ON THE USE OF GURSON'S MODEL IN CONTINUUM DAMAGE MECHANICS

R. Souchet
Association Française de Mécanique

1. Introduction and statement of the problem

Originally, Gurson obtained a yield surface for porous plastic materials under some particular conditions. Here we give a generalised form of the Gurson yield surface (A), viz

(1) $$f(\sigma,x,\varepsilon_M)=(\sigma_{eq}/\sigma_M)^2+2qx\cosh(\tfrac{1}{2}tr\sigma/\sigma_M)-(1+q^2x^2)=0\,,$$

where σ denotes the Cauchy stress tensor, σ_M the flow yield strength, q=3/2 and *x* is a function of the void volume fraction *v*. An essential fact is the presence of the hydrostatic stress [1].

Close relations exist between stress levels in the matrix material, viewed as the effective material, and the porous material, viewed as the damaged material. By example, in CDM, the respective stress tensors (σ_r,σ) are connected by some relation [2]

(2) $$\sigma_r=\sigma/y(x)\,,\quad 0\le y(x)\le 1\,,\quad y(0)=1,\quad y(1/q)=0,$$

where *y(x)* is an unknown decreasing scalar operator. But the introduction of the matrix material by means of (2) leads to the explicit use of the von Mises yield surface $f_r=0$ (B) on the matrix material.

2. The yield surfaces f_r and f

As Gurson, we suppose that the matrix material is rigid-plastic. The void-function *x(v)*is taken as a damage variable and the equivalent plastic strain ε_M as an internal variable. The yield surface (B) is given as a function of the state (σ_r,ε_M) of the matrix material. But this state (σ_r,ε_M) is connected to the state (σ,x,ε_M) of the damaged material by the formula (2); so it is equivalent to express this yield surface in function of the parameters (σ,x,ε_M), obtaining

(3) $$f_r(\sigma,x,\varepsilon_M)=(\sigma_{eq}/\sigma_M)^2-(y(x))^2=0\,.$$

Note that (3) is not a yield surface for the damaged material (except in particular cases).

Now if a mechanical process occurs in the damaged body, then from (1) we have $f\le 0$. But the matrix material undergoes some accompanying process and from (3) we have $f_r\le 0$. So the region $f\le 0$ must be restricted by the region $f_r\le 0$. This is not surprising since the Gurson surface is a necessary condition only, satisfied by the damaged material under the hypothesis $f_r=0$. If we suppose that reversible processes are possible (leaving the rigidity hypothesis), then the domains $f_r\le 0$ and $f\le 0$ generally intersect [2]. The particular case of the strict inclusion "$f\le 0$ implies $f_r\le 0$" is possible, but not the reverse one. This last result is due to the fact that it is not possible to give an a priori evolution of the matrix material since the presence of micro-voids restricts the deformations of the matrix material. Naturally the domains $f\le 0$ and $f_r\le 0$ may coincide.

Due to rigidity hypothesis, it is easy to show that a damage-plasticity effect arises only when the condition $y(x)<y_0(x)=(1-qx)$ is satisfied. Fig 1 describes the two yield curves $f=0$ and $f_r=0$.

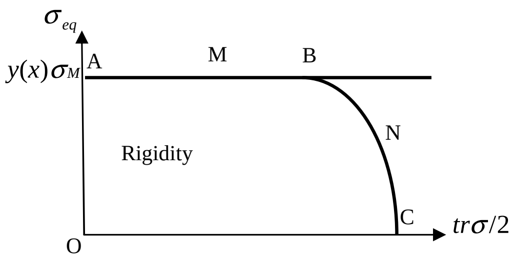

Figure 1. The three parts of the resultant yield curve: AMB,B,BNC

Along AMB, the matrix is plastically strained whereas the micro-voids do not suffer irreversible opening. Along BNC, due to very small elastic strains (rigidity hypothesis) of the matrix, micro-voids suffer opening. Finally, if the stress point rests on the intersection point B, then damage-plasticity arises.

3. Constitutive model and conclusion

We quote below the equations of *damage-plasticity process* only (arising in B, Eq (4)):

(4) $$2qx\cosh(\tfrac{1}{2}tr\sigma/\sigma_M) = 1 + q^2x^2 - y^2(x) \ , \ (\sigma_{eq}/\sigma_M)^2 = y^2(x)$$

(5) $$d_p = 3(\lambda_r + \lambda)\breve{\sigma}/\sigma_M + \lambda qx\sinh(\tfrac{1}{2}tr\sigma/\sigma_M)I$$

(6) $$\dot{\varepsilon}_M = (1-v)^{-1}\sigma_M^{-1}(\sigma : d_p) \ , \ \dot{v} = (1-v)(tr\, d_p) + a_n A(\varepsilon_M)\dot{\varepsilon}_M \, .$$

Eq (5), for the plastic strain rate d_p, is an associative evolution law at the non-smooth point B with two multipliers (λ_r, λ) ($\breve{\sigma}$ is the deviator of σ). Eq (6) are evolution laws of ε_M and v [3,4], where the dot designs time-derivative, a_n is a material constant and A a classical Laplace-Gauss function (to be specified). In the evolution law of the void volume fraction v, the first part represents the geometric growth v_g and the second part the contribution of the void nucleation v_n.

The relations (6) give $\dot{\varepsilon}_M$ and $\dot{v}$ through linear function of d_p, then as linear functions of the two multipliers by using (5). But the two consistency conditions give $\dot{\varepsilon}_M$ and $\dot{v}$ through linear functions of the stress rate ($\dot{\sigma}$), so that the two multipliers may be written as linear functions of ($\dot{\sigma}$). Finally we obtained an expression of d_p in function of the stress rate ($\dot{\sigma}$). Simple examples show the ability of the actual scheme and so, in this work, as a new result, attention was given to the necessity of using simultaneously yield conditions of both the damaged and virgin materials.

4. References

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