Advanced Course on Structural Control and Health Monitoring
SMART'01 – (pp.191–209) – Warsaw, May 22-25, 2001.

The smart structure concept in aeronautics – is it still there?

J.P. NARKIEWICZ

Department of Automatics and Aeronautical Systems
Institute of Aeronautics and Applied Mechanics
Warsaw University of Technology, Warsaw, Poland
e-mail: jnark@meil.pw.edu.pl

An overview of activities in application of smart structures concept in aeronautics is presented. Modelling of slender bodies and control methods mainly for rotorcraft applications application is discussed. The topics of current and prospective research indicate possibility of technology transfer from and to other fields of science and technology.

Key words: *aeronautics, smart structures, rotorcraft*

1. Introduction

1.1. Why the question mark in the title

In the end of eighties smart structure concept attracted great interest in aeronautic community. Vibration suppression, noise reduction, performance improvement, health and usage monitoring systems, avionics integrated with the structures were considered as places for prospective applications. During nineties in various centres and universities the activity in this field was undertaken resulted in a substantial amount of papers presented at conferences.

There are special conferences devoted to smart structures, see for instance [1], where the substantial results of research are presented, but to estimate the place which smart structures take in aeronautics, general aviation conferences should be referred. In Table 1 the number of total papers and the number of papers devoted to smart structures is given for three selected meetings. The Congress of International Council of Aeronautical Sciences [2] is the general aeronautical conference, which may be used as sample for the trend, and the two other conferences show activity in rotorcraft, a specific segment of aeronautics. Both rotorcraft conferences group researchers from all parts of the world, with the bias for the number of participants from the place where the meeting takes place.

Comparing the numbers of papers presented at ICAO Congresses in the year 1998 with 2000, the activity in smart structures seems to slow down. According to the other data in Table 1 the activity in Europe seem to be at least as vivid as in the USA [3], but also not very impressive.

TABLE 1. Papers devoted smart structures in sample aeronautical conferences in 2000.

Conference	*all*	*smart*
American Helicopter Society Forum	250	10
International Congress of Aeronautical Sciences	250	6
European Rotorcraft Forum	110	10

The slowing down activity may be explained by the well known fact, that in application of new ideas, after some time of enthusiasm, among researchers arises disappointment, as the great results expected before are not to be achieved easily. From the other point of view, the decreasing number of published results may stem from the discipline maturing and a stable level of the research interest. Part of the research and development activity has been undertaken by specialists of specialised fields of technology (and papers are presented on non-aeronautical meetings) and by commercial companies, which are not willing to unveils their achievements.

In authors opinion the second point of view is closer to the reality. Smart structures in aeronautics are still the actual and important subject of the research, although commercial results have not been achieved yet.

1.2. Specific aeronautical topics

In aviation three types of flying vehicles may distinguished due to their specific design and operational aspects: spacecraft, fixed wing (airplanes) and rotary wing (helicopters) vehicles.

It seems that space applications of smart structures were considered the first [4], in the middle of 80's. Still problems of vibration suppression, shape and structure control of space vehicles are actual.

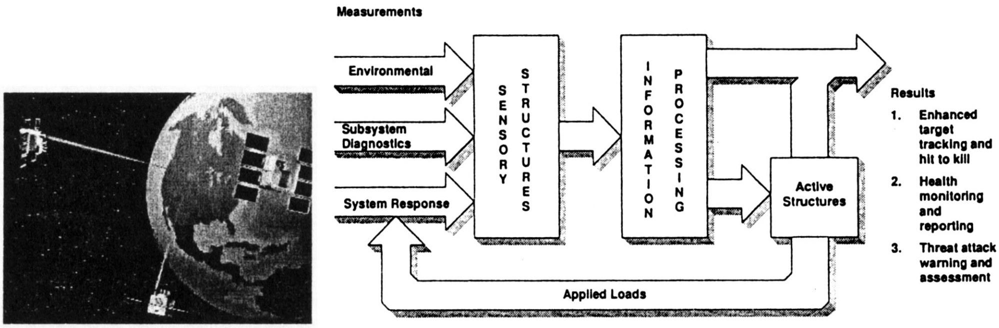


FIGURE 1. Application of smart structures for SDI.

For fixed wing aircraft active suppression of self induced vibrations (flutter) of wing and empennage by application of solid state active elements was investigated [5]. The various possible applications for civil and military aircraft are shown schematically in Fig. 2 [6, 7].

Applications of smart structures to helicopters is illustrated in Fig. 3 [8, 9, 10].

From Figs. 2 and 3 the common features for aeronautical applications (civil and military airplanes, and helicopters) may be selected as engine vibration reduction, avion-

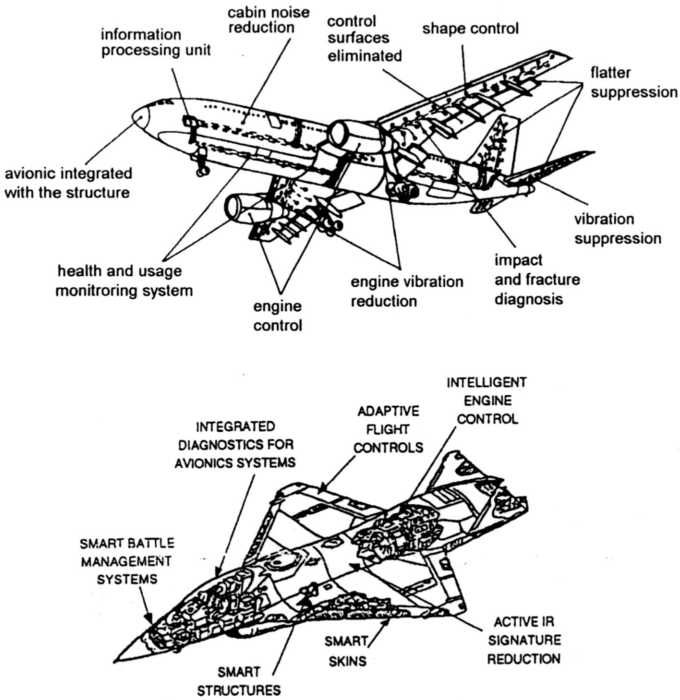


FIGURE 2. Application of smart structures in airplanes.

