

## CONTROLLABILITY FOR THIN LINEARLY ELASTIC SHELLS

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Thanks to the pioneering works of J.L. Lions [1] there exists a general tool for the study of the exact controllability of a distributed system, in particular of various shell models. More precisely let us suppose that it is possible to act on (at least) a part of the boundary of a thin, linearly elastic and isotropic shell with suitable boundary conditions. Then null (or exact) controllability consists in proving that starting from an arbitrary initial state it is possible to steer the shell to rest, by a proper choice of the boundary control, in a finite time. The Hilbert Uniqueness Method (HUM) introduced by J. L. Lions allows to find a control as a minimum of a functional (continuous and convex in a suitable framework) whose coercivity is essentially reduced to the proof of a uniqueness result.

The summation convention is adopted, the greek indices take values in the set 1, 2 and the latin indices in the set 1, 2, 3.

Let  $\omega$  be a bounded connected domain in  $\mathbb{R}^2$  with boundary  $\gamma$  and let  $y = (y_\alpha)$  denote a generic point of  $\bar{\omega}$ . Let  $\theta \in C^\infty(\bar{\omega}; \mathbb{R}^3)$  be an injective mapping such that the vectors  $\mathbf{a}_\alpha(y) := \partial_\alpha \theta(y)$  form the covariant basis of the tangent plane to the surface  $S := \theta(\bar{\omega})$  at the point  $\theta(y)$ ; let  $\mathbf{a}_3(y) = \mathbf{a}^3(y) := \frac{\mathbf{a}_1(y) \wedge \mathbf{a}_2(y)}{|\mathbf{a}_1(y) \wedge \mathbf{a}_2(y)|}$  be the unit normal vector to  $S$ . For any displacement field  $v_i \mathbf{a}^i$  expressed in terms of the contravariant basis the deformed surface is  $\tilde{\theta}(\mathbf{v})(\bar{\omega})$  where  $\tilde{\theta} : \mathbf{v} = (v_i) \rightarrow \theta + v_i \mathbf{a}^i$ . In the framework of linearized theory small displacement  $\mathbf{v}$  are considered. In the Koiter model the membrane and flexural deformation energy are defined by the symmetric forms

$$(1) \quad a_M(\mathbf{u}, \mathbf{v}) = \int_\omega a^{\alpha\beta\lambda\mu} \gamma_{\alpha\beta}(\mathbf{u}) \gamma_{\lambda\mu}(\mathbf{v}) \sqrt{a} dy$$

$$(2) \quad a_F(\mathbf{u}, \mathbf{v}) = \int_\omega a^{\alpha\beta\lambda\mu} \rho_{\alpha\beta}(\mathbf{u}) \rho_{\lambda\mu}(\mathbf{v}) \sqrt{a} dy$$

where  $\gamma_{\alpha\beta}(\mathbf{u})$  and  $\rho_{\alpha\beta}(\mathbf{u})$  are the linearized change of metric and of curvature tensors associated to  $\mathbf{u}$ . Let  $\epsilon \geq 0$  a real parameter and let be  $\mathbf{A}_\epsilon = \mathbf{A}_M + \frac{\epsilon^2}{3} \mathbf{A}_F$  the operator in  $\mathbf{H} = L^2(\omega)^3$  associated to the bilinear form

$$(3) \quad a_\epsilon(\mathbf{u}, \mathbf{v}) = a_M(\mathbf{u}, \mathbf{v}) + \frac{\epsilon^2}{3} a_F(\mathbf{u}, \mathbf{v})$$

defined on a suitable subspace  $\mathbf{V}$  of kinematically admissible displacements (as general references on shell theory see e.g. [2], [3], [4]). Let us consider the evolution problem :

$$(4) \quad \begin{cases} \frac{\partial \mathbf{u}}{\partial t^2}(y, t) + \mathbf{A}_\epsilon \mathbf{u}(y, t) = 0, & \text{for } y \in \omega \text{ and } t > 0 \\ \mathbf{u}(y, 0) = \mathbf{u}^0(y), \frac{\partial \mathbf{u}}{\partial t}(y, 0) = \mathbf{u}^1(y) & \text{for } y \in \omega \\ \mathbf{B}\mathbf{u}(y, t) = \mathbf{v}(y, t) & \text{for } y \in \gamma \text{ and } t \geq 0 \end{cases}$$

where  $\mathbf{B}$  is a suitable system of boundary conditions. The system is *exactly controllable in time  $T$*  if given an initial data  $(\mathbf{u}^0, \mathbf{u}^1)$  it is possible to find a control  $\mathbf{v}$  that can drive the system (4) to rest at time  $T$  i.e.

$$\mathbf{u}(y, T) = \mathbf{0}, \frac{\partial \mathbf{u}}{\partial t}(y, T) = \mathbf{0} \quad \text{for } y \in \omega$$

In [5] it is for instance proved the exact controllability when the middle surface of the shell satisfies a suitable condition (e.g. is not too far from a plane). It is natural to study the dependence of the controllability time  $T$  on  $\epsilon$ . Let us remark that when  $\epsilon \rightarrow 0$  one has a singular perturbation problem ; hence one can expect that the controllability time  $T(\epsilon) \rightarrow \infty$ . This can be proved in similar situations (see e.g. [7]). Moreover one can prove in the case  $\epsilon = 0$  a non-controllability result [6] : there exist some in initial data  $(\mathbf{u}^0, \mathbf{u}^1)$  such that the evolution system (4) is not exactly controllable. In this paper we address the following problem :

*In the case  $\epsilon = 0$  find a space of controllable initial data  $(\mathbf{u}^0, \mathbf{u}^1)$ .*

We will give some preliminary partial results obtained in a joint work with Farid Ammar-Khodja and Arnaud Münch, [8]. The characterization of the controllable initial data depends on the study of the spectral problem associated to the operator  $\mathbf{A}_M$ . This will be illustrated on the example of hemispherical shells and on the example of an arch.

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