SPARSE GRID AND EVOLUTION-TYPE ALGORITHM IN SHAPE OPTIMIZATION FOR BECK'S COLUMN

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1. Introduction

The Beck column is defined as a column clamped at one end and subjected to a certain follower force at the other. The force in this system is always tangent to the deflection line on a free end. After passing critical value of acting force, the Beck column loses its stability in an oscillatory manner. The problem was first investigated theoretically by Pfluger (1950) and solved by Beck (1952). The solution was confirmed by several authors (Pfluger 1950, Leipholtz 1962, Deineko & Leonov 1955, Kordas & Zyckowski 1963). Nowadays the follower force play important role in civil engeneering, vehicle dynamics and earonautics. In this paper a new approach for finding columns with a higher value of the critical force is presented.

2. New approach in optimization

The equation of motion for the Beck column is described in the following manner [2]:

(1) $$(Su'' + Pu)'' = -\rho\ddot{u}\,.$$

where:
$u = u(z,t)$ - the unknown displacement function, P - the given constant longitudinal compressing force, $S = S(z)$ - the bending stiffness, $\rho = \rho(z)$ - the mass density.

For the optimization of the parameters describing the shape, eg.1 has to be studied with appropriate bounadry conditions. We look for the maximum P, such that solution to (1) remain stable, cf. [1], [3], [4]. Well known procedures are the steepest-descent, gradient method and Newton's method. In this paper a new approach for finding optimal shape of a column is applied. The evolution algorithms (EA) [3] are used with sparse grid interpolation to speed up the calculation of objective function. A process of calculating each combination of decision variable is time consuming.

The main idea consists of adapting a form of evolution for finding extrema of functions. At the begining, the algorithm creates an initial population of individuals. Then operations of differentiation are performed on the population. Those operations are selection and mutation. The individuals are then evaluated by the objective function, which is called here the fitness function. The best-fitted individuals are propagated to the next generation. The process is being repeated until suitable stopping criteria are met.

The sparse grids interpolate the objective function with a special discretization technique thus a smooth multivariate function can be approximated with a suitable interpolation formula. The reason for involved spare grids is to speed up the calculation of objective function and restrict the area where the EA searches for a optimal solution.

The object of optimization is the two-segment column jointed with a passive hinge. Below graphs show a plot of interpolated function and the plot of errors function i.e. the difference between real and interpolated values.

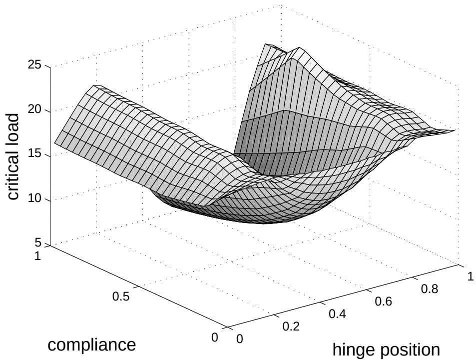

Figure 1. Graph of interpolated critical load vs. position and the compliance of hinge.

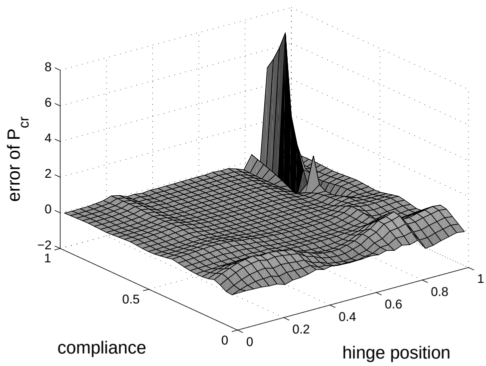

Figure 2. Graph of errors (the difference real and interpolated values) of objective function.

An application of sparse grids with combination of evolution algorithms is a new concept in the optimization of column profile. On the future a column with more number of elements (with mutable cross-section) are going to be analyzed.

3. References

[1] Bogacz, R., Frischmuth K., 2005, Transient Behavior of Columns under Follower Forces, Machine Dynamics Problems, Vol. 29 No. 4, pp. 7–20.

[2] Bogacz R., Janiszewski R.: *Zagadnienia analizy i syntezy kolumn obciążonych siłami śledzącymi ze względu na stateczność*. Warszawska Drukarnia Naukowa, Warszawa 1986

[3] Price K. V., Storn R. M., Lampinen J. A.: *Differential Evolution: A Practical Approach to Global Optimization*. I edition, Springer 2005

[4] Lisowski K., Frischmuth K.: Shape optimization for Beck's column by evolution-type algorithms. Zeszyty Naukowe Wydziału Mechanicznego nr 40, Wydawnictwo Uczelniane Politechniki Koszalińskiej, str. 186-193, Koszalin 2007