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Statics and dynamics of composite structures with embedded shape memory alloys

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The paper illustrates stress-strain relationships for composite structures with embedded SMA fibres and their influence upon certain changes in natural frequencies and thermal buckling of selected composite structures. Governing equations based on the finite element method are formulated for beams, plates, and shells. Active frequency controlling can be used, for example, to avoid resonances in composite structures such as shafts, blades, aircraft wings, etc. The results of calculations demonstrate the potential effectiveness of SMA fiber-reinforcement in composite structural elements in the process of controlling the vibration. The effect of SMA fibers activation on the amplitude of vibration normalized with respect to the amplitude of the uncontrolled vibration can also be analyzed.

1. Introduction

The use of different composite materials has been continuously growing over recent decades. Although composite materials have found many applications during that time, extensive research is still being carried out in order to expand this field. New materials and technologies have been developed enabling more original and more advanced applications for composite materials. One such new application is the integration of composite materials with shape memory materials.

Shape Memory Alloys (SMA) with abilities to change their material properties such as Young's modulus [1, 2] damping capacity [3, 4] and the generation of large internal forces [5], when integrated with composite material structures allow active control of their static and dynamic behaviour. Precise tuning of SMA components [6, 7] enables control of certain static and dynamic characteristics of composite material structures such as maximum deflection and shape, natural frequencies and modes of vibrations, amplitudes of forced vibrations, or damping properties.

SMA components embedded into, or bonded to, composite material structures can be utilised in two different ways. The first implementation is the use of the Active Property Tuning method [6, 7], which exploits only changes in the stiffness of the SMA components during their activation. In the Active Strain Energy Tuning method [6, 7] the shape memory effect is exploited, when activation of previously pseudo-plastically elongated SMA components, integrated with composite material structures, leads to the generation of high recovery stresses.

Different aspects of using SMA components integrated with composite material structures have been investigated by many researchers and published in the literature. Rogers *et al.* [6] presented concepts of using SMA wires for control of natural frequencies and modes of vibrations of simply-supported plates. Both the Active Property Tuning and Active Strain Energy Tuning methods were considered in their work. They also discussed two different techniques of bonding SMA wires to composite structures. For the Active Property Tuning method SMA wires can be directly bonded to structures, while for the Active Strain Energy Tuning method they can be put into sleeves and then attached to the structure at some convenient chosen point, in order to eliminate high shearing stresses arising from their activation process. Rogers *et al.* showed that significant changes in natural frequencies and modes of vibration can be achieved for simply-supported plates with SMA wires, and also stated that the use of the Active Strain Energy Tuning method leads to much better results than the use of the Active Property Tuning method.

Changes in natural frequencies of clamped-clamped composite beams with SMA wires were investigated analytically and experimentally by Baz *et al.* [8]. Baz *et al.* showed that SMA wires embedded into a composite beam can successfully be used for controlling the beam's natural frequencies. The influence of different initial strain levels, as well as temperature effects due to activation of the SMA wires, were also considered in their study.

Baz *et al.* [9] also investigated the use of SMA components for shape control of composite beams. They demonstrated that SMA components in the form of strips, previously trained for the two-way shape memory effect, and then embedded into composite beams, can be used for shape control of such structures. Natural frequencies of composite beams modified in this manner were also significantly affected.

Lee and Lee [10] investigated the buckling and post-buckling behaviour of simply-supported and clamped composite plates with SMA wires. They found that activation of SMA wires can increase the critical load of composite plates, but this effect is also a function of the relative location of the SMA wires and the buckling direction.

Results presented in the literature indicate many possible applications for SMA components in the active control of the static and dynamic behaviour of composite material structures. However, a more detailed study is required for better understanding of this behaviour as results presented in the literature are incomplete and refer to very specific cases.

Rogers *et al.* [6] only examined SMA/Epoxy composite plates, for which the relative volume fraction of SMA wires was very high. The performance of such plates is determined by the high ratio of SMA Young's modulus to epoxy matrix Young's modulus, and also by the high relative volume fraction of SMA wires. The same assumption regarding the high relative volume fraction of SMA components was made by Baz *et al.* [9], who investigated Glass/Epoxy composite beams with SMA strips. Furthermore, Baz *et al.* [8], as well as Baz *et al.* [9] investigated composite beams of very low thickness-to-length ratios. Although such structures are characterised by very good performance when SMA components are activated, there are very few engineering applications due to their low stiffness and low critical loads. Lee and Lee [10] studied the buckling and post-buckling behaviour of composite plates with SMA wires, for which assumed values of recovery stresses during their activation were as high as their ultimate tensile strength.

