VIBRATIONS OF THICK PLATE BY BOUNDARY ELEMENT METHOD

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1. Governing equations

In the theory of thick plates (differently called Reissner-Mindlin plates) is taken into account that transverse strengths and connected with them shear strains have influence on plate deformations. The thicker is the plate, the higher is the influence of transverse strengths - from here name of this theory. In the thick plate theory occurs three independent displacement parameters: deflection w and two rotations φ_α. Additional load of the plate composes moment fields m_α.

Thick plates are described by the following dynamic equilibrium equation system [1]

$$\begin{aligned}
&-H\Delta w - H\varphi_{1,1} - H\varphi_{2,2} + c_w\dot{w} = q - \gamma h\ddot{w},\\
&Hw_{,1} + H\varphi_1 - D\varphi_{1,11} - D\frac{1-\nu}{2}\varphi_{1,22} - D\frac{1+\nu}{2}\varphi_{2,12} + c_1\dot{\varphi}_1 = m_1 - \frac{\gamma h^3}{12}\ddot{\varphi}_1,\\
&Hw_{,2} - D\frac{1+\nu}{2}\varphi_{1,12} + H\varphi_2 - D\varphi_{2,22} - D\frac{1-\nu}{2}\varphi_{2,11} + c_2\dot{\varphi}_2 = m_2 - \frac{\gamma h^3}{12}\ddot{\varphi}_2
\end{aligned} \tag{1}$$

Where H and D mean shear and bending stiffness of the plate, and γ is the mass density.

2. Free vibrations

We transform equations (1) into a form of free harmonic vibrations assuming

$$\begin{aligned}
&q = m_\alpha = 0,\\
&w(\boldsymbol{x},t) = w(\boldsymbol{x})e^{i\omega t}, \quad \varphi_\alpha(\boldsymbol{x},t) = \varphi_\alpha(\boldsymbol{x})e^{i\omega t}
\end{aligned} \tag{2}$$

The equation system (1), written in a convenient form to calculate the fundamental solution, can be written as

$$L_{ij}\bar{u}_{jk} = \delta\delta_{ik} \tag{3}$$

The fundamental solution of the harmonic vibration equation (3) can be found using the Hörmander method. This solution is a function of the parameter ω.

Using the BEM an algebraic equation system with the parameter ω can be obtained.

$$\begin{bmatrix}
\bar{\mathbf{A}}_q^w(\omega) & \bar{\mathbf{A}}_q^{\varphi_1}(\omega) & \bar{\mathbf{A}}_q^{\varphi_2}(\omega) & \bar{\mathbf{A}}_q^{M_1}(\omega) & \bar{\mathbf{A}}_q^{M_2}(\omega) & \bar{\mathbf{A}}_q^{Q_n}(\omega)\\
\bar{\mathbf{A}}_{m_1}^w(\omega) & \bar{\mathbf{A}}_{m_1}^{\varphi_1}(\omega) & \bar{\mathbf{A}}_{m_1}^{\varphi_2}(\omega) & \bar{\mathbf{A}}_{m_1}^{M_1}(\omega) & \bar{\mathbf{A}}_{m_1}^{M_2}(\omega) & \bar{\mathbf{A}}_{m_1}^{Q_n}(\omega)\\
\bar{\mathbf{A}}_{m_2}^w(\omega) & \bar{\mathbf{A}}_{m_2}^{\varphi_1}(\omega) & \bar{\mathbf{A}}_{m_2}^{\varphi_2}(\omega) & \bar{\mathbf{A}}_{m_2}^{M_1}(\omega) & \bar{\mathbf{A}}_{m_2}^{M_2}(\omega) & \bar{\mathbf{A}}_{m_2}^{Q_n}(\omega)
\end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{w}\\ \boldsymbol{\varphi_1}\\ \boldsymbol{\varphi_2}\\ \boldsymbol{M_1}\\ \boldsymbol{M_2}\\ \boldsymbol{Q_n} \end{bmatrix} = 0 \tag{4}$$

Equation system (4), independent of the chosen boundary conditions, can be written in a compact form

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(5) $$\mathbf{A}(\omega)\,\boldsymbol{X} = 0$$

This system has a nonzero solution providing that the determinant of the matrix **A** is equal zero:

(6) $$\det \mathbf{A}(\omega) = 0 \Rightarrow \omega_i, \quad i = 1,2,...$$

3. Forced vibrations

We presuppose the solution of the system (1) in a following form of eigenfunction series:

(7) $$\mathbf{u} = \begin{bmatrix} w \\ \varphi_1 \\ \varphi_2 \end{bmatrix} = \sum_{n=1}^{N} \begin{bmatrix} w_n(\boldsymbol{x}) \cdot T_{wn}(t) \\ \varphi_{1n}(\boldsymbol{x}) \cdot T_{1n}(t) \\ \varphi_{2n}(\boldsymbol{x}) \cdot T_{2n}(t) \end{bmatrix}$$

Equation system (1) separates then into three independent scalar equations of time. Let's write one of them

(8) $$\gamma h \ddot{T}_{wn}(t) + c_w T_{wn}(t) + \gamma h \omega_n^2 T_{wn}(t) = q_n(t)$$

The solution of this equation can be easy found in an analitical way.

A numerical example of the solution of the plate using the upper described routine will be presented during the conference.

4. References

[1] C. M. Wang, S. Kitipornchai and Y. Xiang (1998). *Vibration of Mindlin Plates*, Elsevier Science, Oxford.

[2] K. Myślecki (2004). *Metoda elementów brzegowych w statyce dźwigarów powierzchniowych*, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław.

[3] K. Myślecki and J. Oleńkiewicz (2007). Analysis of free vibration of thin plate by Boundary Element Method (in polish), *LIII Konferencja Naukowa KILiW i KN PZITB*, Krynica.

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