THEORY OF MICROPOLAR THIN ELASTIC CYLINDRICAL SHELLS

S.H. Sargsyan
Gyumri State Pedagogical Institute, Gyumri, Armenia
National Science Acdemy of Armenia, Yerevan, Armenia

1. General

Current developments of mechanics of elastic medium is closely connected with the construction of generalized mathematical models which consider material's particle as a complex object endowed with additional properties describing material's inner structure (unlike the classical theory of elasticity which considers particle as a material unit). Presently, theories of elastic mediums with micro- and nanostructure are successfully cultivated on the basis of micropolar (momental, asymmetrical) theory of elasticity, otherwise, on Cosserate's continuum.

Alongside with the development of the three-dimensional micropolar elastic model, presently, construction of applied theories for micropolar elastic plates and shells is becoming more actual.

In papers [1, 2] on the basis of the asymptotic method linear theories of micropolar thin elastic plates and shells with boundary layer are constructed. Depending on the values of sizeless physical parameters of the plate and shell theories of micropolar elastic plates and shells with independent and constraint rotation and theories with "small shift rigidity" are constructed.

In present paper on the basis of the constructed general theories of micropolar shell [2] mathematical models for micropolar elastic cylindrical shells with independent and constraint rotation and "with small shift rigidity" are studied.

2. System and boundary conditions of micropolar elastic cylindrical (axe symmetrical) shells with independent rotation

Balance equations [2]:

$$\frac{d\mathrm{T}_{11}}{\mathrm{d}\xi}=r\left(q_1^{+}+q_1^{-}\right),\ T_{22}+\frac{dN_{13}}{d\xi}=-r\left(q_3^{+}+q_3^{-}\right),\ \frac{dL_{12}}{d\xi}-r\left(N_{31}-N_{13}\right)=r\left(m_2^{+}+m_2^{-}\right)$$

Elasticity correlations:

$$T_{ii}=\frac{2Eh}{1-\nu^2}\left[\Gamma_{ii}+\nu\Gamma_{jj}\right],\qquad L_{12}=2h(\gamma+\varepsilon)\chi_{12},\quad N_{13}=-2h\frac{4\alpha\mu}{\alpha+\mu}\Gamma_{i3}-\frac{\alpha-\mu}{\alpha+\mu}N_{31},\quad N_{31}=h\left(q_1^{+}-q_1^{-}\right)$$

Geometrical correlations:

$$\Gamma_{11}=\frac{1}{r}\frac{du_1}{d\xi},\quad \Gamma_{22}=-\frac{w}{r},\quad \Gamma_{13}=-\frac{1}{r}\frac{dw}{d\xi}+\Omega_2,\quad \chi_{12}=\frac{1}{r}\frac{d\Omega_2}{d\xi}$$

where T_{11}, T_{22}, N_{13} – are forces averaged along the shell's thickness, L_{12} – is the averaged moment from momental stresses, $\Gamma_{11},\Gamma_{22},\Gamma_{13}$ – are components of tensor deformation, χ_{12} – is the bending in shell's middle surface, u_1, w – are the transitions, and Ω_2 – is the independent rotation of points of the shell's middle surface.

Boundary pivot conditions:

$$w\big|_{\Gamma}=0,\quad T_{11}\big|_{\Gamma}=0,\ L_{12}\big|_{\Gamma}=0$$

http://rcin.org.pl

3. System and boundary conditions of micropolar elastic cylindrical (axe symmetrical) shells with constraint rotation:

Balance equations:

$$\frac{dT_{11}}{d\xi}=r\left(q_1^+ + q_1^-\right),\ T_{22}+\frac{dN_{13}}{d\xi}=-r\left(q_3^+ + q_3^-\right),\ \frac{d}{d\xi}\left(G_{11}-L_{12}\right)-rN_{13}=-r\left(m_2^+ - m_2^-\right)-rh\left(q_1^+ - q_1^-\right)$$

Elasticity correlations:

$$T_{ii}=\frac{2Eh}{1-\nu^2}\left[\Gamma_{ii}+\nu\Gamma_{jj}\right],\qquad L_{12}=2h(\gamma+\varepsilon)\chi_{12},\quad G_{11}=-\frac{2Eh^3}{3\left(1-\nu^2\right)}K_{11}$$

Geometrical correlations:

$$\Gamma_{11}=\frac{1}{r}\frac{du_1}{d\xi},\ \Gamma_{22}=-\frac{w}{r},\quad K_{11}=\frac{1}{r}\frac{d\beta_1}{d\xi},\quad \beta_1=\frac{1}{r}\frac{dw}{d\xi},\ \chi_{12}=\frac{1}{r}\frac{d\Omega_2}{d\xi}\quad \Omega_2=\beta_1$$

Boundary pivot conditions

$$T_{11}\big|_\Gamma=0,\ \left(L_{12}-G_{11}\right)\big|_\Gamma=0,\ w\big|_\Gamma=0$$

4. System and boundary conditions of micropolar elastic cylindrical shells (axe symmetrical) with "small shift rigidity":

Balance equations:

$$\frac{dT_{11}}{d\xi}=r\left(q_1^+ + q_1^-\right),\ T_{22}+\frac{dN_{13}}{d\xi}=-r\left(q_3^+ + q_3^-\right),\ \frac{dG_{11}}{d\xi}-rN_{31}+rh\left(q_1^+ - q_1^-\right)=0,\ \frac{dL_{12}}{d\xi}=r\left(m_2^+ + m_2^-\right)$$

Elasticity correlations:

$$T_{ii}=\frac{2Eh}{1-\nu^2}\left[\Gamma_{ii}+\nu\Gamma_{jj}\right],\qquad L_{12}=2h(\gamma+\varepsilon)\chi_{12},\quad G_{11}=-\frac{2Eh^3}{3\left(1-\nu^2\right)}K_{11},\quad N_{13}=N_{31}-8h\alpha\Gamma_{13}$$

Geometrical correlations:

$$\Gamma_{11}=\frac{1}{r}\frac{du_1}{d\xi},\ \Gamma_{22}=-\frac{w}{r},\quad \Gamma_{13}=-\beta_1+\Omega_2,\ K_{11}=\frac{1}{r}\frac{d\beta_1}{d\xi},\quad \beta_1=\frac{1}{r}\frac{dw}{d\xi},\ \chi_{12}=\frac{1}{r}\frac{d\Omega_2}{d\xi}$$

Boundary pivot conditions:

$$w\big|_\Gamma=0,\ T_{11}\big|_\Gamma=0,\ G_{11}\big|_\Gamma=0$$

On the basis of the above mentioned mathematical models of micropolar elastic cylindrical (axe symmetrical) shells, definition of stress-deformed state in them is brought to final formula and numerical results. On the basis of the numerical results, properties of the constructed applied theories of microplar shells are analyzed, conclusions and recommendations on the application of the micropolar materials are made.

5. References

1. Sargsyan S.H (2008). Boundary problems of thin plates in asymmetrical theory of elasticity, *Applied Mathematics and Mechanics*, **72**, N 1, 129-147, (in Russian).
2. Sargsyan S.H (2008). General theory of thin elastic shells on the basis of asymmetrical theory of elasticity, *Applied Mathematics and Mechanics*, **72**, not published yet.

http://rcin.org.pl