	19,4777	0,2088	93,29	0,000	1,8

```
S = 0,02140     R-Sq = 89,7%     R-Sq(adj) = 89,7%

Analysis of Variance
```

Source	DF	SS	MS	F	P
Regression	5	14,7905	2,9581	6461,22	0,000
Residual Error	3722	1,7040	0,0005		
Total	3727	16,4945			

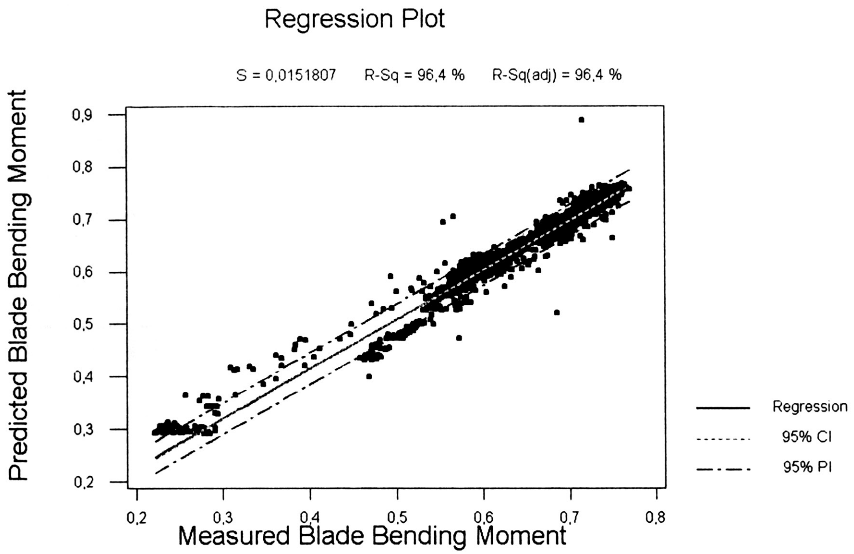

FIGURE 29. Regression model quality for data collected during flight no. 6245.

FIGURE 30. The quality of regression model for flight no. 6421

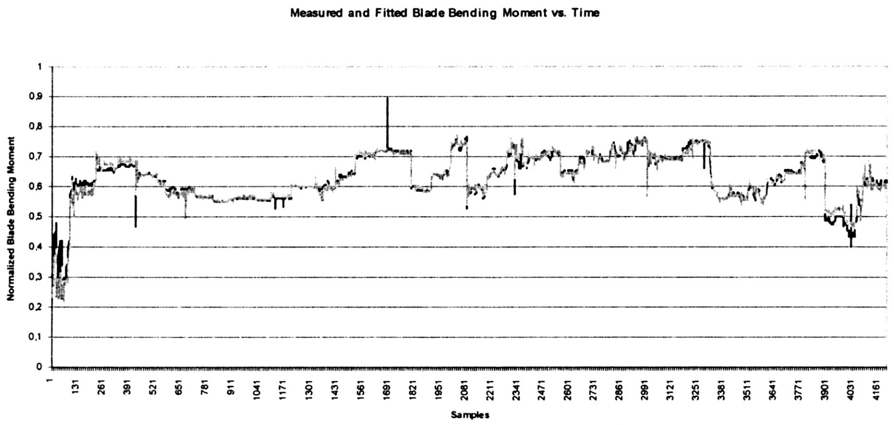

FIGURE 31. Comparison between measured and predicted with regression model blade bending moment for flight no. 6245

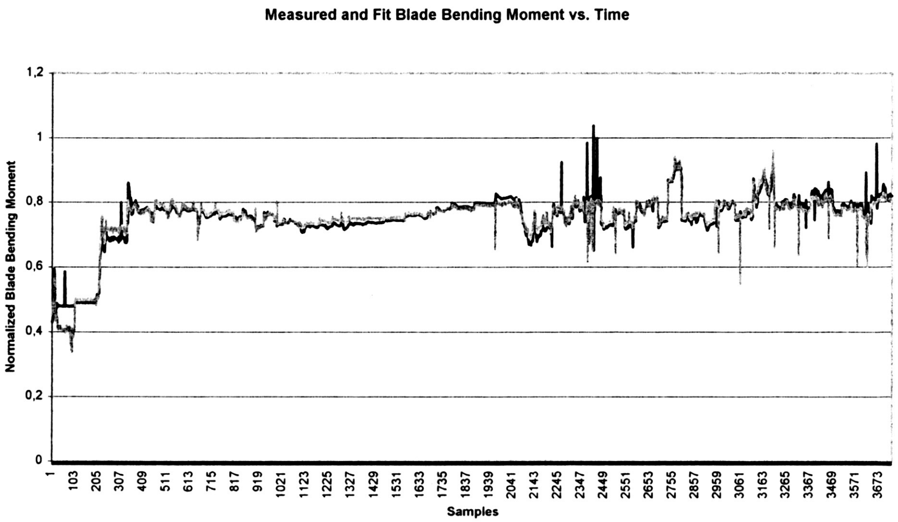

FIGURE 32. Comparison between measured and predicted with regression model blade bending moment for flight no. 6421.

the values of a specified proportion of all future observations of the loading. By loading the parameter blade bending moment or damper moment is meant.

The results for different flight states are presented bellow. For flight no. 6245, that was vertical take off and for flight 6421 that was level flight with speed $v = 180\,\mathrm{km/h}$. For the first case the regression equation has the form shown in Table 4.

The quality of regression function has been tested and obtained results show relatively good fits of experimental data by used regression model. The results of statistical testing of regression model are summarized in table. The regression function is shown in Fig. 29.

The regression model and its statistical quality for light no. 6421 are shown in Table 5.

The regression model and fits for different samples of data are shown in Fig. 30.

The comparison of predicted and measured blade bending moment for both flights is shown in Fig. 31 and 32.

The comparison shows different quality of prediction of bending moment for different flights using regression models which limits application of proposed approach to loads identification using regression model for a case of data acquired from flying helicopter. The main reason is very complex mapping and nonlinear of flight parameters into loads vector. Due to these reason author propose to use artificial intelligence to identify the form of this mapping.

5.3. Conclusions and final remarks

Presented application of regression analysis for load identification of mechanical structures shows a good accuracy of identified model and applicability of the methodology for load identification in this case. The procedure of identification is relatively simple but model order should be selected correctly, this is a big disadvantage of the method. Author applied model order selection based on statistical analysis of quality of regression model. This method gave a good results for the case of helicopter, but in a case of pipeline loda identification that was investigated by author results have been worse.

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