IDENTIFICATION OF BOUNDARY HEAT FLUX ON THE EXTERNAL SURFACE OF CASTING

***E. Majchrzak*[1]*, B. Mochnacki*[2] *and J.S. Suchy*[3]**
[1] *Silesian University of Technology, Gliwice, Poland*
[2] *Czestochowa University of Technology, Czestochowa, Poland*
[3] *AGH, Cracow, Poland*

1. Introduction

The thermal processes in the system casting-mould are considered. In particular, the inverse problem consisting in the estimation of boundary heat flux flowing from casting sub-domain to the mould sub-domain is analyzed. To solve the problem the global function specification method is applied. The additional information necessary to solve an inverse problem results from the knowledge of cooling curves at the point selected from casting sub-domain. The solidification model bases on the equation corresponding to the one domain method. As an example, the 1D system created by steel casting and sand mix mould is considered. On the stage of numerical solution of direct problem and additional one the finite difference method has been applied.

2. Governing equations

The thermal processes proceeding in the casting sub-domain are described by the following energy equation

$$C(T)\frac{\partial T}{\partial t} = \mathrm{div}\left[\lambda(T)\mathrm{grad}T\right]$$

where $C(T) = c(T) - L\,\mathrm{d}f_S/\mathrm{d}T$ [J/m^3·K] is called a volumetric substitute thermal capacity [1], $c(T)$ is a volumetric specific heat of casting material, f_S is a volumetric solid state fraction at the point considered, L is a latent heat. From the mathematical point of view the equation determines the transient temperature field in the entire, conventionally homogeneous casting domain and this approach is called 'a one domain method' [1].

A similar equation, namely

$$c_m(T_m)\frac{\partial T_m}{\partial t} = \mathrm{div}\left[\lambda_m(T_m)\mathrm{grad}\,T_m\right]$$

determines a temperature field in a mould sub-domain (c_m is a volumetric specific heat of mould, λ_m is a thermal conductivity of the mould).

On a contact surface between casting and mould the continuity condition is given

$$x \in \Gamma_c: \; -\lambda\,\bar{n}\cdot\mathrm{grad}\,T = -\lambda_m\,\bar{n}\cdot\mathrm{grad}\,T_m \;\;,\;\; T = T_m$$

while on the fragments of external boundary the Dirichlet, Neumann or Robin conditions can be accepted [1]. The initial temperatures (pouring temperature and initial mould temperature) are also known. The simpler model of heat exchange between casting and mould consists in the approximation of mould influence by the Neumann condition (in this way the mould sub-domain is conventionally neglected).

To determine the time dependent substitute Neumann condition the cooling curves at the points selected from the casting domain are applied and they constitute the additional information necessary to solve the inverse problem considered.

http://rcin.org.pl

3. Global function specification method

It is assumed that the time dependent boundary heat flux $q(t)$ on the external surface of casting is unknown. The time interval $[0, t^F]$ is divided into intervals $[t^{f-1}, t^f]$ with constant step $\Delta t = t^f - t^{f-1}$ and for $t \in [t^{f-1}, t^f]$: $q(t) = q(t^f) = q^f$. In the global function specification method [2] the unknown values $q^1, q^2, \ldots, q^{f-1}, q^f, \ldots, q^F$ are identified simultaneously.

Let us assume that the temperatures T_{di}^f at the points x_i are given. Applying the least squares criterion [2] one obtains

$$S\left(q^1, q^2, \ldots, q^F\right) = \sum_{f=1}^{F} \sum_{i=1}^{M} \left(T_i^f - T_{di}^f\right)^2 \rightarrow \text{MIN}$$

where M is the number of sensors, T_i^f are the calculated temperatures obtained from the solution of the direct problem by using the current available estimate for the unknown values q^f, $f=1, 2, \ldots, F$.

At first the direct problem should be solved under the assumption that $q^f = q^{fk}$, $f=1, 2, \ldots, F$ at the same time q^{fk} for $k=0$ are the arbitrary assumed values of heat fluxes, while for $k>0$ they result from the previous iteration. The solution obtained this means the temperature distribution at the points x_i for times t^f, $f=1, 2, \ldots, F$ will be denoted as T_i^{fk}.

Function T_i^f is expanded into Taylor series at the neighbourhood of this solution, and using the necessary condition of several variables function minimum, after the certain mathematical manipulations one obtains

$$\sum_{f=p}^{F} \sum_{i=1}^{M} \sum_{s=1}^{f} z_i^{f,s} z_i^{f,p} \left(q^s - q^{sk}\right) = \sum_{f=p}^{F} \sum_{i=1}^{M} z_i^{f,p} \left(T_{di}^f - T_i^{fk}\right), \quad p = 1, 2, \ldots, F$$

where $z_i^{f,s} = \partial T_i^f / \partial q^s$, $z_i^{f,p} = \partial T_i^f / \partial q^p$ are the sensitivity coefficients [2]. This system of equations allows to find the values $q^1, q^2, \ldots, q^F$.

4. Example of computations

The 1D system casting - mould is considered. The dimensions of layers corresponding to casting and mould: $2L_1 = 0.03$ m, $L_2 - L_1 = 0.045$ m. Initial temperatures equal $T_p = 1550\,^{\circ}$C (casting) and $T_{m0} = 20\,^{\circ}$C (mould). The remaining data have been taken from [1]. In Figure 1 the cooling curves from casting domain are shown, while Figure 2 illustrates the course of real and identified boundary heat flux.

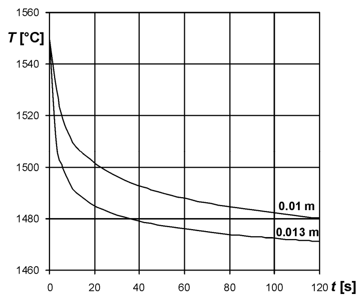

Fig. 1. Cooling curves

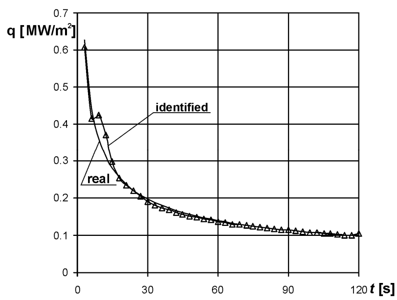

Fig. 2. Real and identified heat flux

5. References

[1] E.Majchrzak, B.Mochnacki (2007). Identification of thermal properties of the system casting - mould, *Materials Science Forum*, 539-543, 2491-2496.

[2] B.Mochnacki, E.Majchrzak, R.Szopa, J.S.Suchy (2006). Inverse problems in the thermal theory of foundry, *Scientific Research of the Institute of Mathematics and Computer Science*, Czestochowa, 1(5), 154-179.

http://rcin.org.pl