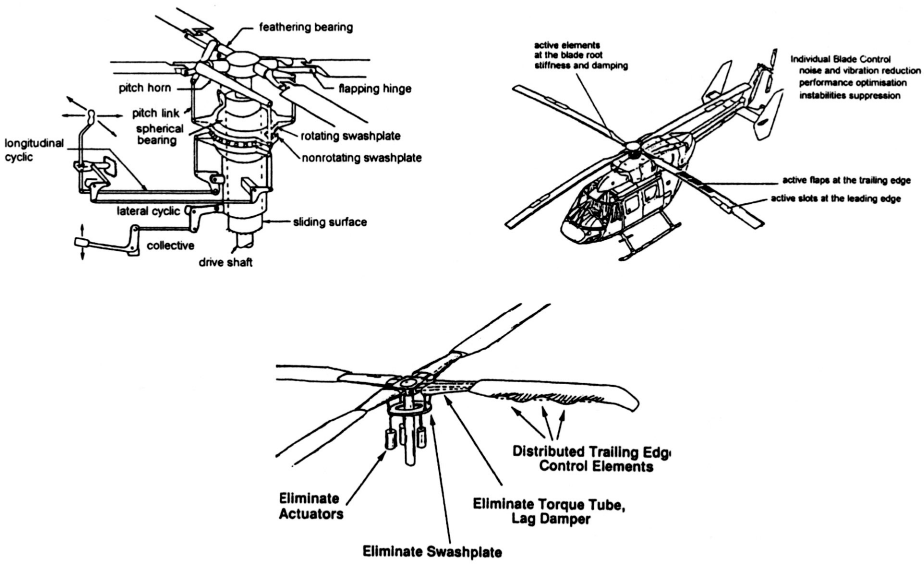


FIGURE 3. Application of smart structures for rotorcraft application.

ics integrated with the structure, health and usage monitoring both for structure and systems. Also the common objectives of smart structure application are performance improvement (in terms of energy savings, extension of flight envelope, stability and manoeuvrability enhancement, internal and external noise reduction.

1.3. Smart structure properties

There are several definitions of smart structure. Generally speaking, the smart structure should wisely adapt to changes in environment and properly react to disturbances [11]. It should combine the load carrying, sensing, actuating and control functions; the smart system classification is shown in Fig. 4.

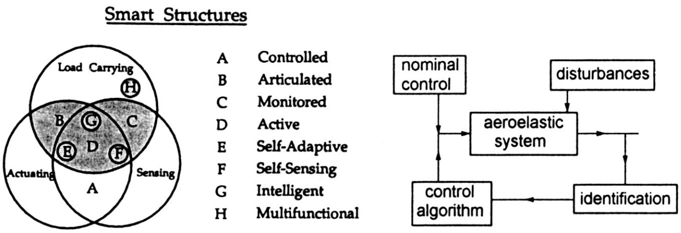


FIGURE 4. Combining functions in smart structures.

A structure to be "smart" should be a solid state (integrated) device, which performs self diagnostics and self repairing. From the short overview above it may be concluded that there are many places in aeronautics where the concept of smart structure may be applied. But it seems to be difficult to obtain all these features in one system, especially a mechanical one.

A good example of this activity may be a shape control of antennas of space vehicles [12]. An antenna is an important part of the spacecraft system allowing receiving signals from outside and information transfer from the station. Due to various systems

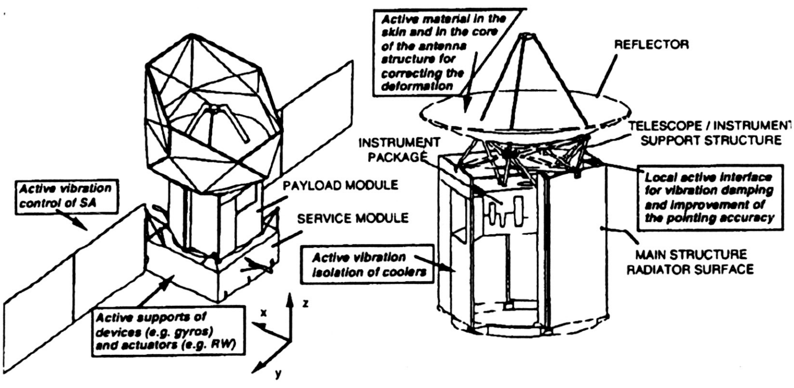


FIGURE 5. Controlling antenna of space station.

on board several antennas of different shapes and sizes are needed. The main restriction in aeronautics is the weight of the vehicle, so reducing the number of antennas and its weight is very important. To save energy and have good signal transfer some antennas should direct signals into the fixed point adjust their shape to radio-waves propagation and there should be no shape disturbances due to heating and cooling or vibrations and support movement. These goals may be achieved by controlling the shape of antennas by actuators embedded into structure Fig. 5.

The other approach is embedding antennae into structure, saving weight. Such smart skin [4, 13] may perform also other useful functions.

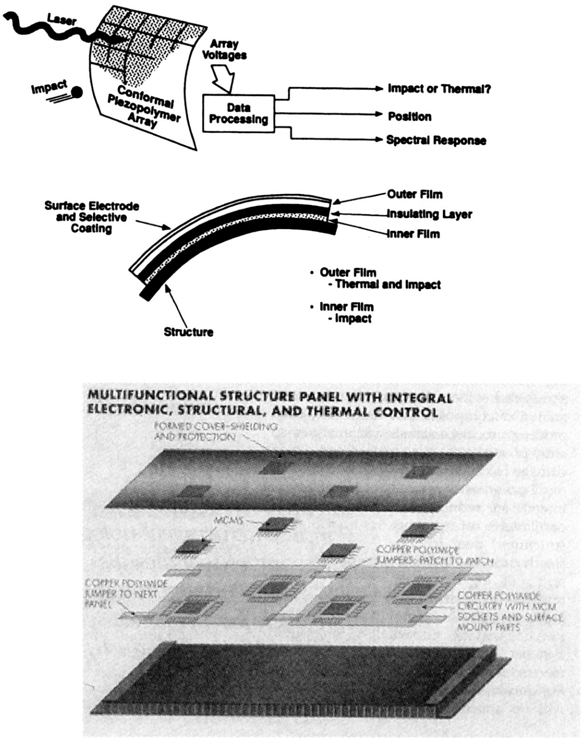


FIGURE 6. Smart skin of a vehicle with embedded electronic chips.

1.4. Contents of the paper

The short overview above indicates from application point of view topics which may be of interest to also other fields of technology.

Loads acting on an aeronautical structure are: aerodynamic (A) and dynamic composed of inertial (I) (gravity included) and elastic (E) Fig. 7. Coupling of these loads may lead to instabilities. Methods for suppressing these adverse phenomena may be passive or active. Interactions between aircraft structural dynamics, aerodynamics, and automatic flight control system is an important design consideration. This interaction is called aeroservoelasticity and is illustrated in Fig. 7. Therefore, the term "aeroser-

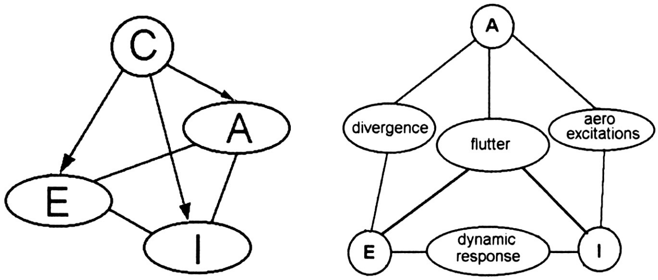


FIGURE 7. Aeroservoelastic loads and aeroelastic instabilities.

voelastic methods" in aeronautics denote methods applied to improve characteristics of aeroelastic systems by feedback control.

