LONGITUDINAL WAVE RESPONSE OF A CHIRAL SLAB INTERPOSED BETWEEN MICROPOLAR SOLID HALF-SPACES

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ABSTRACT

Consider a chiral slab of uniform thickness and interposed between two different micropolar elastic solid half-spaces. Let B_1 be the plane boundary between the micropolar half-space (in which the incident wave is assumed to travel) and the chiral slab, and B_2 be the plane boundary between the chiral slab and the other micropolar elastic half-space. A plane longitudinal displacement wave propagating through one of the micropolar elastic solid half-spaces is assumed to be incident on the chiral slab. A part of the energy carried by the incident wave will be reflected back from the boundary B_1 into the incident medium and rest will be transmitted into the elastic chiral slab. A portion of the incident energy transmitted into the chiral slab will proceed to interact with the boundary B_2. Here, again some part of the energy will be reflected and rest will be transmitted into the micropolar medium (not the incident one). These reflected waves will go back to interact with the boundary B_1 and the process will repeat.

The equations of motion without body force and body couple densities are given by
For micropolar elastic solid (see Eringen [1])

$$(\lambda+2\mu+K)\nabla\nabla\cdot \boldsymbol{u} - (\mu+K)\nabla\times\nabla\times\boldsymbol{u} + K\,\nabla\times\,\boldsymbol{\Phi} = \rho\ddot{\boldsymbol{u}},$$
$$(\alpha+\beta+\gamma)\nabla\nabla\cdot\boldsymbol{\Phi} - \gamma\nabla\times\nabla\times\boldsymbol{\Phi} + K\,\nabla\times\mathbf{u} - 2K\boldsymbol{\Phi} = \rho J\ddot{\boldsymbol{\Phi}}.$$

For chiral medium (see Nowacki [2])

$$(\lambda+2\mu)\nabla\nabla\cdot\boldsymbol{u} - \mu\nabla\times\nabla\times\boldsymbol{u} + C_3\,\nabla\nabla\cdot\boldsymbol{\Phi} - C_3\nabla\times\nabla\times\boldsymbol{\Phi} = \rho\ddot{\boldsymbol{u}},$$
$$(\alpha+\beta+\gamma)\nabla\nabla\cdot\boldsymbol{\Phi} - \gamma\nabla\times\nabla\times\boldsymbol{\Phi} + 2C_3\,\nabla\times\boldsymbol{\Phi} + C_3\nabla\nabla\cdot\boldsymbol{u} - C_3\nabla\times\nabla\times\boldsymbol{u} = \rho J\ddot{\boldsymbol{\Phi}}.$$

The constitutive relations describing force stress tensor τ_{ij} and the couple stress tensor m_{ij} (i, j =1,2,3) are given by
For micropolar elastic solid (see Eringen [1])

$$\tau_{ij} = \lambda u_{k,k}\,\delta_{ij} + \mu\,(u_{i,j} + u_{j,i}) + K(u_{j,i} - e_{ijk}\Phi_k),$$
$$m_{ij} = \alpha\,\Phi_{k,k}\delta_{ij} + \beta\,\Phi_{i,j} + \gamma\,\Phi_{j,i},$$

For chiral solid (Lakes and Benedict [3])

$$\tau_{ij} = \lambda u_{k,k}\,\delta_{ij} + \mu\,(u_{i,j} + u_{j,i}) + C_3\Phi_{j,i},$$
$$m_{ij} = \alpha\,\Phi_{k,k}\delta_{ij} + \beta\,\Phi_{i,j} + \gamma\,\Phi_{j,i} + C_3(u_{j,i} - e_{ijk}\Phi_k).$$

We shall discuss two–dimensional problem in *x-z* plane so that second-component of displacement vector, first and third components of the micro-rotation vector will not enter into the analysis.

