OPTIMAL STABILLIZATION OF POSTBUCKLING PATH FOR CONICAL SHELLS UNDER EXTERNAL PRESSURE

J.Krużelecki and D.Trybuła
Cracow University of Technology, Cracow, Poland

A postbuckling path for a conical shell under compressive radial pressure is unstable one. It means that loss of stability of a shell can be associated with a snap-through, which can lead to very large deflections and finally, to destruction of a structure

A standard problem of structural optimization under stability constraints is usually formulated as maximization of the instability load for a prescribed volume of a design element. Very often a standard optimal structure has unstable postbuckling behaviour and it is very sensitive to imperfections. That is weakness of the design and it indicates that the combination of geometrically nonlinear analysis with the design becomes necessary, especially from the practical point of view. Postbuckling constraints of a special form added to formulation of the optimization problem permit to modify the postbuckling path and the stable postbuckling path can be created, even in the case of unstable behaviour of a reference structure.

The effect of modification of the postbuckling behaviour in most cases has been obtained by changing sizing variables, which are usually dimensions of the design elements. This type of problems were considered, for example, by Bochenek [1, 2]. In this paper an alternative concept is applied, namely stabilization of the postbuckling path is obtained by application of additional loadings acting on a shell without changing of a shape and sizes of the optimized structure. These loadings can be either active forces applied to a structure or passive ones (reactions of the additional supports), or both active and passive forces acting simultaneously. Such problems, for finite-degree-of-freedom of rod system that models the behaviour of a real shell structure, was considered by Bochenek and Krużelecki [3]. On the other hand Krużelecki and Król [4] examined the real cylindrical shells with different geometrical parameters whereas Krużelecki and Trybuła [5] investigated such shells under twisting moment. These papers showed that axial loadings can stabilize an initially unstable postcritical path.

In this paper stabilization of a postbuckling path for a simply supported (different variants of supports are considered) truncated conical shell under radial compressive pressure is formulated as a certain modified non-standard problem of optimization. From mentioned above types of stabilizing loadings only an active axial force is investigated here. Calculations are performed using ANSYS code for elastic shells of different length, thickness and and semi-vertex angle.

The problem of optimization is stated as follows. The minimum value of the axial load N, which leads to the stable behaviour of a shell is looked for

$$\textit{Minimize} \quad N$$

$$\textit{subject to} \quad \frac{\partial p}{\partial f}\left(f^{*}, N\right) = \frac{\partial^{2} p}{\partial f^{2}}\left(f^{*}, N\right) = 0$$

where f denotes a displacement of a wall of a shell. The displacement f^{*} refers to the horizontal inflexion point at the equilibrium path, Fig.1. The above conditions lead to elimination of the snap-through and finally, one obtains the stable postbuckling path even the original equilibrium path is unstable one. It is shown in Fig.1, where the thick line refers the stable equilibrium path under the minimum axial load N. This formulation of the optimization problem contains only one design variable N and two constraints, which are imposed on the postbuckling state. They ensure the stable behaviour of the cylindrical shell under external pressure. A condition of a constant volume of the structure is automatically fulfilled because that formulation does not take into account modification of the shell geometry.

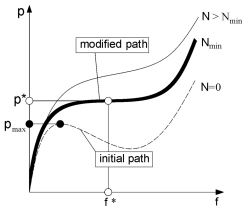

Fig.1. Equilibrium paths for N=0 and N≠0

Calculations were performed for a semi-vertex angle from 5 to 45 degrees, three different length and thickness of a shell assuming only elastic deformations. The numerical analysis showed that the active forces can improve the resistance of the radially compressed truncated conical shells against buckling. The critical pressure for elastic shells under additional axial loading can be much higher in comparison with a structure loaded by external pressure only. The active force can stabilize the initially unstable postbuckling path for elastic shells under external pressure. Then, the optimal shells do not lose their stability at all.

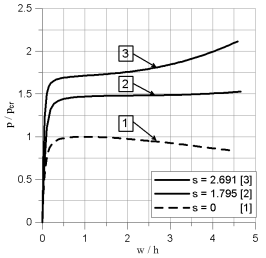

Fig.2. Equilibrium paths for the shell with semi-vertex angle equals to 15^0, R_{sr}/L=2, h/R_{sr}=0.005

In Fig.2 the postbuckling paths are presented for the shell with semi-vertex angle equals to 15^0, R_{sr}/L=2 h/R_{sr}=0.005, where R_{sr} means the mean value of the shell radius. The postbuckling paths are shown in dimensionless coordinates: p/p_{cr}, w/h, where p_{cr} denotes the critical pressure (maximum pressure at the equilibrium path for N=0) and w stands for the maximum radial displacement. As a measure of applied active force we took $s = \sigma_z / \sigma_0$, where σ_z denotes the axial stress $\sigma_z = N/A$. The dash lines (1) represent the postbuckling path for s=0, the line (2) corresponds to the active force, which stabilizes the postbuckling path and the line (3) is connected with the active forces, which is larger than the minimal stabilizing one.

[1] B. Bochenek (1977). Optimization of geometrically nonlinear structures with respect to buckling and postbuckling constraints, *Engineering Optimization*, **29**, 401-415.

[2] B. Bochenek (2001). Problems of optimal design of structures for postbuckling behaviour, *Zeszyty Naukowe Podstawowe Nauki Techniczne No.22, Politechnika Krakowska*, Kraków.

[3] B. Bochenek, J. Krużelecki (2001). A new concept of optimization for postbuckling behaviour. *Engineering Optimzation*, **33**, 503-522

[4] J. Krużelecki, M. Król (2006). Optimization of postbuckling path for cylindrical shells under external pressure. In *Proc. III European Conference on Computational Mechanics; Solids, Structures and Coupled Problems in Engineering*, edited. by C.A. Mota Soares, J.A.C. Martin, H.C. Rodrigues, J.A.C. Ambrosio, Springer (CD-ROM only).

[5] J. Krużelecki, D. Trybuła (2007). Optimal stabilization of postbuckling path for cylindrical shells under torsion, In *Proc. Seventh World Congress on Structural and Multidisciplinary Optimization* (CD-ROM only).