In this paper only selected topics may be reviewed, and the two selected are modelling of elongated structures and control methods mainly for rotorcraft applications. This selection stems from authors interest and experience.

2. Model of a continuous structure

General mathematical model of a continuous structure may be described as a boundary value problem for nonlinear unsteady partial differential operator:

$$A\mathbf{w} = \mathbf{F}, \qquad \mathbf{w} \in \Omega \tag{1}$$

$$B_j \mathbf{w}|_{\partial\Omega} = \mathbf{g}_j \qquad j = 1, \dots, s, \tag{2}$$

where: $\mathbf{w}$ – vector of state variables, A – nonlinear, unsteady differential operator, $\mathbf{F}$ – vector of external loads, B_j – boundary value operator, $\mathbf{g}_j$ – functions defining the boundary conditions.

A general description (1), (2) is useful in problem formulation but it must be adjusted to situation to allow practical results. In many cases, complex distribution of mass and stiffness may by simplified, to allow analytical approach to simulation and control. Slender (elongated) structures (like wings and rotor blades) are usually modelled as beams, fins / empennages – as plates and fuselage surface (skin) – as shells. In this paper modelling of elongated structures is discussed in detail.

3. Modelling of slender, elongated structures

3.1. General

Usually slender, elongated structures are modelled as beams. Due to complex shape, stiffness and mass distribution as well as the aim of modelling, various beam models are developed. The beam modelling is especially important for rotorcraft due to high aspect ratio of blades and complex load distribution.

3.2. Typical cross section with trailing edge flap

The typical cross section approach is utilised in preliminary studies or as a test-bed for new aerodynamic load models and control methods. A rotor blade or an airplane wing are modelled as a two dimensional section (airfoil), which properties are selected to describe inertia, stiffness and damping of the real structure. The trailing edge tab may also be included into analysis Fig. 8. In such a model there are three degrees of freedom: vertical translation, rotation (pitch) and tab deflection.

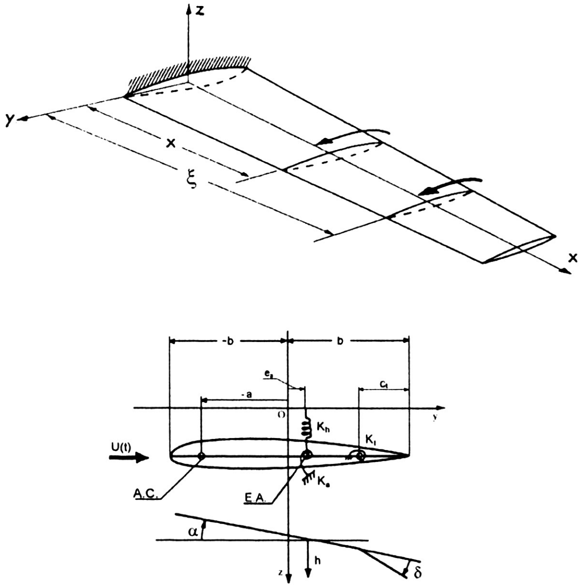


FIGURE 8. Typical cross section approach for modelling blade with trailing edge flap.

The mathematical model of a particular system depends on the way aerodynamic loads are described. Unsteady aerodynamic effects may be modelled (see for instance [14]) as additional flow state variables $\mathbf{y}_i$ fulfilling the system of ordinary differential equations for instance in the form [15]:

$$\frac{d^M y_i}{dt^M} + \sum_{m=1}^{M-1} b_m \frac{d^m y_i}{dt^m} + b_0 y_i = Q(t), \qquad i = 1,2,3, \qquad M = M_1, M_2, M_3. \tag{3}$$

The system of equations of motion cross section has the form:

$$\mathbf{B}\ddot{\mathbf{x}}_\mathbf{P} + \mathbf{D}\dot{\mathbf{x}}_\mathbf{P} + \mathbf{K}\mathbf{x}_\mathbf{P} = \mathbf{F}_\mathbf{A}(\mathbf{x}_\mathbf{P}, \dot{\mathbf{x}}_\mathbf{P}, \ddot{\mathbf{x}}_\mathbf{P}, \mathbf{u}, \mathbf{y}_i), \tag{4}$$

where $\mathbf{x}_\mathbf{P} = [x_{P_i}] = [h, \alpha, \delta]$, $i = 1,2,3$ and $\mathbf{u} = [u] = [\delta_s]$ are vectors of generalised coordinates and control variable respectively; $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{K}$ are mass, damping and stiffness matrices.

In this model the right hand sides of equations of motion contain the second derivatives of the degrees of freedom, so special numerical procedures for solving these equations must be applied.

3.3. Cantilever bending beam

The "standard" model for elongated structures is a one-dimension cantilever bending beam. Partial differential equation of motion, when the damping is neglected, has the form:

$$\mu(x)\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}\right] = P_z(x,t), \tag{5}$$

where $w(x,t)$ – beam deflection; μ – mass distribution along the span, $EI(x)$ – bending stiffness distribution along the beam span, $P_z(x,t)$ – external load distribution, t – time, x – spatial variable.

For a cantilever beam the boundary conditions have the form:

$$w(0,t) = 0, \qquad \frac{\partial w(0,t)}{\partial x} = 0, \qquad \frac{\partial^2 w(l,t)}{\partial x^2} = 0, \qquad \frac{\partial^3 w(l,t)}{\partial x^3} = 0, \tag{6}$$

where l is the beam length.

For the rotating beam, such as helicopter rotor blades, centrifugal forces give additional stiffness in bending due to tension along the blade (Fig. 9).

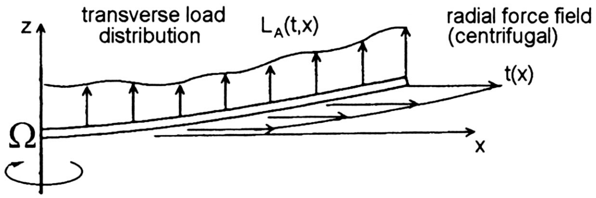


FIGURE 9. Cantilever rotating beam.

The equations of motion have the form:

$$\mu(x)\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 w}{\partial x^2}\right) - \frac{\partial}{\partial x}\left(t(x)\frac{\partial w}{\partial x}\right) = P_z(t,x) \tag{7}$$

where $t(x) = \Omega^2 \int_x^R xm(x)dx$ is a centrifugal tension loads due to rotation.

