NINE-NODE ASSUMED STRAIN SHELL ELEMENT WITH DRILLING ROTATION

P. Panasz , K. Wisniewski
Institute of Fundamental Technological Research, Warsaw, Poland

1. Introduction

Nine-node shell elements are more complex than four-node elements, but are advantageous in some applications, involving dominant in-extensional bending.

In the current paper, we develop a finite-rotation 9-node shell element with drilling rotation, which is based on the Reissner's kinematics and the Green strain. The basic 9-node isoparametric Lagrangian shell element suffers from the transverse shear and membrane locking. Several techniques of avoiding them were proposed and tested in the literature. One of the most effective is the assumed strain (AS) method, which consists of sampling of strain components at certain points, and extrapolating these values over the element. We apply and modify this method, for details see [1].

2. Basic shell equations

Classical papers on the subject are restricted to two-parameter rotations, which have to be defined in the local basis and transformed to the reference basis. We define an *extended configuration space* in terms of the deformation function $\boldsymbol{\chi}$ and rotations $\mathbf{Q} \in SO(3)$

(1) $$\mathcal{C}_{\text{ext}} \doteq \{(\boldsymbol{\chi}, \mathbf{Q}) : \ B \to R^3 \times SO(3) \quad | \quad \boldsymbol{\chi} \in \mathcal{C}\},$$

which includes the drilling rotation, so, in effect, we have 3 rotational parameters per node, and the rotation vector can be directly assumed in the reference basis. The rotations are included in the formulation using the rotation constraint (RC) equation: $\text{skew}(\mathbf{Q}^T\mathbf{F}) = \mathbf{0}$, where $\mathbf{F} \doteq \nabla\boldsymbol{\chi}$, see [2]. The formulation is based on the following 3D two-field functional

(2) $$F_2(\boldsymbol{\chi}, \mathbf{Q}) \doteq \int_B \left[\mathcal{W}(\mathbf{F}^T\mathbf{F}) + \frac{\gamma}{2}\text{skew}(\mathbf{Q}^T\mathbf{F}) \cdot \text{skew}(\mathbf{Q}^T\mathbf{F})\right] \mathrm{d}V + F_{\text{ext}},$$

where $\mathcal{W}$ is the strain energy, and $\gamma \in (0, \infty)$ is the regularization parameter.

The shell kinematics is based on the Reissner's hypothesis, with the current position vector expressed as follows,

(3) $$\mathbf{x}(\xi^\alpha, \zeta) = \mathbf{x}_0(\xi^\alpha) + \zeta\, \mathbf{Q}_0(\xi^\alpha)\, \mathbf{t}_3(\xi^\alpha),$$

where $\mathbf{x}_0$ is the current position of the reference surface, and $\mathbf{Q}_0$ is a rotation of the reference surface, and $\mathbf{t}_3$ is the shell director normal to the reference surface in the initial configuration. Besides, $\xi^\alpha \in [-1, +1]$ and $\zeta \in [-h/2, +h/2]$, where h is the initial shell thickness.

The strain energy is assumed in the Saint Venant-Kirchhoff's form, $\mathcal{W} \doteq \frac{1}{2}\lambda\,(\text{tr}\mathbf{E})^2 + G\,\text{tr}(\mathbf{E}^2)$, where $\mathbf{E} \doteq \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$ is the Green strain, and λ, G are Lamé constants.

3. Features of our 9-node shell element

The developed 9-node shell element has the following features: (a) the drilling rotation is included, yielding 3 rotational dofs/node, (b) the rotations are parameterized by the canonical rotation vector, and are unrestricted, (c) the Reissner's kinematics is applied, so the transverse shear energy is included.

To eliminate the transverse shear and membrane locking, the AS method is applied. A modification of the AS method is proposed, consisting in treating the sampling and the numerical integration together, which results in 6 sampling points being replaced by two sampling lines, see Fig.1. This change facilitates the implementation and significantly improves efficiency of differentiation, for details see [1]. The two-level approximation is applied to strain components in the ortho-normal basis at the element's center.

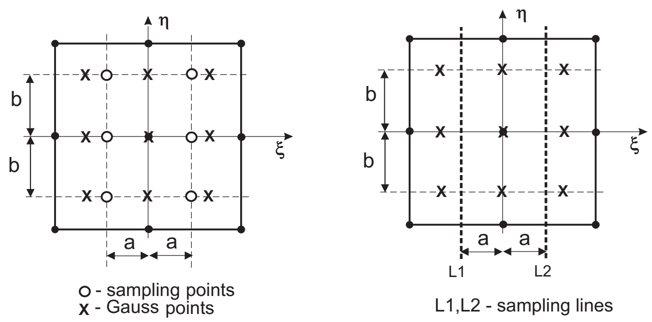

Figure 1. Location of sampling points and lines, $a = \sqrt{\frac{1}{3}}$, $b = \sqrt{\frac{3}{5}}$.

4. Numerical tests

The developed 9-AS shell element is subjected to a range of benchmark tests, to establish the sensitivity to mesh distortion, the coarse mesh accuracy, and to confirm the lack of locking. One of the tests is the analysis of deployment of a ring, with the deformation shown in Fig.2. Our results are compared with results obtained by the MITC9 element of ADINA and the S9R5 element of ABAQUS.

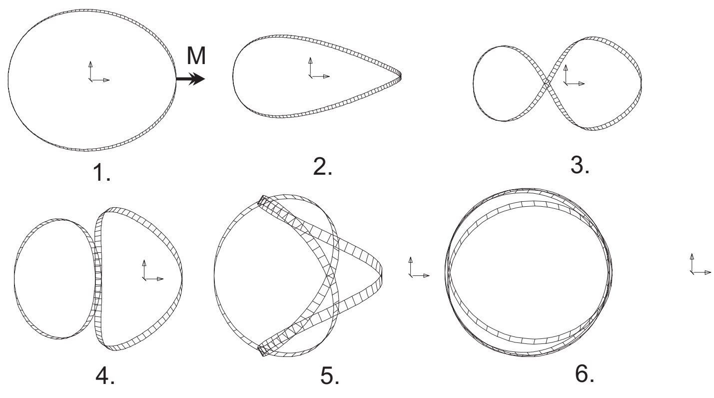

Figure 2. Deployable ring. Progressing stages of deformation.

5. References

[1] Panasz P., Wisniewski K.: *Nine-node shell elements with 6 dofs/node based on two-level approximations.* Submitted (2007)

[2] Wisniewski K., Turska E.: *Second order shell kinemtaics implied by rotation constraint equation,* J. Elasticity, Vol.67, 229–246 (2002).