It can be expected that the performance of composite material structures with SMA components strongly depends on such factors as the ratio of the SMA Young's modulus to the Young's modulus of the reinforcing fibres (Glass, Kevlar, Graphite, Boron, etc.), the relative volume fraction of the SMA components, the relative volume fraction of the reinforcing fibres, the structure thickness-to-length ratio (slenderness), the location and orientation of the SMA components within the structure, temperature effects and the influence of moisture, for example.

2. Mechanical and physical properties of shape memory alloys

Mechanical and physical properties of shape memory alloys strongly depend on temperature and initial stress [11, 12]. Changes in temperature and initial stress involve changes in the volume fraction of martensite in the alloys. During the martensite transformation a recovery stress appears. This recovery stress is not only a function of alloys temperature but also depends on initial strain ε.

In order to model accurately the behaviour of composite structures with embedded shape memory alloy components three literature models have been examined, and the special attention has been paid to the improved Brinson model described by the following equation:

$$(\sigma - \sigma_0) = E(\xi)\,\varepsilon - E(\xi_0)\,\varepsilon_0 + \Omega(\xi)\,\xi_S - \Omega(\xi_0)\,\xi_{0S} + \Theta(T - T_0), \tag{1}$$

where σ, σ_0 describe stress and initial stress, ε, ε_0 strain and initial strain, T and T_0 temperature and initial temperature, Θ thermoelastic coefficient. It is assumed that the Young's modulus E, as well as the phase transformation coefficient Ω of a shape memory alloy, are functions of the martensite volume fraction ξ. The function to describe the martensite volume fraction ξ is defined as the sum of two fractions:

$$\xi = \xi_S + \xi_T, \tag{2}$$

where ξ_S and ξ_T describe stress-induced and temperature-induced martensite volume fractions.

All the required material properties for the Brinson model are presented in Table 1.

TABLE 1. Material properties of the Nitinol alloy

Moduli, Density	Transformation Temperatures	Transformation Constants	Maximum residual strain
$E_A = 67.0 \times 10^3$ MPa	$M_F = 9.0°$C	$C_M = 8.0$ MPa/°C	$\varepsilon_L = 0.067$
$E_M = 26.3 \times 10^3$ MPa	$M_S = 18.4°$C	$C_A = 13.8$ MPa/°C	
$\Theta = 0.55$ MPa/°C	$A_S = 34.5°$C	$\sigma_S = 100.0$ MPa	
$\rho = 6448.1$ kg/m^3	$A_F = 49.0°$C	$\sigma_F = 170.0$ MPa	

In Fig. 1 SMA recovery stress versus SMA temperature for initial strain ($\varepsilon_0 = 0.005$) is presented. It can be easily noticed that an increase in the initial strain involves correspondingly higher recovery stress in the SMA wires, and simultaneous changes in the temperatures of phase transformation are also observed.