The solution of (7) is not straightforward and requires using numerical methods.

3.4. Coupled bending-bending-torsion beam model

In many cases the simple bending beam model is not adequate for describing behaviour of the aeronautical structure. In helicopter rotor blades bending deflections are usually coupled with torsion, which must be accounted in astructural model. As aerodynamic loads depend highly on an angle of incidence, bending torsion coupling may be used to avoid aeroelastic instability. The mechanism of such stabilising effect is explained in Fig. 10 [16]. The procedure of selecting useful wing/blade dynamic and aerodynamic properties to avoid aeroelastic instabilities is named aeroelastic tailoring.

The approach to model helicopter rotor blade as a slender beam subjected to coupled deflections in two directions and twist is presented below [17, 18].

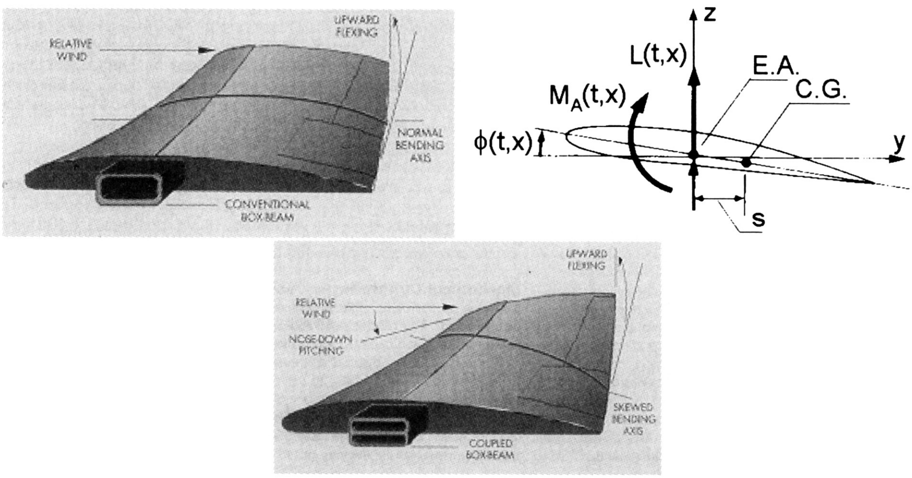


FIGURE 10. Twist bending coupling.

The crucial step in formulation elastic loads Q_E in equations of motion is expressing the structural operator as variation of potential energy U of elastic deformation, according to the formulae:

$$Q_E = \int_{t_1}^{t_2} \delta U \, dt \tag{8}$$

For a slender body potential energy of elastic deformation may be assumed in the form:

$$U = \frac{1}{2} \int_0^R \int_A \left(E\varepsilon_{11}^2 + 2G(\varepsilon_{12}^2 + \varepsilon_{13}^2) \right) dA \, dR, \tag{9}$$

where elastic strain components ε_{22}, ε_{33} and ε_{23} have been neglected.

Co-ordinate systems used in blade modelling, shown in Fig. 11, are used to define sequential transformations from inertial into blade fixed coordinate system. The first coordinate system (not shown in the figure) is an inertial system, the next system is fixed to the rotor hub and it is rotating with angular velocity Ω. The axis of this system are denoted as x, y, z with the versors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$. An elastic axis of the undeformed blade coincides with the x-axis.

The blade deformations are described using system of coordinates connected to the blade root. The transformation of coordinate system of each point of elastic axis due to blade deformation results in transformation of the triad of versors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ into the triad $\mathbf{e}_\xi$, $\mathbf{e}_\eta$, $\mathbf{e}_\varsigma$ can be found in [18].

Assuming that the blade cross-section neither does not warp, vector $\mathbf{r}$ describing the position of a beam point after deformation is given as:

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{u} + \mathbf{T}\xi, \tag{10}$$

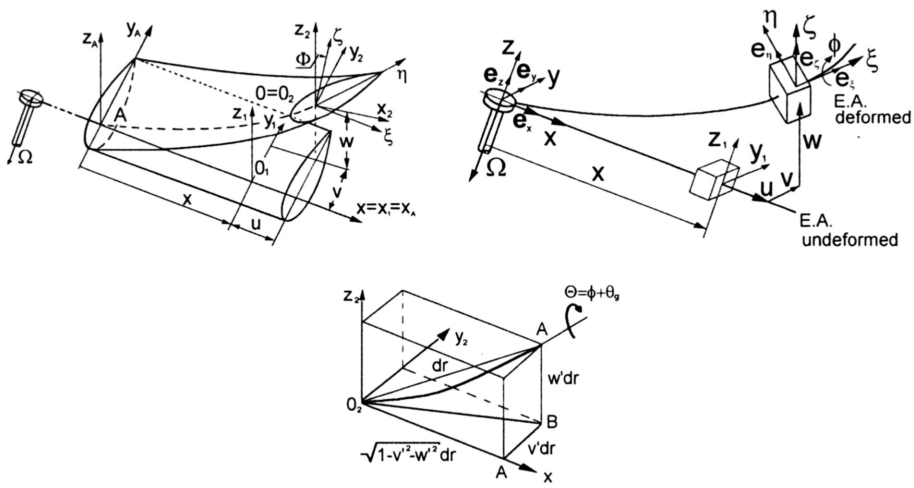


FIGURE 11. Deformation of beam elastic axis.

where: $\mathbf{r}_0 = [x,0,0]^T$ – vector of section placement along the span, $\mathbf{u} = [u,v,w]^T$ – vector of elastic deformation $\xi = [\xi,\eta,\varsigma]^T$ – vector of coordinates in the blade section, $\mathbf{T}$ – rotation matrix due to blade deformation.

The strain/displacement relation is obtained in the form:

$$\begin{aligned} \varepsilon_{11} &= u' + \frac{v'^2}{2} + \frac{w'^2}{2} + \left(\frac{\phi'^2}{2} + \phi'\theta_g'\right)(\eta^2+\xi^2) \\ &\quad -v''\big[\eta\cos(\theta_g+\phi) + \varsigma\sin(\theta_g+\phi)\big] - w''\big[-\eta\sin(\theta_g+\phi) + \varsigma\cos(\theta_g+\phi)\big], \\ &\qquad \varepsilon_{12} = \frac{1}{2}\phi'\varsigma\varepsilon_{13} = -\frac{1}{2}\phi'\eta, \end{aligned} \tag{11}$$

where $\theta_g(x)$ is geometrical twist angle along the span.

