EVALUATION OF THE PERTURBATION SENSITIVITY AND THE LIMIT LOADS OF SHELLS BY THE PERTURBATION ENERGY CONCEPT

J. Pontow and D. Dinkler

Institute for Structural Analysis, Braunschweig, Germany

1. General

The perturbation sensitivity and the limit loads of shells are widely discussed phenomena. Both phenomena may be classified with respect to the time und the type of a perturbation. Contrary to other methods, the perturbation energy concept enables to describe the buckling process in a very natural way and to analyse the perturbation sensitivity as well as the limits loads by a uniform approach.

2. Perturbation energy concept

Basic idea of the perturbation energy concept is the identification of a critical state belonging to a fundamental state F [1]. The difference of strain energy between both states is an indicator for the stability of the fundamental state and referred to as the perturbation energy Π_{cr}. As several critical states may exist, the identification of the critical state which is relevant for the stability of the fundamental state is interpreted as an optimisation problem,

$$(1) \quad f(\mathbf{z}_F, \Delta \mathbf{z}) = \Pi_{cr} \rightarrow Min.$$

In this problem, the fundamental state is represented by the state variables $\mathbf{z}_F$ and the distance as well as the direction between the fundamental and the critical state are denoted by the change $\Delta \mathbf{z}$ of the state variables. The kind of the critical state depends on the distribution in time of the perturbation. For a kinetic perturbation, the state N characterised by vanishing first variation of the incremental elastic potential and by no change of the fundamental load is the critical state, compare figure 1. In case of a static perturbation, the state M of vanishing second variation of the potential is the critical state. These conditions constrain the optimisation problem (1) whose solution may be found by nonlinear eigenvalue problems. Thereby, the order of nonlinearity with respect to the eigenvalue and the eigenvector, respectively, is governed by the formulation of the potential.
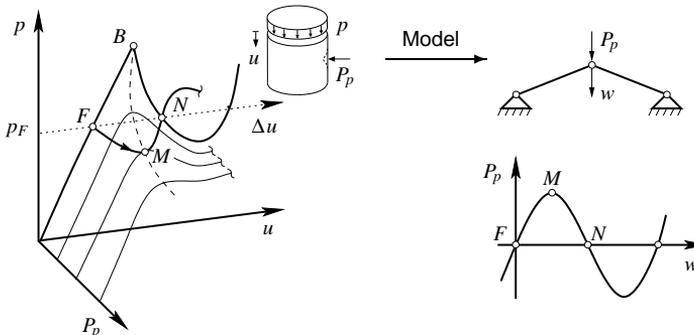

Figure 1. Load-deformation behaviour of a perturbation-sensitive shell

In general, non-initial load perturbations P_p are necessary to reach a critical state. Non-initial perturbations concerning other parameters of the model equations, such as the bedding modulus and the wall thickness, influence the topology of the energy surface and emphasise the similarity between the perturbation energy concept and the perturbation theory. The effect of initial perturbations is

measured by the associated change of the fundamental energy Π_F as well as the change of the perturbation energy. For identifying the static limit loads of different shells by one energy value, the perturbation energy is normalised by the bending stiffness of the shell continuum as bending energy is the dominating part of the perturbation energy. The reference value of the normalised perturbation energy, $\pi_{cr,M} = 2.7\,\%$, represents a proper indicator for realistic static limit loads. Kinetic limit loads are determined with respect to a fundamental state and by the degree of stability [2]. Furthermore, the optimisation of the perturbation sensitivity of a shell in terms of a certain load level and the design parameter x may be described by the objective function

$$(2) \quad f(x, \mathbf{z}_F(x), \Delta \mathbf{z}(x)) = \Pi_{cr} - \Pi_{cr}^{req} = 0$$

where Π_{cr}^{req} represents the required perturbation sensitivity. The computation of the solution of this load-level-specific optimisation is based on the linearisation of the objective function which is performed by the forward difference scheme. In general, the solution is found after few iterations. During the optimisation, the energy surface and so the fundamental state as well as the distance and the direction between the fundamental and the critical state are changing. Furthermore, the optimisation influences the fundamental energy and the load level of singular points in the primary load-deformation path. The advantage of the proposed optimisation procedure over a systematic change of the design parameter becomes obviously especially for high-dimensional systems and several design parameters.

3. Numerical results

For cylindrical and spherical shells, the perturbation sensitivity and static limit loads are analysed including different loadings, geometries, boundary conditions and material parameters. The results indicate that spherical shells under radial pressure are nearly as perturbation-sensitive as cylindrical shells subjected to axial pressure. For these buckling cases, limit loads according to the reference value of the normalised perturbation energy are in good agreement with those corresponding to the ECCS-Recommendations, but differ for elasto-plastic material behaviour significantly to those according to DIN 18800 [3]. The limit loads calculated for conical shells under meridional pressure are for different meridional angles similar to those corresponding to DASt-Richtlinie 013. In addition, buckling cases less intensively discussed in the design rules are analysed. But it is not possible to specify realistic limit loads of all these buckling cases by the reference value of the normalised perturbation energy due to the absence of a problem-specific scaling of the perturbation energy. Nevertheless, for fibre reinforced composites consisting of uniform UD-layers an adequate scaling is feasible.

The stability of a shell against a kinetic perturbation load depends on the energy induced into the system by the perturbation load. Therefore, the influence of the distribution in time and space of the perturbation load on the stability of a spherical shell is investigated. The results highlight the importance of the perturbation energy concept not only for the evaluation of the buckling resistance but also for the determination of an unfavourable perturbation load and the kinetic limit load, repectively.

4. References

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