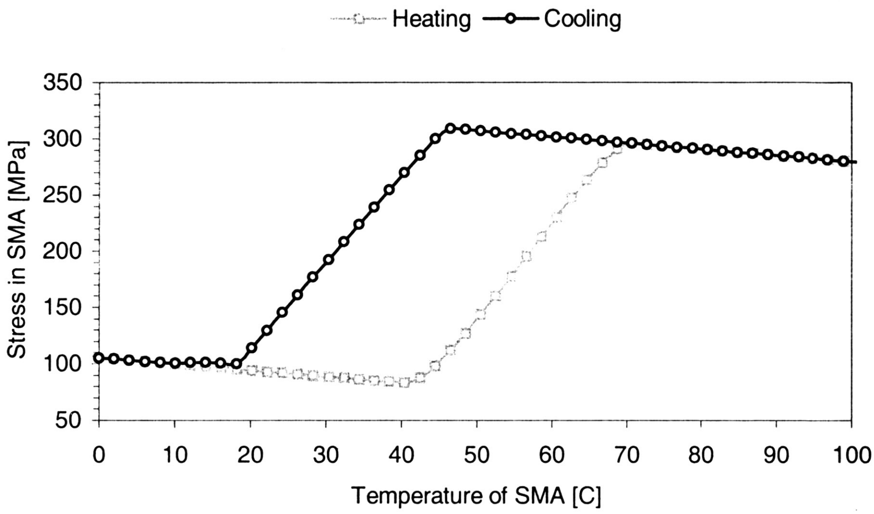

Figure 1. Changes in the internal stress σ as a function of the temperature T during heating and cooling of the Nitinol sample at the constant initial strain $\varepsilon_0 = 0.005$.

3. Beams and plates with shape memory alloy components

Shape memory components in the form of wires, strips or foil embedded or bonded to elements of structures enable active control of their static and dynamic characteristics [6, 12]. This is possible due to the fact that certain mechanical properties of shape memory alloys can be very precisely controlled and changed in a required manner according to the particular implementation. Certain static and dynamic characteristics of structural elements such as the maximum deflection and shape, natural frequencies and modes of vibrations, amplitudes of forced vibrations or damping properties can be controlled by the use of shape memory alloy components [13, 14].

Shape memory alloy components embedded, or bonded, to elements of structures can be utilised in two different ways.

The first implementation is the use of the Active Property Tuning technique. This technique is based on changes in the stiffness of shape memory alloy components embedded into, or bonded to, structural elements over the transformation from martensite-to-austenite. In this case shape memory alloy wires, strips, or foil, can be epoxied into the elements, or placed into sleeves, and then attached to the structures at key points [1].

The second implementation is a technique called Active Strain Energy Tuning. In this case components made of shape memory alloy are placed in a residual strain state generating large internal forces during their activation as well as changes in their stiffness. Both epoxy and sleeve methods can be used for bonding shape memory alloy components into structural elements [1].

3.1. Composite beam with shape memory alloy wires

The use of both the Active Property Tuning and the Active Strain Energy Tuning methods in the case of a cantilever composite beam with shape memory alloy wires, as presented in Fig. 2, has been investigated by Żak, Cartmell, and Ostachowicz [15]. The beam under consideration is of the following dimensions: length 500 mm, width 30 mm, and thickness 9 mm. It is assumed that the beam consists of 12 layers of composite material, 2 SMA/Epoxy layers, and 10 Graphite/Epoxy layers. The orientation of the reinforcing Graphite fibres and the SMA wires for each layer is defined by the angle α. It is also assumed that the SMA/Epoxy layers are placed symmetrically across the cross-section of the beam, as shown in Fig. 2, in the form 2 outer layers. The thickness

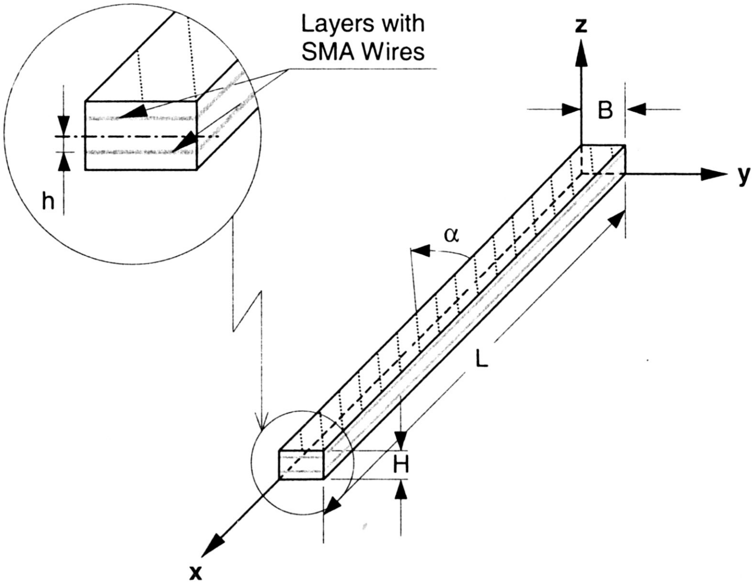

FIGURE 2. Nomenclature for a multi-layered, composite beam with shape memory alloy wires.

