R-FUNCTIONS METHOD APPLYING TO LARGE DEFLECTION ANALYSIS OF ORTHOTROPIS SHALLOW SHELLS ON ELASTIC FOUNDATION

L. Kurpa, K. Lyubitsky
National Technical University ”Kharkov Polytechnic Institute”, Kharkov, Ukraine

Shallow shells are structural elements of many modern constructions, in particular case if they made from composite materials. Investigation of such shells is connected with large mathematical difficulties due to complexity of mathematical statement.

In the present work a geometric non-linear bending of orthotropic shallow shells with complex planform resting on an elastic foundation is studied. To solve the system of the governing equations the theory of R-functions [1], variational methods and step-by-step method are used. A principal advantage of this approach is the possibility to investigate the shallow shells of an arbitrary planform and consider the different boundary conditions. It should be noted that desired solution is found in analytical form.

Governing equations for large deflections of shallow shells on base of the classical theory [2] are:

$$
\begin{aligned}
&L_1(D_{ij})W + \nabla_k(\Phi) - L(W,\Phi) = q - p \\
&L_2(A_{ij})\Phi + \nabla_k(W) + \frac{1}{2}L(W,W) = 0\,,
\end{aligned} \tag{1}
$$

where W is the deflection function, Φ is the stress function, q is the transverse loading, p is the foundation pressure. The equilibrium system is supplied by corresponding boundary conditions.

For Winkler foundation p can be defined mathematically by

$$p = rW\,. \tag{2}$$

To solve nonlinear system (1) let us linearize it. One of the known method of linearization is the step-by-step method, which was proposed by Vlasov and was developed by his followers [3]. As result the given system (1) is reduced to the following linear system:

$$
\begin{aligned}
&L_2(A_{ij})\delta\Phi + \nabla_k(W_i) + L(W_i,\delta W) = 0 \\
&L_1(D_{ij})\delta W + \nabla_k(\Phi_i) - L(\delta W,\Phi_i) - L(W_i,\delta\Phi) = Q_i - r\delta W
\end{aligned} \tag{3}
$$

where δW and $\delta\Phi$ are increments of the unknown functions on the present loading step.

On every i-th step this system are solved by variational Ritz’s method. Unknown functions are presented as expansions in series with help of coordinate functions satisfying the given boundary conditions. Problems of constructing such sequences for shells of an arbitrary shape have been solved by RFM (R-Functions Method). Note, that RFM allows to describe the domain boundary as uniform analytical expression, and to receive, as result, a solution in an analytical form.

The sought for solution on k-th step may be presented as

$$W^{(k)} = \sum_{i=1}^{k}\delta W_i\,,\ \Phi^{(k)} = \sum_{i=1}^{k}\delta\Phi_i\,,\ Q^{(k)} = \sum_{i=1}^{k}\delta Q_i \tag{4}$$

To exact the approximate solution method by Newton-Rafson is applied.

In the present study to find the upper and lower buckling loads the algorithm is worked out.

Proposed approach allows to build a whole deflection curve of shallow shell.

The numerical implementation of the offered approach is carried out in framework of a system POLE-RL [4]. Its reliability was checked out by test examples. To demonstrate possibilities of the proposed method there would be solved the non-linear bending problem shallow shell ($\bar{k}_1$=0, $\bar{k}_2$=20) with complex plan form (figure 1) and mixed boundary conditions. It is supported that shell is made from glass-epoxy material with following mechanical characteristics: E_1/E_2=3, G_{12}/E_2=0.6, v_{12}=0.25. Boundary conditions are assumed to be sliding simply supported on the sides $x = \pm a/2$ and sliding clamped on another parts of the boundary. Foundation modulus is *r*=20

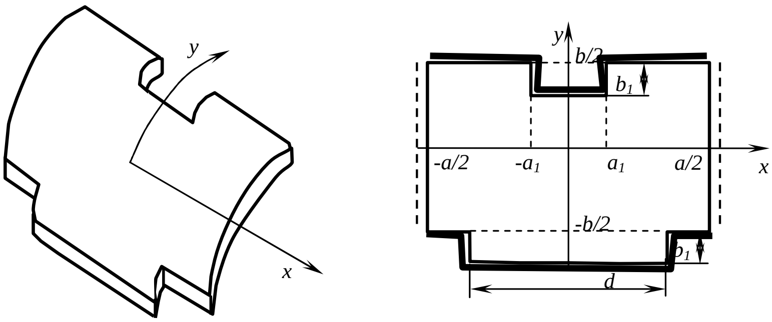

Fig. 1 Planform of a glass-epoxy shallow shell (*b/a*=0.5, a_1/a=0.4, b_1/a=0.2, *d/a*=1)

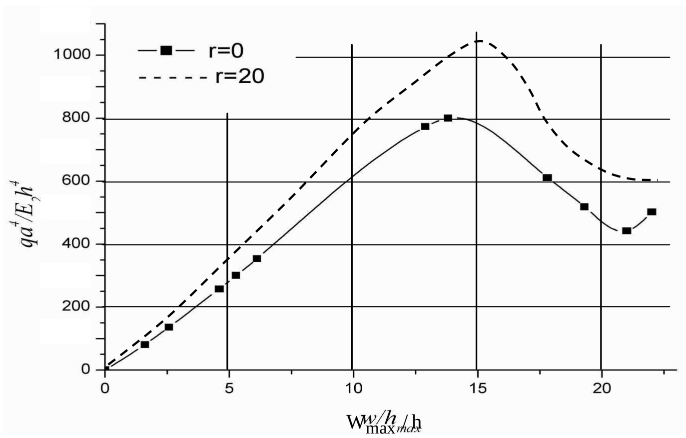

Fig. 2

On fig.2 the dependence of load-maximum deflection is presented.

References

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