STRESS STATE OF NONTHIN NONCIRCULAR ORTHOTROPIC CYLINDRICAL SHELLS WITH VARIABLE THICKNESS UNDER DIFFERENT TYPES OF BOUNDARY CONDITIONS.

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1. Basic assumptions

The abstract addresses the static problems for nonthin noncircular orthotropic shells using refined Timoshenko-type model based on the hypothesis of a straight line .

Let the shell mid surface be referred to the orthogonal coordinate system s, θ, where s and θ are the coordinates along the generatrix and directrix, respectively. Let γ be normal coordinate to the surface s, θ.

The first quadratic form of the mid surface is $dS^2 = A_1^2 ds^2 + A_2^2 d\theta^2$, $(0 \leqslant s \leqslant l, \theta_1 \leqslant \theta \leqslant \theta_2)$, where $A_1 = 1$ and $A_2 = A_2(\theta)$ are the Lame coefficients.

According to the hypothesis accepted, the displacements of the shell can be represented as

$$(1) \quad u_s(s,\theta,\gamma) = u(s,\theta) + \gamma\psi_s(s,\theta), \quad u_\theta(s,\theta,\gamma) = v(s,\theta) + \gamma\psi_\theta(s,\theta), \quad u_\gamma(s,\theta,\gamma) = w(s,\theta),$$

where u, v and w are the displacements of points of the coordinate surface along the directions s, θ, γ, respectively; ψ_s and ψ_θ are the total angles of rotation of the rectilinear element.

The strains can be expressed as

$$(2) \quad \begin{aligned} e_s(s,\theta,\gamma) &= \varepsilon_s(s,\theta) + \gamma\varkappa_s(s,\theta), \quad e_\theta(s,\theta,\gamma) = \varepsilon_\theta(s,\theta) + \gamma\varkappa_\theta(s,\theta), \\ e_{s\theta}(s,\theta,\gamma) &= \varepsilon_{s\theta}(s,\theta) + \gamma 2\varkappa_{s\theta}(s,\theta), \quad e_{s\gamma}(s,\theta,\gamma) = \gamma_s(s,\theta), \quad e_{\theta\gamma}(s,\theta,\gamma) = \gamma_\theta(s,\theta), \end{aligned}$$

where

$$(3) \quad \begin{aligned} \varepsilon_s &= \frac{\partial u}{\partial s}; \ \varepsilon_\theta = \frac{1}{A_2}\frac{\partial v}{\partial \theta} + kw; \ \varepsilon_{s\theta} = \frac{1}{A_2}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial s}; \ \varkappa_s = \frac{\partial \psi_s}{\partial s}; \ \varkappa_\theta = \frac{1}{A_2}\frac{\partial \psi_\theta}{\partial \theta} - k\varepsilon_\theta; \\ 2\varkappa_{s\theta} &= \frac{1}{A_2}\frac{\partial \psi_s}{\partial \theta} + \frac{\partial \psi_\theta}{\partial s} - \frac{k}{A_2}\frac{\partial u}{\partial \theta}; \ \gamma_s = \psi_s - \vartheta_s; \quad \gamma_\theta = \psi_\theta - \vartheta_\theta; \\ \vartheta_s &= -\frac{\partial w}{\partial s}; \ \vartheta_\theta = -\frac{1}{A_2}\frac{\partial w}{\partial \theta} + kv; \end{aligned}$$

k is the directrix curvature.

The equilibrium equations are:

$$(4) \quad \begin{aligned} \frac{\partial N_s}{\partial s} + \frac{1}{A_2}\frac{\partial N_{\theta s}}{\partial \theta} + q_s &= 0, \quad \frac{1}{A_2}\frac{\partial N_\theta}{\partial \theta} + \frac{\partial N_{s\theta}}{\partial s} + kQ_\theta + q_\theta = 0, \\ \frac{\partial Q_s}{\partial s} + \frac{1}{A_2}\frac{\partial Q_\theta}{\partial \theta} - kN_\theta + q_\gamma &= 0, \quad \frac{\partial M_s}{\partial s} + \frac{1}{A_2}\frac{\partial M_{\theta s}}{\partial \theta} - Q_s = 0, \quad \frac{1}{A_2}\frac{\partial M_\theta}{\partial \theta} + \frac{\partial M_{s\theta}}{\partial s} - Q_\theta = 0, \end{aligned}$$

