THREE-DIMENSIONAL STRESS-STRAIN STATE OF ROLLER-SHAFT SYSTEM IN CONDITIONS OF CONTACT INTERACTION AND NON-CONTACT BENDING OF SHAFT

S. Sherbakov
Department of Mechanics and Mathematics, Belarusian State University, Minsk, Belarus

1. Introduction

In the mechanics of deformable rigid body a wide class of problems is devoted to the research of contact interaction of rigid bodies [1]. One of the most essential conditions accepted for the solution of contact problems is fast attenuation of stresses and strains if the distance between the considered point and contact area increases.

In addition to the solutions for contact problems mechanics of a deformable rigid body have well developed methods of studying of stress-strain state without considering local effects in the areas of load application (see for example [2]).

Mechanical systems known as active systems [3] operate in conditions of contact interaction and loaded by non-contact forces. Mechanical and mathematical model of three-dimensional stress state of typical roller-shaft active system is considered in the present work. Calculation results show significant difference between such stress state and traditional contact and non-contact stress states.

2. Stress state

Roller-shaft active system is loaded by contact F_N and non-contact Q forces (figure 1). Thus stresses caused by the action of distributed normal $p(x, y)$, tangential $q(x, y)$ contact tractions and non-contact load (figure 1) should be taken into account:

(1) $$\sigma_{ij} = \sigma_{ij}^{(n)} + \sigma_{ij}^{(\tau)} + \sigma_{ij}^{(b)}, \quad i, j = x, y, z,,$$

where $\sigma_{ij}^{(n)}$ – stresses caused by normal contact traction, $\sigma_{ij}^{(\tau)}$ – stresses caused by tangential contact traction (force of friction), $\sigma_{ij}^{(b)}$ – stresses caused by non-contact loads.

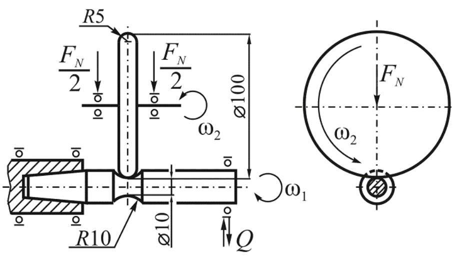

Figure 1. Loading scheme of roller-shaft system

Calculation of stresses $\sigma_{ij}^{(n)}$ in any point of the half-space under the surface when $z < 0$ caused by the action of normal contact tractions $p(x, y)$ is carried out numerically using Boussinesq problem solution $\sigma_{ij}^{(B)}$ [1] (determination of stress components in the half-space caused by unit normal force) [4,5]:

(2) $$\sigma_{ij}^{(n)}(x, y, z) = \iint\limits_{S(\xi,\eta)} p(\xi, \eta)\sigma_{ij}^{(B)}(\xi - x, \eta - y, z)d\xi d\eta,$$

Calculation of stress state $\sigma_{ij}^{(\tau)}$ under the action of friction force modeled by the distribution of tangential tractions $q(x, y)$ is also carried out numerically using Cerruti problem solution $\sigma_{ij}^{(C)}$ [1] (determination of stress components in the half-space caused by unit tangential force) [4,5]:

(3) $$\sigma_{ij}^{(\tau)}(x,y,z)=\iint\limits_{S(\xi,\eta)} q(\xi,\eta)\sigma_{ij}^{(C)}(\xi-x,\eta-y,z)\,d\xi d\eta\,,$$

Stresses caused by non-contact loads are defined applying particular approaches (for example bending theory):

(4) $$\sigma_{ij}^{(b)}=\sigma_{ij}^{(M)}+\sigma_{ij}^{(N)}+\sigma_{ij}^{(Q)}\,,$$

where indexes *M*, *N* and *Q* correspond to internal moment, normal and shear forces.

Since model (1) is constructed as the superposition of components $(\sigma_{ij}^{(n)},\sigma_{ij}^{(\tau)},\sigma_{ij}^{(b)})$ of stress there is a possibility of analysis of both general solution and any of special cases.

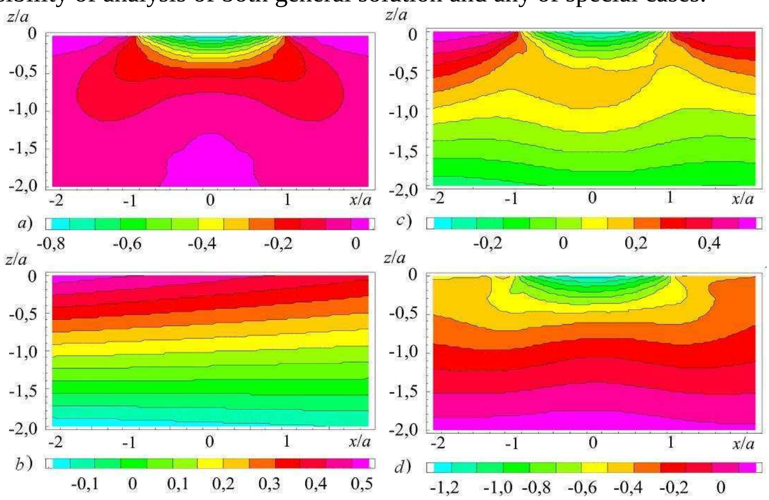

Figure 2. Distributions of stresses $\sigma_{xx}^{(n)}$ (*a*), $\sigma_{xx}^{(b)}$ (*b*), $\sigma_{xx}^{(n)}+\sigma_{xx}^{(b)}$ $(Q>0)$ (*c*), $\sigma_{xx}^{(n)}-\sigma_{xx}^{(b)}$ $(Q<0)$ (*d*), normalized by maximum Hertz stress p_0 in the neighborhood of contact area (y = 0, a / b = 0.5), (a and b are the greater and smaller semi-axes of contact ellipse)

It is easy to see from the distributions presented in figure 2 that the stress state in the active system (figures 2.*c* and 2.*d*) strongly differs (qualitatively and quantitatively) from traditionally studied stress states under contact or bending (figures 2.*a* and 2.*b* respectively). Using (1) it is possible on the one hand to investigate how the field of the stresses (strains) caused by volume deformation is disturbed in some local area where the field of contact stresses (strains) simultaneously occurs. On the other hand it is possible to investigate how the field of local stresses (strains) changes when the field of the stresses (strains) caused by volume deformation is imposed on it.

3. References

[1]. K.L. Johnson. (1985). Contact Mechanics, Cambridge University Press, Cambridge.

[2]. S.P. Timoshenko, J.N. Goodier (1970). Theory of Elasticity. McGraw-Hill, NY.

[3]. L.A. Sosnovskiy Tribo-Fatigue. Wear-fatigue damage and its prediction (Foundations of engineering mechanics) (2004). Springer.

[4]. L.A. Sosnovskiy, S.S. Shcharbakou (2005). New Class of Contact Problems. *Proceedings of the International Mechanical Engineering Congress and Exposition*, Orlando, 6 pp.

[5]. L.A. Sosnovskiy, S.S. Sherbakov (2006). Special Class of Contact Problems and the Calculation of the Sstate of Stress of Wheel/Rail System Elements. *Proceedings of the 7th International Conference on Contact Mechanics and Wear of Rail/Wheel Systems*, Brisbane, 93–104.