AN ASYMPTOTIC APPROACH TO PROBLEMS OF SCATTERING ACOUSTIC WAVES BY ELASTIC SHELLS

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1. Scattering of stationary acoustic waves by elastic shells is considered. A procedure is proposed for constructing an approximate solution, based on matching the expansions for different asymptotic models of the interaction of the shell with the acoustic medium. In the vicinities of zero frequency the refined Kirchhoff-Love theory of fluid-structure interaction is applied [1]. This model takes into consideration transverse compression of a shell by a fluid and some other phenomena. In the vicinities thickness resonance frequency long-wave high-frequency approximations are employed [2,3]. They describe small number resonance of higher order Lamb waves. Outside the vicinities of zero frequency and thickness resonance frequency vibrations of a shell correspond to short-wave motions. Here a flat layer model is used [2,3]. It is shown for different parameters of material in a shell that the flat layer model has overlap regions both the refined Kirchhoff-Love theory and the theories associated with long-wave high-frequency approximations. A comparison of numerical data corresponding to asymptotic and exact solutions cylindrical and spherical shells shows that the proposed procedure is highly efficient.

2. Let the plane acoustic wave $p_i = p_0 \exp[-i(k\xi + \omega t)]$ be scattered either by a circular cylindrical shell or by a spherical shell. We introduce the following parameters characterizing the scattering process:

$$\kappa = \rho/\rho_1, \quad \beta_i = c_i/c \;\; (i = 1,2), \quad \gamma_0 = c_2/c_1, \quad k = \omega/c.$$

Here c_1 and c_2 are the dilatation and distortion wave speeds in the material of the shell, respectively, ρ_1 is the mass density of the shell, c is the sound speed in the fluid, ρ is the mass density of the fluid, ω is the circular frequency, p_i is the pressure in the incident wave, p_0 is a constant. The incident pressure p_i and the scattered pressure p_s have to satisfy the Helmholz equation. In addition, the scattered pressure p_s should obey the radiation condition at infinity.

Let (r,θ) be cylindrical or spherical coordinates (the problem do not depend upon the axial coordinate in the case of a cylindrical shell and upon the angle along parallel in the case of a spherical shell), the radius of the shell be equal R, and the half-thickness of the shell be equal h. The incident pressure can be written as $p_i = p_0 \sum_{n=0}^{\infty} E_n (-i)^n f_n(kr) F_n(\theta)$.

Here for a cylindrical shell $E_0 = 1$, $E_n = 2$ $(n \ge 1)$, $f_n = J_n$, $g_n^{(1)} = H_n^{(1)}$, $F_n(\theta) = \cos n\theta$, $N = n$, J_n is a cylindrical Bessel function of the first kind, $H_n^{(1)}$ is a Hankel function of the first kind; for a spherical shell sphere $E_n = 2n+1$, $f_n = j_n$, $g_n^{(1)} = h_n^{(1)}$, $F_n(\theta) = F_n(\cos\theta)$, $N = n + 1/2$, j_n is a spherical Bessel function of the first kind, $h_n^{(1)}$ is a spherical Hankel function of the first kind, P_n is a Legendre polynomial. The solution for the scattered pressure in the case of normal incidence of the plane acoustic wave has the form

$$p_s = p_0 \sum_{n=0}^{\infty} E_n (-i)^n B_n g_n^{(1)}(kr) F_n(\theta) \qquad (1)$$

The coefficients B_n have to be defined by solving the contact problems for the equations describing the motion of the shell. We introduce the relative half-thickness of the shell $\eta = h/R$. Let us consider three approximate models mentioned above.

The regions in which the refined asymptotic model and the model based on classical Kirchhoff–Love theory can be used are limited by the inequalities $\omega R/c_2 << \eta^{-1}$, $\omega R/c_2 << \eta^{-1/2}$, respectively [1]. Thus, both of these theories describe only the order Lamb-type waves S_0 and A_0 or the fluid-born wave A. The relevant mode numbers lie in the ranges $n << \eta^{-1}$ and $n << \eta^{-1/2}$ for the refined asymptotic model and the Kirchhoff–Love theory, respectively.

The first modes of higher order Lamb-type waves correspond to long-wave high-frequency vibrations of fluid-loaded shells. There are two types of the long-wave high-frequency approximations [2,3]. The transverse approximation is to use in the vicinities of the thickness stretch resonance frequencies. In the vicinities of the thickness shear resonance frequencies the tangential approximation should be used. The model formulated above is applicable only for the small values of the parameter n ($n << \eta^{-1}$). But series (1) only begin to converge when $n \sim x \sim \eta^{-1}$, where $x = ka$, i.e. solution contains short-wave components as well. Consequently, when calculating the scattered pressure using formula (1) the long-wave high-frequency approximations must be used together with the flat elastic layer model that will be considered below.

The flat elastic layer model is developed in references [2,3]. The equations for this model are valid under following conditions: $\partial/\partial\zeta \sim \partial/\partial\theta \sim \omega R/c_2 \sim \eta^{-1}$, i.e. for short-wave motions of the shell. In this case the equations of elasticity written in curvilinear coordinates can be replaced by those in plane problem of elasticity presented in Cartesian coordinates, in doing so the radial coordinate is “frozen” on the mid-surface of a shell.

The results of the synthesis of the form function in the far field ($r \to \infty$) in the case of backscattering ($\theta = 0$) are presented in [2,3]. Here

(2) $$p = G\left|\sum_{n=0}^{\infty} E_n B_n (-1)^n\right|$$

The long-wave high-frequency approximation is applied beginning with the first thickness resonance frequency and only for $n < 10$. The rest of series (2) is evaluated by the flat layer model. A numerical analysis demonstrates advantages of the chosen scheme. The calculations for different parameters of material in a shell and value parameter η ($1/69 \le \eta \le 1/17$) show that there exist overlap regions, therefore, the proposed method give a possibility to describe both the resonance components of the partial modes and the scattered pressure with high accuracy.

3. References

[1] V.A. Kovalev (2002). Application a refined asymptotic model in scattering of a plane acoustic waves by a spherical shell, *Izv. AN MTT* (*Mechanics of solids*). **37** (2), 155–162.

[2] J.D. Kaplunov, V.A. Kovalev, M.V. Wilde (2003). Matching of asymptotic models in scattering of a plane acoustic wave by an elastic cylindrical shell, *J. of Sound and Vibration*. **264** (3), 639–655.

[3] V.A. Kovalev (2002). Synthesis of acoustic pressure scattered by an elastic cylindrical shell basis on acoustic waves by elastic spherical shell, *J. of Appl. Maths. and Mechs.* **66** (4), 581–590.