THE INFLUENCE OF HISTORY OF PRECRITICAL LOADING ON BIFURCATION OF PROCESS OF DEFORMATION OF ELASTIC-PLASTIC BODIES

Yu.Chernyakov*, *V.Shneider* and *D. Teslenko

Dnepropetrovsk National University, Dnepropetrovsk, Ukraine

1. Introduction

The foundations of bifurcation theory outside elasticity were laid in works [1, 2]. Further investigations in this direction have shown that bifurcation of deformation process is closely connected with the problem of singularity of yield surface [3]. The question of systematic investigation of stability outside the limits of elasticity at complex precritical loading remains insufficiently studied. For the solving of such problems it is necessary to use theory of plasticity that adequately describes mechanical behaviour of polycrystals at arbitrary loading. In the present work the theory of micro strains [4, 5, 6] is used. This theory leads to singular yield surface and allows describing deformation of metals at complex loading. In [7] it was shown that the theory is capable to describe ratcheting – the accumulation of inelastic strain under the cyclic loading. This accumulation can cause the bifurcation in some situations.

2. Problem statement

Let's consider elastic-plastic body occupying volume V_0 in the init state. On a part of a surface Ω_u we shall specify rate of displacements v_i, and on a part of a surface Ω_σ – rate of surface pressure $\dot{p}_i(t)$. We shall assume, that during some moment of time $t = t_{cr}$, alongside with the basic solution of a boundary problem v_i^0 there is other solution v_i^b. The problem of stability for a difference of solution is reduced to following problem of optimization in a class of cinematically possible differences of velocities:

$$\int_{V_0} \Delta\dot{\mathrm{p}} : \delta(\nabla(\Delta\mathbf{v}))dV = 0\,,\ \Delta\mathbf{v}(\mathbf{x}) = 0 \text{ при } \mathbf{x} \in \Omega_u\,,$$

where $\dot{\boldsymbol{\pi}}$ is rate of first Piola-Kirchhoff tensor, $\nabla\mathbf{v}$ is tensor of rate gradient and $\Delta(\cdot) = (\cdot)_b - (\cdot)_a$.

Structure of constitutive relations of theory of micro strains is such that at active pre critical loading the domain of directions of full loading, within limits of which constitutive relations are linearized, exists. At that we have following inequality

$$(\boldsymbol{\sigma}_b^J - \boldsymbol{\sigma}_a^J) : (\mathbf{d}_b - \mathbf{d}_a) \geq (\mathbf{d}_b - \mathbf{d}_a) : \mathbf{G}_0 : (\mathbf{d}_b - \mathbf{d}_a)\,,$$

where $\boldsymbol{\sigma}^J$ - Jaumann derivative of Cauchy stress tensor, $\mathbf{d}$ - rate deformation tensor, $\mathbf{G}_0$ - stiffness matrix of linearized comparison body, which coincide with stiffness matrix of theory of micro strains at full loading. It is worth to note that stiffness matrix $\mathbf{G}_0$ is functional of precritical loading process.

Last inequality allows reducing the problem of bifurcation of deformation process to Euler stability problem of linearized comparison body:

$$\nabla \cdot \dot{\boldsymbol{\pi}} = 0\,,$$

$$\dot{\boldsymbol{\pi}} = \dot{\boldsymbol{\sigma}}^J + \mathbf{R} : \nabla \mathbf{v}$$

$$\dot{\boldsymbol{\sigma}}^J = \mathbf{G}_0 : \mathbf{d},$$

with boundary conditions

$$\mathbf{v}(\mathbf{x}) = 0, \text{ при } \mathbf{x} \in \Omega_u,$$

$$\mathbf{N} \cdot \dot{\boldsymbol{\pi}} = 0 \text{ at } \mathbf{x} \in \Omega_\sigma .$$

This system of equations with homogeneous boundary conditions allows to investigate bifurcation of elastic-plastic bodies at complex loading.

3. Results

In the present work it is considered the problems of localization of plastic deformation and stability of stripe at plane strain, and also the stability of thick plate and surface of half-space at bi-axial precritical loading. The analysis of influence of type of applied loading trajectory on bifurcation of deformation process was done. Bi-axial trajectories of loading and trajectories of complex cyclic loading with ratcheting were considered. It is shown that the history of loading can essentially influence on critical parameters of problem.

6. References

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