A GRADIENT-ENHANCED COUPLED DAMAGE-PLASTICITY MODEL IN LARGE STRAIN FORMULATION

T. Żebro [1]***, K. Kowalczyk-Gajewska*** [2]***, J. Pamin*** [1]
[1] *Cracow University of Technology, Cracow, Poland*
[2] *Institute of Fundamental Technological Research, Warsaw, Poland*

1. General

Coupled damage-plasticity models are relatively simple if isotropy is assumed, while they have all features necessary for the numerical modelling of composites: void or crack growth, irreversible deformations and stiffness degradation can be represented. However, if applied in localized failure simulations, the models require regularization which can be performed as a non-local enhancement, having either a gradient or integral form.

The aim of the paper is to present a damage-plasticity model at large strain, based on a free energy and dissipation potentials decomposed into elasto-damage and plastic parts. The model incorporates a gradient-type averaging equation for the strain energy which is a driving force of elastic damage coupled to irreversible deformations. The paper is based on the concepts presented in the paper by Liebe and Steinmann [4], extending the theory with a coupling to plasticity. An implementation in the FEAP finite element package is performed. Numerical simulations contain one-element tests and the one-dimensional tensile bar benchmark.

2. Local model

The model is based on the multiplicative split of the deformation gradient $\boldsymbol{F}$ into elastic and plastic parts. We adopt the Helmholtz free energy in the form, cf. [7, 1]:

(1) $$\Psi = (1-D)\Psi^{\mathrm{e}}(\boldsymbol{b}^{\mathrm{e}}) + \Psi^{\mathrm{p}}(\kappa^{\mathrm{p}}),$$

where D is the scalar damage parameter growing from 0 for the intact material to 1 for complete damage, $\boldsymbol{b}^{\mathrm{e}} = \boldsymbol{F}^{\mathrm{e}}\boldsymbol{F}^{\mathrm{eT}}$ the elastic left Cauchy-Green tensor and κ^{p} the internal variable (plastic strain measure).

The elastic part of the Helmholtz potential is the strain energy composed of the volumetric and deviatoric parts, respectively:

(2) $$\Psi^{e} = W = \Psi''^{\mathrm{e}}(J) + \Psi'^{\mathrm{e}}(\boldsymbol{b}'^{\mathrm{e}})$$

where $J = \det(\boldsymbol{F})$ and $\boldsymbol{b}'^{\mathrm{e}} = J^{-2/3}\boldsymbol{b}^{\mathrm{e}}$ is the isochoric elastic left Cauchy-Green tensor. When the Kirchhoff stress $\boldsymbol{\tau}$ is derived from Ψ in a usual manner, the effective Kirchhoff stress tensor $\hat{\boldsymbol{\tau}}$ occurs:

(3) $$\boldsymbol{\tau} = (1-D)\hat{\boldsymbol{\tau}}$$

The definitions of Ψ'^{e} and Ψ''^{e} are based on [7] and lead to a relation between the Hencky strains (logarithmic stretches) and principal effective Kirchhoff stresses which resembles the classical linear Hooke's law. The plastic part of the Helmholtz potential is standard.

Further, the dissipation potential is postulated in a decoupled form

(4) $$\Phi(\boldsymbol{\tau}, q, Y; D) = \Phi^{\mathrm{p}}(\hat{\boldsymbol{\tau}}, q) + \Phi^{\mathrm{d}}(Y)$$

In the associative case the first part is equal to the yield function $\Phi^{\mathrm{p}} \leq 0$ that depends on the effective Kirchhoff stress $\hat{\boldsymbol{\tau}}$, while parameter q represents the yield strength with isotropic hardening. In the simplest case linear hardening $q = \sigma_{\mathrm{y}} + h\kappa^{\mathrm{p}}$ and the Huber-Mises yield function are used. The second part of the dissipation potential is the damage loading function $\Phi^{\mathrm{d}} = Y - \kappa^{\mathrm{d}} \leq 0$, in which Y is the thermodynamic force conjugated to damage, equal to the strain energy W. Both the yield and damage conditions are subject to respective Kuhn-Tucker conditions. The damage parameter is computed as a function of the current damage history parameter κ^{d}:

$$D = f^{\mathrm{d}}(\kappa^{\mathrm{d}}), \quad \kappa^{\mathrm{d}} = \max_{-\infty<s<t}(Y(s), \kappa_0^{\mathrm{d}})$$

with the initial damage threshold κ_0^{d}. This function can for instance be exponential [4] or based on the model of Lemaitre [7].

To integrate the nonlinear problem in time, we follow the approach pioneered by Simo in order to preserve the convenient small-strain structure of return mapping algorithm, see [2].

3. Gradient-enhancement

The introduction of gradient-enhancement requires the selection of a non-local parameter and the formulation of a corresponding averaging equation. Within elastic damage models there is an energy gradient formulation with a non-local stored energy $\bar{W}$ serving as an independent variable, and a damage gradient formulation with damage parameter D serving as an independent variable and its gradient $\mathbf{D}$ entering the free energy function [4]. There are also gradient enhanced theories with non-local damage parameter $\bar{D}$ [1]. In the case of ductile damage models, a kinematic non-local variable $\bar{z}$ is introduced, having its local kinematic counterpart, e.g. the equivalent plastic strain [3].

Here the first option is adopted, called in [4] the Energy Gradient Formulation. The damage driving force W is substituted by its non-local counterpart $\bar{W}$ in the damage condition, cf. [6]:

$$\bar{W} = W - \mathrm{Div}\mathbf{W} \quad \rightarrow \quad \Phi^{\mathrm{d}} = \bar{W} - \kappa^{\mathrm{d}} \leq 0, \quad \kappa^{\mathrm{d}} = \max_{-\infty<s<t}(\bar{W}(s), \kappa_0^{\mathrm{d}}) \tag{5}$$

where a damage flux $\mathbf{W}$ is introduced. If the damage flux is derived from the non-local energy by $\mathbf{W} = -c\nabla_X \bar{W}$ with c related to the square of an internal length scale, an implicit formulation ensues which resembles the computationally convenient concept of averaging [5]. It remains to decide whether the averaging should be performed in the initial configuration as above (Lagrange averaging, cf. [6]) or in the current configuration (Euler averaging, cf. [3]). This issue is discussed and the results of one-dimensional tensile bar benchmark are presented.

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