It is assumed that material has linear isotropy and Hooke's law is applicable, the blade is cantilevered, the curvature and deflections of the deformed blade are small, the blade elastic axis (E.A) forms a straight line. Including tension loads into the analysis, the coupled bending torsion equations of motion are obtained:

$$\begin{aligned} &-\Big\{\Big[GJ + Tk_A^2 + EB_1\left(\theta_g'\right)^2\Big]\phi' - EB_2\theta_g'\left(v''\cos\theta_g + w''\sin\theta_g\right)\Big\}' \\ &+Te_A\left(v''\sin\theta_g - w''\cos\theta_g\right) + \Omega^2 mxe\left(-v'\sin\theta_g + w'\cos\theta_g\right) + \Omega^2 me\sin\theta_g v \\ &+\Omega^2 m\Big[\left(k_{m_2}^2 - k_{m_1}^2\right)\cos 2\theta_g + ee_o\cos\theta_g\Big]\phi + mk_m^2\ddot{\phi} - me\left(\ddot{v}\sin\theta_g - \ddot{w}\cos\theta_g\right) \\ &\qquad = M + \left(Tk_A^2\theta_g'\right)' - \Omega^2 m\Big[\left(k_{m_2}^2 - k_{m_1}^2\right)\sin\theta_g\cos\theta_g + ee_0\sin\theta_g\Big], \end{aligned} \tag{12}$$

$$\begin{aligned}
&\big[(EI_2 - EI_1)\sin\theta_g\cos\theta_g w'' \\
&+ \left(EI_1\sin^2\theta_g + EI_2\cos^2\theta_g\right)v'' + Te_A\phi\sin\theta_g - EB_2\theta_g'\phi'\cos\theta_g\big]'' \\
&-(Tv')' + \left(\Omega^2 mxe\phi\sin\theta_g\right)' + \Omega^2 me\phi\sin\theta_g + m\left(\ddot{v} - e\ddot{\phi}\sin\theta_g\right) - \Omega^2 mv \\
&\qquad = L_y + (Te_A\cos\theta_g)'' + \left(\Omega^2 mxe\cos\theta_g\right)' + \Omega^2 m\left(e_o + e\cos\theta_g\right),
\end{aligned} \tag{12 [cont.]}$$

$$\begin{aligned}
&\Big[\left(EI_1\cos^2\theta_g + EI_2\sin^2\theta_g\right)w'' + (EI_2 - EI_1)\sin\theta_g\cos\theta_g v'' - Te_A\phi\cos\theta_g \\
&-EB_2\theta_g'\phi'\sin\theta_g\Big]'' - (Tw')' - \left(\Omega^2 mxe\phi\cos\theta_g\right)' + m\left(\ddot{w} + e\ddot{\phi}\cos\theta_g\right) \\
&\qquad = L_2 + (Te_A\sin\theta_g)'' + \left(\Omega^2 mxe_A\sin\theta_g\right)',
\end{aligned}$$

where: m – the mass distribution along the blade, e – distance from tension axis to elastic axis, $(')$ – denotes differentiation with respect to beam length.

It may be noticed, that neglecting the geometrical twist and coupling terms, the simple beam equations may be obtained.

3.5. Thin-walled beams

Thin-walled beams are also used extensively in aerospace structures, both as direct load carrying members and as stiffening elements in panel constructions.

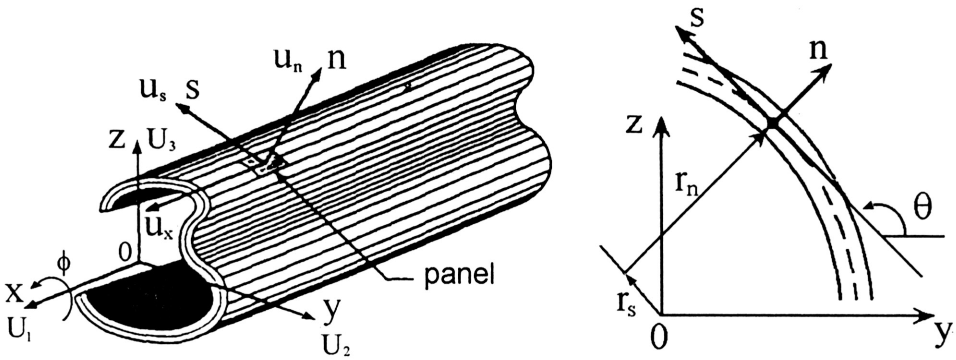


FIGURE 12. Coordinate systems and deformation variables.

Two basic assumptions provide the foundation for a linear engineering theory of thin walled beams made from isotropic materials: the contour of cross-section does not deform in its plane, which implies, that the strain ε_s in the contour direction is small compared to the strain ε_x parallel to the beam axis and the shearing strain ε_{xs} in the middle plane of a plate element is zero for each plate element. The equations of beam deformation may be found in [19]. In thin walled beams the warping is important especially, when "smart actuators" are embedded into structure.

4. Beams with actuators

To control beam deflections actuators embedded into structure. Two ways of actuating beam deflections may be considered: embedding solid state elements or active fibers.

4.1. Model of rotating blade with active fibers

To evaluate the influence of including active fibers embedded into the rotor blade model, the model described in Section 3.3 was modified, assuming that the tension force $t(x)$ is a sum of two parts:

$$t(x) = t_T(x) + t_a(x), \tag{13}$$

passive $t_T(x)$ due to centrifugal force, and active $t_a(x)$ in the form:

$$t_a(x) = \int\limits_{A_a} E_a (u_a)' dA_a \tag{14}$$

After including into (11) and following the same derivation procedure as in passive blade case, the longitudinal strain it is obtained as:

$$\begin{aligned}\varepsilon_{11} = \varepsilon_T + \varepsilon_a + \left(\eta^2 + \zeta^2 - k_T^2\right) \phi' \theta_g' - \big[(\eta - \eta_T)\cos(\theta_g + \phi) \\ + (\zeta - \zeta_T)\sin(\theta_g + \phi)\big] v'' - \big[(\eta - \eta_T)\sin(\theta_g + \phi) - (\zeta - \zeta_T)\cos(\theta_g + \phi)\big] w''.\end{aligned} \tag{15}$$

where ε_a is the active strain and ε_T the passive strain.

For active fibers made from piezoelectric material, the active strain depends of the electrical field according to the formulae:

$$\varepsilon_a(x) = \frac{du_a}{dx} = \frac{d_{31}}{\varepsilon^E} U(x,t), \tag{16}$$

where U – electric voltage, ε^E – dielectric constant, d_{13} – piezoelectric constant.