There are two sets of boundary conditions possible at the micropolar - chiral interfaces, i.e., at B_1 and B_2.
Set-I:
(i) the continuity of displacement, (ii) the continuity of traction (normal component of stress), and (iii) the continuity of micro-rotation, i.e.,

$At\ B_1:$

$$u^{(1)}_{(inc)} + u^{(1)}_{(ref)} = u^{(2)}_{(ref)} + u^{(2)}_{(tr)},\ \hat{e}_z \cdot (\tau^{(1)}_{(inc)} + \tau^{(1)}_{(ref)}) = \hat{e}_z \cdot (\tau^{(2)}_{(ref)} + \tau^{(2)}_{(tr)}),\quad \Phi^{(1)}_{(ref)} = \Phi^{(2)}_{(ref)} + \Phi^{(2)}_{(tr)},$$

$At\ B_2:$

$$u^{(2)}_{(ref)} + u^{(2)}_{(tr)} = u^{(3)}_{(tr)},\quad \hat{e}_z \cdot (\tau^{(2)}_{(ref)} + \tau^{(2)}_{(tr)}) = \hat{e}_z \cdot \tau^{(3)}_{(tr)},\quad \Phi^{(2)}_{(ref)} + \Phi^{(2)}_{(tr)} = \Phi^{(3)}_{(tr)},$$

Set-II:
(i) the continuity of displacement, (ii) the continuity of traction (normal component of stress), and (iii) the continuity of the normal component of couple stress, i.e.,

$At\ B_1:$

$$u^{(1)}_{(inc)} + u^{(1)}_{(ref)} = u^{(2)}_{(ref)} + u^{(2)}_{(tr)},\ \hat{e}_z \cdot (\tau^{(1)}_{(inc)} + \tau^{(1)}_{(ref)}) = \hat{e}_z \cdot (\tau^{(2)}_{(ref)} + \tau^{(2)}_{(tr)}),\ \hat{e}_z \cdot (m^{(1)}_{(inc)} + m^{(1)}_{(ref)}) = \hat{e}_z \cdot (m^{(2)}_{(ref)} + m^{(2)}_{(tr)}),$$

$At\ B_2:$

$$u^{(2)}_{(ref)} + u^{(2)}_{(tr)} = u^{(3)}_{(tr)},\ \hat{e}_z \cdot (\tau^{(2)}_{(ref)} + \tau^{(2)}_{(tr)}) = \hat{e}_z \cdot \tau^{(3)}_{(tr)},\ \hat{e}_z \cdot (m^{(2)}_{(ref)} + m^{(2)}_{(tr)}) = \hat{e}_z \cdot m^{(3)}_{(tr)},$$

where the superscript (1) indicates the incident micropolar elastic medium, the superscript (2) indicates the interposed chiral medium and the superscript (3) indicates the other micropolar elastic medium.

Using Helmholtz decomposition of vectors into scalar and vector potentials, the equations of motion can be reduced to some wave equations (coupled and uncoupled). Assuming the appropriate form of potentials in the respective medium and employing the above boundary conditions, it can be seen that each set of boundary conditions gives a non-homogeneous system of eighteen equations in eighteen unknown. Both the sets enable us to determine the amplitude ratios of various reflected and transmitted waves. For each set of boundary conditions, the equations satisfying them are solved numerically to obtain the reflection and transmission coefficients. Each set is found to exhibit different expressions of the reflection and transmission coefficients. Various reflection and transmission coefficients are found to be the functions of the angle of incidence, frequency of the incident wave, elastic parameters of the media and the thickness of the interposed layer. The variations in the modulus of the amplitude ratios with the angle of incidence and with the frequency ratio are computed for a peculiar model and depicted graphically. The effects of the chirality parameter and the thickness of the chiral slab on various amplitude ratios are also studied. Numerical results reveal that for very thin slab, the variations in all the amplitude ratios with the angle of incidence are found to be smooth enough. But as the thickness of the chiral slab becomes significant, we obtain more and more fluctuations in the variations of these coefficients with the angle of incidence. Comparisons in the modulus of the respective amplitude ratios obtained from the two possible sets of boundary conditions are also observed and are depicted graphically. Some results of earlier researchers [4] and [5] have also been reduced as special cases of the present formulation.

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