where N_s, N_θ, $N_{s\theta}$, and $N_{\theta s}$ are the tangential forces; Q_s, Q_θ are the shear forces; M_s, M_θ, $M_{s\theta}$, and $M_{\theta s}$ are the bending and twisting moments; q_s, q_θ and q_γ are the components of the surface load. Elastic relations for orthotropic shells,which are symmetrical with respect to the chosen coordinate surface, have the form

$$(5) \quad \begin{aligned} N_s &= C_{11}\varepsilon_s + C_{12}\varepsilon_\theta, \ N_\theta = C_{12}\varepsilon_s + C_{22}\varepsilon_\theta, \ N_{st} = C_{66}\varepsilon_{s\theta} + 2kD_{66}\varkappa_{s\theta}, \\ N_{\theta s} &= C_{66}\varepsilon_{s\theta}, \ M_s = D_{11}\varkappa_s + D_{12}\varkappa_\theta, \ M_\theta = D_{12}\varkappa_s + D_{22}\varkappa_\theta, \\ M_{\theta s} &= M_{s\theta} = 2D_{66}\varkappa_{s\theta}, \ Q_s = K_1\gamma_s, \ Q_\theta = K_2\gamma_\theta, \end{aligned}$$

where C_{ij}, D_{ij}, K_1, and K_2 are the parameters that depend on the material properties and shell thickness.

2. Resolving technique and its application

Choosingthe displacements u, v, w, and the total angles of rotation ψ_s, ψ_θ as unknown functions and using (3)–(5) the resolving system of partial differential equation describing the stress state of orthotropic non circular cylindrical shells can be presented as follows [2]:

$$(6) \quad L\overline{y} = 0,$$

where L is the linear differential operator of the second order and $\overline{y} = \{u, v, w, \psi_s, \psi_\theta\}$ is the desired vector-function. Adding to (6) boundary conditions on ends and boundary conditions on rectalinear contours in the case of a closed shell or symmetry conditions, if a shell is open, we obtain two-dimensional boundary-value problem, whose solution can be presented in the following form:

$$(7) \quad \overline{y} = \Phi\overline{y}_*,$$

where $\overline{y}_* = \{u_0(\theta),\ldots,u_N(\theta),\, v_0(\theta),\ldots,v_N(\theta),\, w_0(\theta),\ldots,w_N(\theta),\, \psi_{s0}(\theta),\ldots,\psi_{sN},\, \psi_{\theta0}(\theta),\ldots,\psi_{\theta N}(\theta)\}$ is unknown vector-function and components of matrix Φ, which satisfy various boundary conditions on ends, are linear combinations of cubic B-splines on a uniform mesh. Substituting (7) into (6) and boundary or symmetry conditions, we require that they would be held at the $N+1$ points of collocation s_i along the generatrix. As a result, we obtain one-dimensional boundary-value problem

$$(8) \quad \frac{d\overline{z}}{d\theta} = A\overline{z} + \overline{f}, \quad B_1\overline{z} = \overline{b}_1 \ (\theta = \theta_1), \ B_2\overline{z} = \overline{b}_2 \ (\theta = \theta_2),$$

where $\overline{z} = \{\overline{y}_*, \overline{y}'_*\}$ is the vector-function of θ; $\overline{f}$ is the vector of right-hand sides; A is the square matrix whose elements depend on θ; B_1 and B_2 are the matrices of boundary conditions, $\overline{b}_1$ and $\overline{b}_2$ are the corresponding vectors. The one-dimensional boundary-value problem (8) can be solved by the discrete-orthogonalization method [1]. Substituting $\overline{y}_*$ into (7), we obtain the solution of the two-dimensional boundary-value problem.

On the basis of the approach proposed, we have solved the set of problems related to the stress-strain state of orthotropic cylindrical shells with an elliptical and corrugated cross-section. Analysis of displacement and stress fields under different boundary condition is carried out.

[1] R. Bellman and R. Kalaba (1965). *Quasilinearization and nonlinear boundary-value problems*, Elsevier, 218 p.

[2] Ya.M. Grigorenko and S.N. Yaremchenko (2004). Stress Analysis of Orthotropic Noncircular Cylindrical Shells of Variable Thickness in a Refined Formulation , *Int. Appl. Mech.*, **40**, 266-274.