Finally including (16) into (14) it is found that

$$t_a(x) = \int\limits_{A_a} E_a \frac{d_{31}}{\varepsilon^E} U(x,t) dA_a = K_a(x) U(x,t) \tag{17}$$

where

$$K_a(x) = \int\limits_{A} E_a \frac{d_{31}}{\varepsilon^E} dA.$$

The operator of elastic loads in the equations of motion has the form

$$\mathbf{Q}_E = \mathbf{Q}_p \mathbf{q} + \mathbf{h}_p + (\mathbf{Q}_a \mathbf{q} + \mathbf{h}_a) U \tag{18}$$

where the matrices and vectors of elastic loads were decomposed into passive $\mathbf{Q}_p$ and $\mathbf{h}_p$ and active $\mathbf{Q}_a$ and $\mathbf{h}_a$ components, given in [ ].

It may be evaluated from (17), that the active strain is less then the passive one by orders of magnitude. It cannot be expected then, that active extensions will be large enough to allow influencing properties of rotating beam directly. Utilisation of active composite for shape control of rotating blades needs redesigning of rotor blades with respect to more effective composite action. The latest work on this was published in [21] and the experimental blade is shown in Fig. 13.

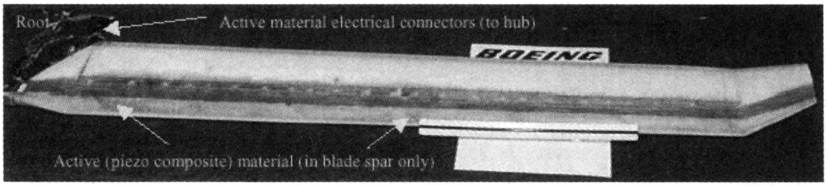


FIGURE 13. Blade with active composites.

There is also other approach to control blade shape – extending the beam model and make use from secondary effects like especially by extending the blade model by inclusion of the second order effects, which can extensively change the blade properties [22].

4.2. Embedding active elements

To control deformations, also solid active elements may be embedded into structure. This approach may be used for exciting: torsion by placing two elements at 45° angle with respect to the longitudinal axis (Fig. 14). Placing elements along the beam axis extension or bending may be obtained depending on the elements elongation (in phase, out of phase), (Fig. 14).

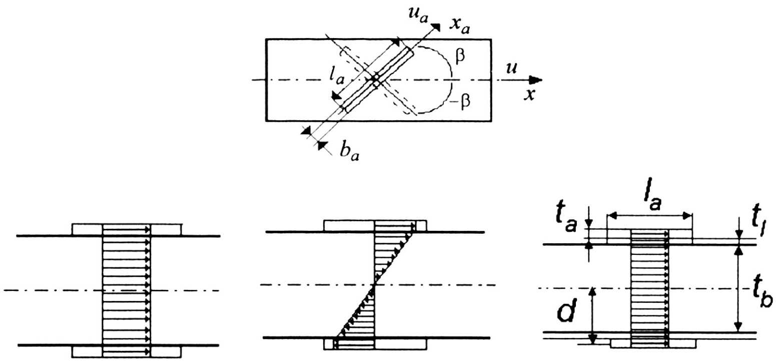
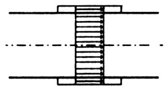
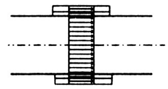
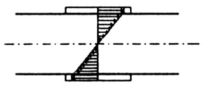
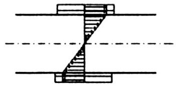
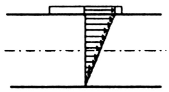
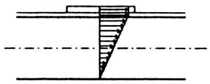
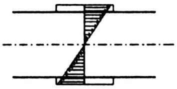
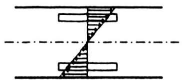


FIGURE 14. Active element placement on the beam surfaces for different deflections.

Interaction of active element with beam is modelled in various ways. For surface mounted element the strain for different models are summarised in Table 2 based on [23-27].

TABLE 2. Beam-active element interaction.

<table>
<tr><th colspan="4">Uniform stress distribution in beam</th></tr>
<tr><td colspan="2">without bonding layer</td><td colspan="2">with bonding layer</td></tr>
<tr><td colspan="2">$$\varepsilon_s(\bar{x}) = \varepsilon_a(\bar{x}) = \frac{\alpha}{\alpha+\psi}\Lambda(\bar{x})$$</td><td colspan="2">$$\varepsilon_a(\bar{x}) = \frac{\alpha}{\alpha+\psi}\left[1+\frac{\psi}{\alpha}\frac{\cosh(\Gamma\bar{x})}{\cosh(\Gamma)}\right]\Lambda(\bar{x})$$ $$\varepsilon_s(\bar{x}) = \frac{\alpha}{\alpha+\psi}\left[1-\frac{\cosh(\Gamma\bar{x})}{\cosh(\Gamma)}\right]\Lambda(\bar{x})$$ $$\tau_s(\bar{x}) = G_l\left(\frac{t_l}{l_a}\right)^{-1}\frac{\sinh(\Gamma\bar{x})}{\Gamma\cosh(\Gamma)}\Lambda(\bar{x})$$</td></tr>
<tr><td colspan="4">symmetric excitation</td></tr>
<tr><td>$\psi = \frac{(EA)_s}{(EA)_a}, \quad \alpha = 2$</td><td></td><td>$\psi = \frac{(EA)_s}{(EA)_a}, \quad \alpha = 2$</td><td></td></tr>
<tr><td colspan="4">antisymmetric excitation</td></tr>
<tr><td>$\psi = \frac{12(EI)_s}{t_s^2(EA)_a}, \quad \alpha = 6$</td><td></td><td>$\psi = \frac{12(EI)_s}{t_s^2(EA)_a}, \quad \alpha = 6$</td><td></td></tr>
<tr><td colspan="4">one side excitation</td></tr>
<tr><td>$\psi = \frac{(EA)_s}{(EA)_a}, \quad \alpha = 4$</td><td></td><td>$\psi = \frac{(EA)_s}{(EA)_a}, \quad \alpha = 4$</td><td></td></tr>
<tr><th colspan="4">Linear stress distribution in the beam, no bonding layer, antisymmetric excitation</th></tr>
<tr><td colspan="2">actuators on the surface

$$T = \frac{t_s}{t_a}, \qquad \psi = \frac{(EA)_s}{(EA)_a} = \frac{Eb_s t_s}{Eb_a t_a},$$ $$\varepsilon_s(\bar{x}) = \varepsilon_a(\bar{x}) =$$ $$= \frac{12\,(T+1)}{(6+\psi)T^2+12T+8}\left(\frac{1}{t_a}\right)\Lambda(\bar{x})z$$</td><td colspan="2">actuators embedded into structure

$$I_a = A_a\left(d^2+dt_a+\frac{t_a^3}{3}\right),$$ $$\varepsilon_s(\bar{x}) = \varepsilon_a(\bar{x}) =$$ $$= \frac{(EA)_a(t_a+2d)}{E_s(I_s-2I_a)} + 2(EA)_a\Lambda(\bar{x})z$$</td></tr>
</table>

Symbols used in Table 2:
I – cross section surface inertia moment;
α – parameter depending on the way excitation is applied, $\alpha = 2, 4, 6$;
$\bar{x}$ – coordinate with respect to beam length;
lower indices: a – active element, s – structure, l – bonding layer.
Parameter Γ is calculated as:

$$\Gamma^2 = \left(\frac{G_l}{E_a}\right)\left(\frac{A_l}{A_a}\right)\left(\frac{t_l}{l_a}\right)^{-2}\left(\frac{\psi+\alpha}{\psi}\right).$$

Other quantities defined in Fig. 14.

