ON COMPARISON OF THEORY OF MICROSTRAINS WITH THEORIES, BASED ON THE CONCEPTION OF SLIDING

Yu. Chernyakov , A. Polishchuk
Dnepropetrovsk National University, Dnepropetrovsk, Ukraine

Over the last decades a number of theories capable of describing mechanical response of different materials under a complex loading were created. These theories are usually formulated at the micro structural level and the most widely-used of them are the theories of plasticity for mono- and poly-crystalline materials which are based on the concept of sliding. The main assumption of such theories is that the plastic deformation of the representative volume is some aggregate of deformations of sliding of single-crystals on the planes of different orientation. Such direction in the theory of plasticity was developed in the works of Batdorf, Budiansky, Bazhant, Hutchinson, Hill, Leonov, Malmeister, etc. In contrast to sliding theories, in the theory of microstrains (Chernyakov, Kadashevich, Novozhilov) a representative volume is considered as an aggregate of the variously oriented grains. It is assumed that every micro-particle (grain) is characterized by an orientation tensor $\boldsymbol{\mu}$, determining the direction of micro-plastic deformation, and by the local yield limit τ. In general case the theory of microstrains assumes, that $\boldsymbol{\mu}$ is the arbitrary normed deviator i.e.

$$\boldsymbol{\mu} : \boldsymbol{\mu} = 1,\ \mathbf{tr}\,\boldsymbol{\mu} = 0, \tag{1}$$

where ":" denotes the contraction of tensors by two indexes and "**tr** " means the trace of a tensor. In the papers [1-4] it was proved that other types of tensor $\boldsymbol{\mu}$ allow to obtain a number of micro structural theories of plasticity on the base of the theory of microstrains. In particular, choice of $\boldsymbol{\mu}$ like following

$$\boldsymbol{\mu} = \frac{1}{\sqrt{2}}\left(\vec{n}\otimes\vec{m} + \vec{m}\otimes\vec{n}\right),\ |\vec{n}| = 1,\ |\vec{m}| = 1,\ \vec{n}\cdot\vec{m} = 0 \tag{2}$$

allows to derive a number of sliding theories.

Results, which concern possibility of obtaining different microstructural theories, had analytical character, and only a possibility of a high-quality approximation was shown, however a quantitative side of the question was not examined due to lack of computational power.

Nowadays with intensive development of the computational engineering, a possibility of numerical implementation of integration scheme of the constitutive relations of the microstrains theory appeared. This allows us to conduct quantitative comparison of different mechanical theories which use microstuctural approach.

We offer the new method of representation of the directional tensor $\boldsymbol{\mu}$ in the following form:

$$\boldsymbol{\mu} = \rho(\xi)\left[\left(-\frac{\cos\xi}{\sqrt{6}} - \frac{\sin\xi}{\sqrt{2}}\right)\vec{j}_1\vec{j}_1 + \left(-\frac{\cos\xi}{\sqrt{6}} + \frac{\sin\xi}{\sqrt{2}}\right)\vec{j}_2\vec{j}_2 + \left(\frac{\cos\xi}{\sqrt{6}}\right)\vec{j}_3\vec{j}_3\right], \tag{3}$$

where $\vec{j}_k$, $k = 1..3$ are three principal directions of the tensor, which are defined as arbitrary oriented orthogonal unit vectors and can be expressed through two spherical angles ϕ, ψ and one auxiliary angle θ and ξ is a type-angle parameter (measure of a third invariant) for the tensor $\boldsymbol{\mu}$.

In the case $\rho(\xi) = 1$ the representation (3) fulfils conditions (1) and leads to the known variant of the theory of microstrains [2]. Choosing $\rho(\xi) \neq 1$ allows us to obtain the variants of theory which are capable of describing materials whose mechanical response is different under the compression, tension and shear. In this case holds $\boldsymbol{\mu} : \boldsymbol{\mu} = \rho(\xi)$, i.e. the norm of $\boldsymbol{\mu}$ is not identity like in

classical variant of theory. This fact required a proper generalisation of constitutive relations which was successfully made. Also, choosing $\xi = const$, in particular $\xi = \pi/6$, makes (3) equivalent to (1). So the representation (3) of the orientation tensor $\boldsymbol{\mu}$ is rather general and can be used to obtain different cases.

A unified numerical integration algorithm for the constitutive relations of the theory of microstrains based on the new representation of the directional tensor (3) was created. This approach allows to unify numerical implementation for different theories based on the framework of the microstrains theory. It is now possible to investigate a number of different cases of microstrains theory and their connection with other microstructural theories. However an emphasise was made on comparison of theory of microstrains to the theories, based on conception of sliding. And it is shown that distinction of deformations, expected in the theory of microstrains and sliding theories in a material subjected to complex loading has an order of distinction of initial yield conditions, i.e. conditions of Huber-Mises and Treska

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