ON PHASE TRANSITIONS
IN THERMOELASTIC AND THERMOVISCOELASTIC SHELLS

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Phase transitions (PT) play an important role in different problems of continuum mechanics. Equilibrium conditions of elastic thin-walled structures (plates and shells) undergoing PT of martensitic type were formulated in [1, 2] within the dynamically and kinematically exact theory of shells presented in books [3, 4]. From experimental data we know that PT depending on strain rates and inelastic effects may considerably influence the stress state of the solid.

The aim of this contribution is to extend the results of [1, 2] by taking into account thermal and viscoelastic effects of the shell material phases under quasistatic loading.

1. Basic relations of nonlinear shell thermostatics

The 2D local *equilibrium equations* as well as the *energy balance* and the *Clausius-Duhem inequality* satisfied at any part of the undeformed base surface M of the shell are [1, 2]

$$Div_s \mathbf{N} + \mathbf{f} = \mathbf{0}, \quad Div_s \mathbf{M} + ax\left(\mathbf{N}\mathbf{F}^T - \mathbf{F}\mathbf{N}^T\right) + \mathbf{c} = \mathbf{0},$$

(1)
$$\rho \frac{d\varepsilon}{dt} = \rho\left(q^+ + q^- + q_\Pi\right) - Div_s \mathbf{q} + \mathbf{N} \bullet \mathbf{E}^\circ + \mathbf{M} \bullet \mathbf{K}^\circ,$$

$$\rho \frac{d}{dt}\eta \geq \rho\left(\frac{q^+}{T_{ext}^+} + \frac{q^-}{T_{ext}^-} + \frac{q_\Pi}{T}\right) - Div_s\left(\frac{1}{T}\mathbf{q}\right).$$

Here $\mathbf{N}$ and $\mathbf{M}$ are the internal surface stress resultant and couple stress tensors of the nominal type, $\mathbf{F} = Grad_s \mathbf{y}$ is the surface gradient of shell deformation $\mathbf{y} = \boldsymbol{\chi}(\mathbf{x}) = \mathbf{x} + \mathbf{u}$, $ax(\mathbf{A})$ denotes the axial vector associated with the skew tensor $\mathbf{A}$, while $Grad_S$ and Div_S are the surface gradient and divergence operators on M, respectively. Additionally, ε is the surface internal energy density, η is the surface entropy density, $\mathbf{q}$ is the surface influx vector, $q^\pm$ are the heat influxes through the upper (+) and lower (-) shell faces, q_Π is the internal surface heat supply, while by T_{ext}^+ and T_{ext}^- we denote temperatures of the external media surrounding the shell from above and below.

2. Constitutive equations of thermoviscoelastic shells

The surface stress measures $\mathbf{N}, \mathbf{M}$ depend only on prehistories of the surface natural strain and bending tensors $\mathbf{E}, \mathbf{K}$, [4], as well as on the temperature T. We split the surface stress measures and their constitutive equations into elastic (equilibrium) and inelastic (dissipative) parts, $\mathbf{N} = \mathbf{N}_E + \mathbf{N}_D$ and $\mathbf{M} = \mathbf{M}_E + \mathbf{M}_D$, where

(2)
$$\mathbf{N}_E = \rho\frac{\partial\psi}{\partial\mathbf{E}}, \quad \mathbf{M}_E = \rho\frac{\partial\psi}{\partial\mathbf{K}}, \quad \mathbf{N}_D = \mathbf{N}_D\left(T, \mathbf{E}, \mathbf{K}, \dot{\mathbf{E}}, \dot{\mathbf{K}}\right), \quad \mathbf{M}_D = \mathbf{M}_D\left(T, \mathbf{E}, \mathbf{K}, \dot{\mathbf{E}}, \dot{\mathbf{K}}\right).$$

Here $\psi = \varepsilon - T\eta = \psi\left(T, \mathbf{E}, \mathbf{K}\right)$ is the surface free energy density, and $\mathbf{N}_D = \mathbf{N}_D\left(T, \mathbf{E}, \mathbf{K}, \mathbf{0}, \mathbf{0}\right) = \mathbf{0}$, $\mathbf{M}_D = \mathbf{M}_D\left(T, \mathbf{E}, \mathbf{K}, \mathbf{0}, \mathbf{0}\right) = \mathbf{0}$.

3. Continuity conditions and kinetic equation

The phase interface in the shell may be modelled by a smooth surface curve $C \subset M$. Then along C the 1D local *equilibrium and thermodynamic continuity conditions* are [5]

(3)
$$\left[\mathbf{N}\mathbf{v}\right] = \mathbf{0}, \quad \left[\mathbf{M}\mathbf{v}\right] = \mathbf{0},$$
$$V\left[\rho\varepsilon\right] + \left[\mathbf{N}\mathbf{v}\cdot\mathbf{v}\right] + \left[\mathbf{M}\mathbf{v}\cdot\mathbf{w}\right] - \left[\mathbf{q}\cdot\mathbf{v}\right] = 0, \quad V\left[\rho\eta\right] - \left[\frac{1}{T}\mathbf{q}\cdot\mathbf{v}\right] \equiv -\delta^2 \leq 0.$$

Here the expression $[...] = (...)^{+} - (...)^{-}$ means the jump at C, $\mathbf{v} = \dot{\mathbf{u}}$ is the translational velocity vector, $V = \dot{\mathbf{x}}_C \cdot \mathbf{v}$ is the normal velocity tangent to $T_x M$ of the phase curve C with the unit outward normal $\mathbf{v}$ and $\dot{\mathbf{x}}_C$ the velocity of C kinematically independent on $\mathbf{v}$, $\mathbf{w} = ax\left(\dot{\mathbf{Q}}\mathbf{Q}^T\right)$ is the angular velocity vector, and δ^2 represents creation of entropy at the interface C.

Transforming (3) it can be shown that the surface Eshelby tensor $\mathbf{C}$ introduced in [1] satisfies the *thermodynamic continuity condition* $\left[\mathbf{v}\cdot\mathbf{C}\mathbf{v}\right] = 0$ in the case of equilibrium. For the *coherent* interface, when the independent translation $\mathbf{u}$ and rotation $\mathbf{Q}$ fields are supposed to be continuous at C, we obtain $\mathbf{C} \equiv \mathbf{C}_c = \rho\psi\mathbf{A} - \mathbf{N}^T\mathbf{F} - \mathbf{M}^T\mathbf{K}$. For the interface *incoherent in rotations*, when only $\mathbf{u}$ is continuous at C and continuity of $\mathbf{Q}$ may be violated, we have $\mathbf{C} \equiv \mathbf{C}_i = \rho\psi\mathbf{A} - \mathbf{N}^T\mathbf{F}$. From the thermodynamic point of view the expression $d\left[\mathbf{v}\cdot\mathbf{C}\mathbf{v}\right]/ds$ is the configurational force acting on the phase interface, with s the length along C.

Thermodynamically consistent *kinetic equation* along C is given by the relation

(4)
$$V = K\left(d\left[\mathbf{v}\cdot\mathbf{C}\mathbf{v}\right]/ds\right),$$

where K is a non-negative definite kinetic function. The equation (4) describes motion of the phase interface C under quasistatic deformations of the shell. It generalizes the balance equations on the equilibrium phase interface obtained in [1, 2].

During Solmech2008 we present solutions of some model problems for two-phase viscoelastic shells under quasistatic loading.

4. References

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