5. Control methods

5.1. General

There are various objectives to be obtained by controlling aeronautical structures, but the most often goal is vibration control. Vibrations may be suppressed by passive or active systems or hybrid active-passive systems. In aeronautics passive methods are based on selecting dynamic properties of structures to avoid instabilities, which, with structure optimisation, is being developed recently very intensively.

As the control theory for continuous systems is not yet sufficiently developed for direct applications, a spatial discretization of continuous structures is performed using coupled deformation mode shapes. After spatial discretization the system of ordinary differential equations is obtained.

The control of structures subjected to periodic excitation is considered which has direct relation to controlling helicopter rotor blade. Helicopter rotor blade is inherently a nonlinear system, varying periodically in time. This periodic variations stems from primary blade control for achieving the desired flight conditions and nonlinearity is due to aerodynamic loads, even for small deflections.

The mathematical model of a helicopter rotor blade in the form of nonlinear system of ordinary differential equations in state variables has the form:

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{u}) \tag{19}$$

There are a few methods for controlling nonlinear systems, so the blade equations of motion (19) are linearised about selected motion $\mathbf{x}_d(\mathrm{t})$ and control $\mathbf{u}_d(t)$ leading to linearised system in the form:

$$\begin{gathered}\dot{\mathbf{x}} = \mathbf{A}(\mathbf{t})\mathbf{x} + \mathbf{B}(t)u(t) + \mathbf{R}(\mathbf{x}, \mathbf{x}_d, u(t), u_d(t), t)\mathbf{A} = [A_{ij}] = \left[\frac{\partial f_i}{\partial x_j}\right]_{\mathbf{x}_d, u_d}, \\ \mathbf{B} = [B_i] = \left[\frac{\partial f_i}{\partial u}\right]_{\mathbf{x}_d, u_d}.\end{gathered} \tag{20}$$

In modern systems the control is applied through digital devices, so the system should be discretized in time. It may be done by approximating the time derivative by the forward finite difference

$$\dot{\mathbf{x}} = \frac{\mathbf{x}(t + \Delta t) - \mathbf{x}(t)}{\Delta t}. \tag{21}$$

Inserting (21) into (20) the linearized equations are transfered to the discrete time domain system:

$$\mathbf{x}(t + \Delta t) = [\mathbf{I} + \mathbf{A}(t)\Delta t]\mathbf{x} + \mathbf{B}(t)\Delta t\, u(t) + \mathbf{R}(\mathbf{x}, \mathbf{x}_d, u(t), u_d(t), t)\Delta t. \tag{22}$$

The two operation i.e. linearisation and discretization are needed for transferring the system to the form

$$\mathbf{x}(k + 1) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)u(k) + \mathbf{d}(k). \tag{23}$$

The system has periodically varying coefficients and there are a few (if any ?) methods effectively dealing with such kind of systems. This was the reason for searching "nonclassical control methods" to apply in the considered case and the learning algorithms were applied by the author [28].

5.2. Nonclassical fixed wing control

For wings where there is no periodic coefficients, the system describing aeroelastic phenomena in the linear range in the state variables has the form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{24}$$

where $\mathbf{x}(n \times 1)$ – vector of state variables, $\mathbf{u}(m \times 1)$ – vector of control variables, and $\mathbf{A}$ and $\mathbf{B}$ are constant state and control matrices of appropriate dimensions.

Two methods for controlling such systems will be outlined.

5.3. Optimal control

The objective of control is minimisation of performance index in the form:

$$I = \frac{1}{2}\int\limits_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}\right)\,dt, \tag{25}$$

where $\mathbf{Q}(n \times n)$ is non-negative and $\mathbf{R}(m \times m)$ is positive definite symmetric weighting matrix.

Minimisation of performance index is achieved by the feedback control law in the form:

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}\mathbf{x}(t). \tag{26}$$

A positive-definite symmetric matrix $\mathbf{K}$ in the feedback gain matrix is obtained as a solution of matrix algebraic Riccati equation

$$\mathbf{K}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K} - \mathbf{K}\mathbf{A} - \mathbf{A}^T\mathbf{K} - \mathbf{Q} = \mathbf{0}. \tag{27}$$

Generally solution of (27) requires sophisticated numerical methods. The resulting closed-loop system has the form:

$$\dot{\mathbf{x}} = \mathbf{L}\mathbf{x}, \qquad \mathbf{L} = \mathbf{A} + \mathbf{B}\mathbf{F}, \tag{28}$$

where the feedback control matrix has the form: $\mathbf{F} = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}$. For a controllable system such a solution yields to a stable closed-loop system, i.e. the eigenvalues $\lambda_j(\mathbf{L})$, $j = 1, \ldots, n$ of $\mathbf{L}$,

$$\operatorname{Re}\lambda_j(\mathbf{L}) < 0, \tag{29}$$

lie in left-half plane of the complex plane.

Using this method systems may be stabilized in the range of parameters essential for the system application.

5.4. Modal control

In this method named also eigenstructure assignment or pole placement technique, the control law is a linear function of the state vector (here the full state feedback is considered):

$$\mathbf{u} = -\mathbf{G}\mathbf{x}. \tag{30}$$

The feedback gain matrix $\mathbf{G}$ is selected for to obtain in a closed-loop system the desired placement of eigenvalues λ_i and shapes eigenvectors $\mathbf{v}_i$, i.e. the eigenstructure $\{\lambda_i, \mathbf{v}_i\}$ is calculated. Substituting (30) to (24) yields the closed-loop system:

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{G})\mathbf{x}. \tag{31}$$

Assuming that the vector $\mathbf{w}_i$ satisfies the equation:

$$[\lambda_i \mathbf{I} - \mathbf{A}, \mathbf{B}] \begin{Bmatrix} \mathbf{v}_i \\ \mathbf{w}_i \end{Bmatrix} = \mathbf{0} \tag{32}$$

the feedback gain is chosen to satisfy:

$$\mathbf{G}\mathbf{v}_i = \mathbf{w}_i. \tag{33}$$

Using the Eqs. (32) and (33) the equation is obtained:

$$[\lambda_i \mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{G})]\, \mathbf{v}_i = \mathbf{0}, \tag{34}$$

from which eigenvectors $\mathbf{v}_i$ for desired eigenvalues λ_i are obtained as a closed-loop solution. Finally knowing the *modal matrices* $\mathbf{V}$ and $\mathbf{W}$ matrix $\mathbf{G}$ may be calculated from the relation:

$$\mathbf{G}\mathbf{V} = \mathbf{W}. \tag{35}$$

Some algorithms based on this methodology are presented in [29, 30].