TABLE 2. Mechanical properties of composite material components and SMA wires.

Material	Young's Modulus	Poisson Ratio	Density
Epoxy Resin	3.43 GPa	0.35	1250.0 kg/m^3
Aluminium	70.0 GPa	0.33	2800.0 kg/m^3
SMA – Martensite	26.3 GPa	0.30	6448.1 kg/m^3
SMA – Austenite	67.0 GPa	0.30	6448.1 kg/m^3
Glass Fibres	65.5 GPa	0.23	2250.0 kg/m^3
Kevlar Fibres	130.0 GPa	0.22	1450.0 kg/m^3
Graphite Fibres	275.6 GPa	0.20	1900.0 kg/m^3
Boron Fibres	399.6 GPa	0.21	2580.0 kg/m^3

of each SMA/Epoxy layer is 0.5 mm, and the corresponding relative volume fraction of the SMA wires is 0.57. The relative volume fraction of the Graphite fibres within the inner Graphite/Epoxy layers is 0.5, and the thickness of each Graphite/Epoxy layer is 0.8 mm. The ply stacking sequence of the beam is $[0°/(\pm 45°)_5/0°]$.

Key mechanical properties of certain composite material components and typical SMA wires are shown in Table 2.

The results of specific numerical calculations are shown in Figs. 3-5.

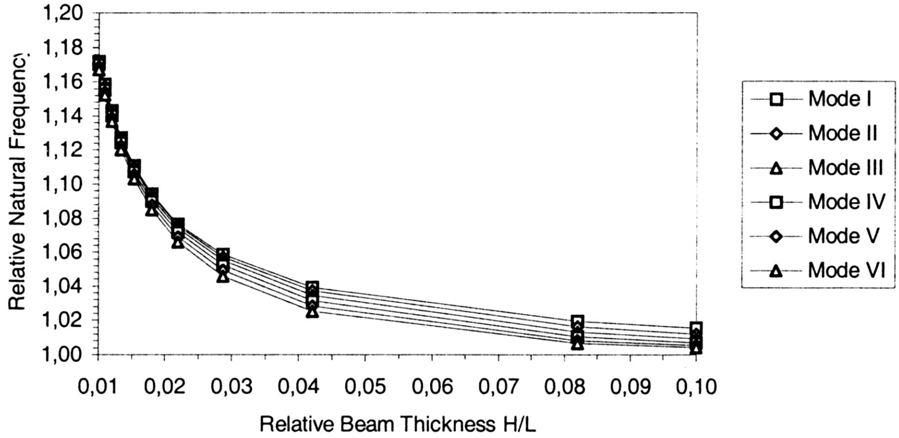

FIGURE 3. The influence of the relative beam thickness on the natural frequencies of a cantilever, multi-layered, composite beam with SMA wires for the Active Property Tuning method.

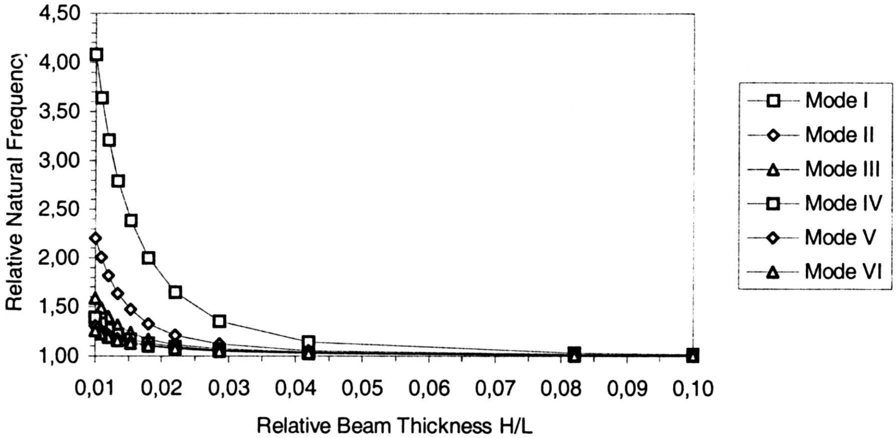

FIGURE 4. The influence of the relative beam thickness on the natural frequencies of a cantilever, multi-layered, composite beam with SMA wires for the Active Strain Energy Tuning method.