6. Conclusions

In aeronautics various applications of "smart structures" are considered. For the last decade this discipline has matured and specific topics, crucial for applications emerged in each segment of aeronautics: space, fixed and rotary wing technologies. Modelling of slender bodies is important for airplane wings and helicopter rotor blades analysis. This models are modified to include active elements. The important aspect is interaction of the structure with active element, where various models have been developed. Selected methods of modern control theory useful for active influence on structure behaviour are discussed.

These two topics were discussed in a more detailed way in the paper.

Acknowledgements

This paper was prepared under grant from Polish State Committee of Scientific Research (KBN) "Integration of control and navigation systems in moving objects".

References

1. `www.aiaa.org`
2. `www.icao.org`
3. `www.vtol.org`

4. M. Obal and J.M. Sater, *Adaptive Structures Programs for the Strategic Defense Initiative Organization*, AIAA-92-2338-CP.

5. E.F. Crawley and J. de Luis, *Use of Piezoelectric Actuators as Elements of Intelligent Structures*, AIAA J., Vol.25, No.10, pp.1373-1385, October 1987.

6. W. Schmidt and Chr. Boller, *Smart Structures. A Technology for Next Generation Aircraft*, AGARD/SMP Specialist's Meeting on Smart Structures for Aircraft And Spacecraft, Lindau, Germany, 5-7 October 1992.

7. J. Kudva, C. Marantidis, C. Van Way, J. Alper and C. Paul, *Guidelines for Smart Structural Health Monitoring Systems for Military Aircraft – Lessons Learned from the SSCORE Contract*, SPIE Vol.2191**.

8. R.W. Prouty, *Helicopter Performance, Stability and Control*, PWS Engineering, Boston 1986.

9. R.A. Ormiston, *Can Smart Materials Make Helicopter Better?*, Fourth Workshop on Dynamics and Aeroelastic Stability Modelling of Rotorcraft Systems, The University of Maryland, 1991.

10. H. Strelhow and H. Rapp, *Smart Materials for Helicopter Rotor Active Control.*

11. E.F. Crawley, *Intelligent Structures for Aerospace: a Technology Overview and Assessment*, AIAA Journal, Vol.32, No.8, August 1994.

12. Charon W., Baier H., *Active Mechanical Components as a Step Towards Adaptive Structures in Space*, SPIE Vol.2191 AGARD/SMP Specialist's Meeting on Smart Structures for Aircraft and Spacecraft, Lindau, Germany, 5-7 October 1992.

13. E.D. Flinn, *A Smart Way to Reshape Antennas*, Aerospace America, February 1999.

14. D.A. Peters and M.J. Johnson, *Finite State Airloads for Deformable Airfoils on Fixed and Rotating Wings*, Aeroelasticity and Fluid Structure Interaction Problems, Proceedings of the ASME Winter Annual Meeting, Chicago, November 6-11, 1994, AD-Vol.44, pp.1-28.

15. J. Narkiewicz, A. Ling and G.T.S. Done, *Unsteady Aerodynamic Loads on an Aerofoil with a Deflecting Tab*, Aeronautical Journal, Vol.99, No.987, pp.282-292, September/October 1995.

16. M. Garfinkle, *Twisting smartly in the wind*, Aerospace America, July 1994.

17. J.C. Houbolt and G.W. Brooks, *Differential Equations of Motion for Combined Flapwise Bending, Chordwise Bending and Torsion of Twisted Nonuniform Rotor Blades*, NACA Rept.1346, 1958.

18. J. Narkiewicz, *Rotorcraft Aeromechanical and Aeroelastic Stability* (in Polish), Scientific Reports of Warsaw University of Technology, Mechanics, No.158, 1994.

19. E.C. Smith and I. Chopra, *Formulation and Evaluation of an Analytical Model for Composite Box-Beams*, Journal of the American Helicopter Society, pp.23-35, July 1991.

20. J. Narkiewicz, *Application of smart structure concept to rotorcraft additional control* (in Polish), Scientific Reports of Warsaw University of Technology, Mechanics, No.169, 1998.

21. R. Derham, D. Weems, M.B. Mathew and R. Bussom, *The Design Evolution of an Active Materials Rotor*, AHS Forum and Technology Display, Washington, May 2001.

22. E.C. Smith and I. Chopra, *Formulation and Evaluation of an Analytical Model for Composite Box-Beams*, Journal of the American Helicopter Society, pp.23-35, July 1991.

23. E.F. Crawley and J. de Luis, *Use of Piezoelectric Actuators as Elements of Intelligent Structures*, AIAA Journal, Vol.25, No.10, pp.1373-1385, October 1987.

24. E.F. Crawley and E.H. Anderson , *Detailed Models of Piezoceramic Actuations of Beams*, Journal of Intelligent Material Systems And Structures, Vol.1, No.1, January 1990.

25. Ch. Park, C. Waltz and I. Chopra, *Bending and Torsion Models of Beams with Induced Strain Actuators*, SPIE Conference, Albuquerque, NM, 1993, Smart Materials and Structures, Vol.5, No.1, 1996.

26. P.C. Chen and I. Chopra *Induced Strain Actuation of Composite Beams and Rotor Blades with Embedded Piezoceamic Elements*, Smart Materials and Structures, Vol.5, No.1, 1996.

27. CH. PARK and I. CHOPRA, *Modelling Piezoceramic Actuation of Beams in Torsion*, AIAA-94-1781-CP, 35th Structures, Structural Dynamics and Materials Conference and Adaptive Structures Forum, Hiltin Head, NC, April 1994. AIAA Journal, Vol.34, No.12, December 1996.

28. J. NARKIEWICZ and G.T.S. DONE, *Application of a Learning Control Algorithm to a Rotor Blade Tab*, AIAA Journal of Guidance, Control and Dynamics, Vol.19, No.6, pp.1383-1386, November-December 1996.

29. J. PIETRUCHA, *Modification of aircraft dynamic properties via feedback control-a unified formulation*, Aviation, Vol.3, pp.63-69, Vilnius "Technika" 1998.

30. J. PIETRUCHA and Z. SZEWCZYK, *Modal control method and its application to the stabilization of helicopter flight* (in Polish), Mech. Teoret. i Stosowana, Vol.4, No.14, pp. 571-584, 1976.