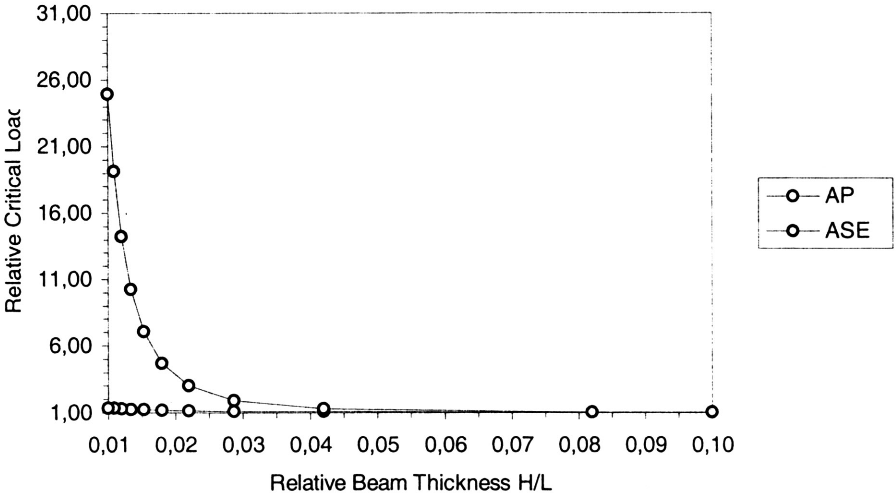

FIGURE 5. The influence of the relative beam thickness on the critical load of a cantilever, multi-layered, composite beam with SMA wires.

3.2. Composite plate with shape memory alloy wires

The use of both the Active Property Tuning and the Active Strain Energy Tuning methods in the case of a simply supported composite plate with shape memory alloy wires as presented in Fig. 6, has been investigated by Żak, Cartmell, and Ostachowicz [15]. In the case of the Active Property Tuning method changes in the stiffness of the shape memory alloy wires influence the plate stiffness, and then affects its static and dynamic behaviour. When the Active Strain Energy Tuning method is investigated activation of the shape memory alloy wires influences both the plate stiffness, as well as generates large in-plane forces, which are characterised by the in-plane load N_x.

A plate of the following dimensions is now considered: length 500 mm, width 500 mm and thickness 9 mm. It is assumed that the plate is made of 12 layers of composite material: 2 SMA/Epoxy layers and 10 Graphite/epoxy layers. The orientation of the reinforcing Graphite fibres and the SMA wires for each layer is defined by the angle α. It is also assumed that the SMA/Epoxy layers are placed symmetrically across the cross-section of the plate, as shown in Fig. 6, in the form of 2 outer layers. The thickness of each SMA/Epoxy layer is 0.5 mm, and the corresponding relative volume fraction of the SMA wires is 0.57. The relative volume fraction of the Graphite fibres within the inner Graphite/Epoxy layers is 0.5, and the thickness of each Graphite/Epoxy layer is 0.8 mm. The ply stacking sequence of the plate is $[0°/(\pm 45°)_5/0°]$.

The results of these numerical calculations are given in Figs. 7-9. Generally, similar to the case of the multi-layered composite beam, greater performance for both the natural frequencies and the critical loads is observed for the ASET (Active Strain Energy Tuning) method, than for the APT (Active Property Tuning) method. The performance of the plate is not only a function of modes of vibrations, but also a function of boundary conditions. Contrary to the multi-layered composite beam, the greatest changes in

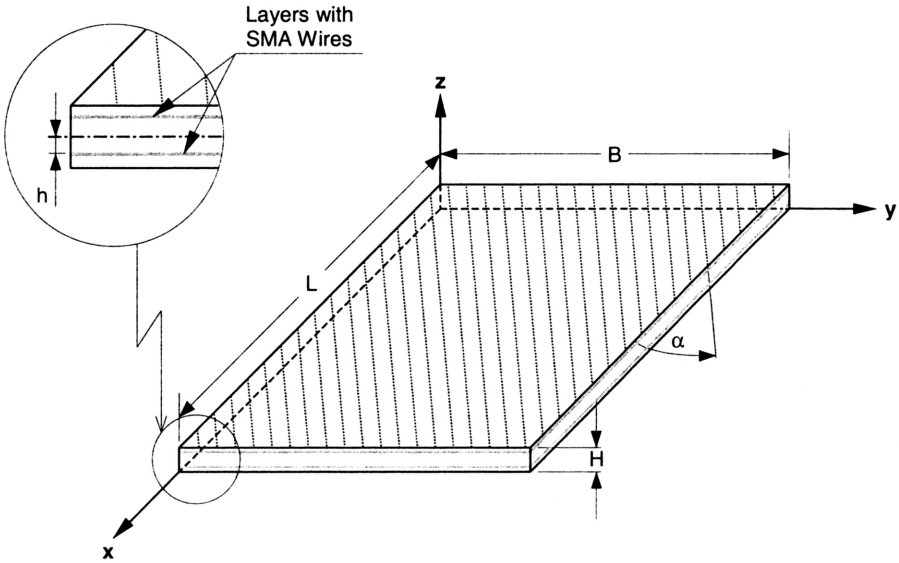

FIGURE 6. Nomenclature for a multi-layered, composite plate with shape memory alloy wires.

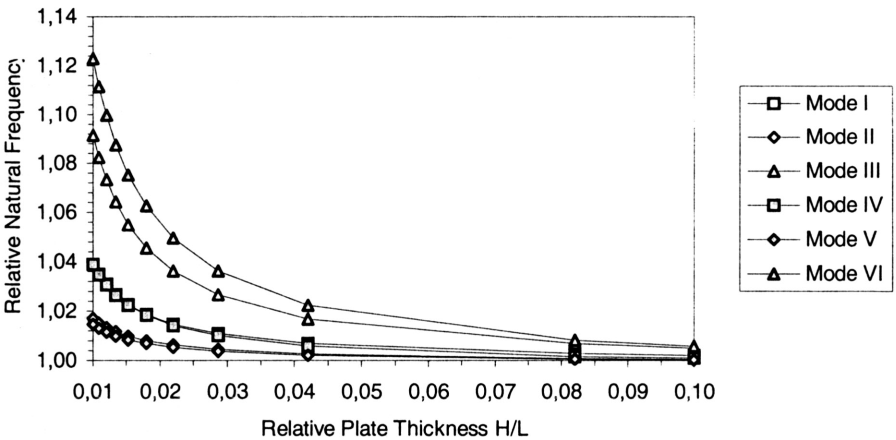

FIGURE 7. The influence of the relative plate thickness on the natural frequencies of a simply-supported, multi-layered composite plate with SMA wires for the Active Property Tuning method.

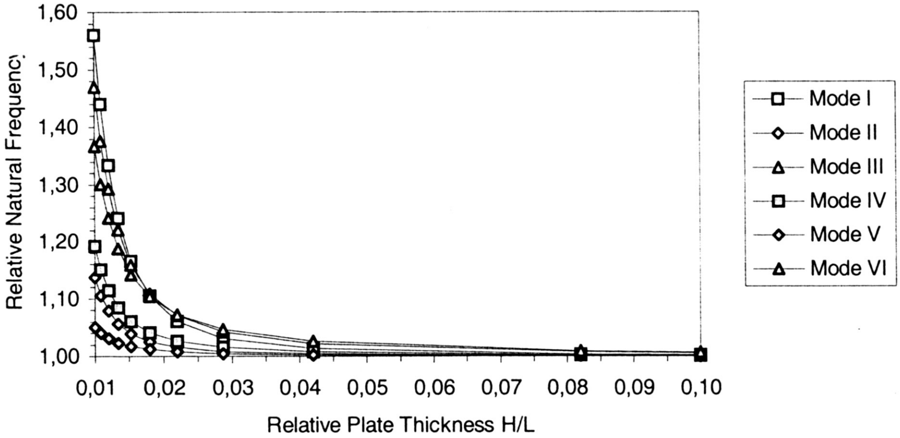

Figure 8. The influence of the relative plate thickness on the natural frequencies of a simply-supported, multi-layered composite plate with SMA wires for the Active Strain Energy Tuning method.

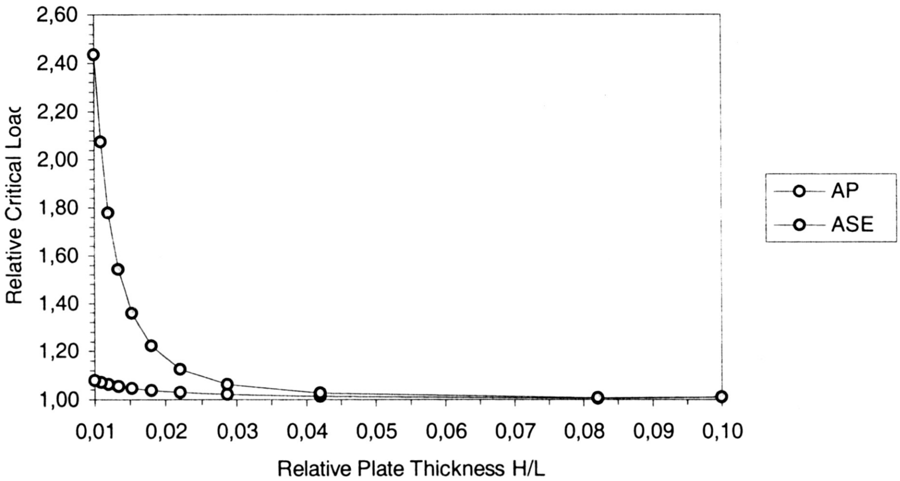

Figure 9. The influence of the relative plate thickness on the critical load of a simply-supported, multi-layered composite plate with SMA wires.

natural frequencies and the critical load are observed not only for lower modes of vibrations, but generally for the modes of vibrations where the nodal lines are perpendicular to the orientation angle of the SMA wires. This behaviour can be explained by the fact that changes in the plates stiffness in the case of the APT method, or the ASET method, due to activation of the SMA wires are maximal in the direction of the SMA wires.

The number of constraints imposed by different types of boundary conditions also influences the results. For more flexible types of boundary conditions smaller number of constraints is imposed, and then both the natural frequencies and the critical load of the plate are lower. As a consequence the greatest changes in the natural frequencies and the critical load are observed in the case of the two-side-clamped type of boundary conditions, while the lowest changes are observed in the case of the fully clamped type of boundary conditions.

The results show that for both the APT and ASET methods the plate performance increases when the relative volume fraction of the Graphite fibres within the inner Graphite/Epoxy layers decreases. This effect is directly linked to the ratio of longitudinal Young's modulus of the outer SMA/Epoxy layers to the longitudinal Young's modulus of the inner Graphite/Epoxy layers. In the case when the relative volume fraction of the Graphite fibres is low, or equal to 0, the stiffness contribution of the outer SMA/Epoxy layers of the plate becomes dominant, and the observed changes in the natural frequencies and the critical load are maximal.

Thermal and hygrothermal effects are not taken into account in this study, however it should be noted that in many practical applications they can have a great influence on the results obtained. These effects were considered in other publications, notably [16, 17].

4. Conclusions

Based on the results of numerical calculations the following conclusions can be drawn.

Greater performance is observed for modification of the natural frequencies and the critical load for the ASET method than for the APT method. However, it should be noticed that in engineering practice the use of the Active Strain Energy Tuning method is very limited. This is due to the fact that additional boundary conditions for the SMA wires are required (independent of structural boundary conditions), in order to produce tensile recovery stresses during activation of the SMA wires. Otherwise, the recovery stresses produced during activation of the SMA wires are compressive and may greatly reduce the natural frequencies and the critical load of a structure. Moreover, high recovery stresses (tensile or compressive) produced during activation of the SMA wires may also generate high shearing stresses within the structure leading to damage.

For both the Active Property Tuning and Active Strain Energy Tuning methods the results shown here indicate that there is a significant overall performance influence of the thickness-to-length ratio, as well as the relative volume fraction of the reinforcing Graphite fibres. However, it should be noticed that in most engineering applications the thickness-to-length ratio of structural elements made of composite material is determined by parameters such as the lowest natural frequency, or the critical load, of a structure.

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