

ADIABATIC MICRODAMAGE ANISOTROPY IN DUCTILE MATERIALS

A. Glema¹, T. Łodygowski¹, P. Perzyna² and W. Sumelka¹

¹Poznan University of Technology, Institute of Structural Engineering, Poznan, Poland

²Institute of Fundamental Technological Research, Warsaw, Poland

1. Introduction

The main aim of the following discussion is the analysis of the adiabatic anisotropic process during fast tension test, where the rate of strains reaches nearly 10^4s^{-1} . An anisotropy is induced by the evolution of the intrinsic microstructure and affects on all stages of the analysis. The problem is defined in terms of the continuum mechanics in the framework of thermodynamics. The essential role, in the formulation, plays the definition of the temperature evolution due to the influence of the intrinsic microdamage. The microdamage introduces an additional term to the temperature evolution law, thus an identification of all of its components is needed.

The microdamage is incorporated into the constitutive structure as a component of the postulated internal state vector, and is described by the second order, symmetric tensorial field - called strictly *microdamage field* [2]. Microdamage field governs the influence of the evolution of the microvoids, microcracks etc., in micro level, on the macro material structure - what in turn can lead to the failure (the loss of the continuity in macro level).

The proposed material model is implemented in the Abaqus commercial finite element code, using the capability of the user subroutine interface.

2. Evolution of temperature

Let us assume that the free energy function ψ exists and takes the form

$$(1) \quad \psi = \varepsilon - \vartheta \eta,$$

where ε is the density of the internal energy, ϑ denotes the absolute temperature and η is an entropy. The first law of thermodynamics, after assuming that thermal energy is transferred through the surface only and keeping Eq. (1), has the local form

$$(2) \quad \frac{1}{\rho_{Ref}} \boldsymbol{\tau} : \mathbf{d} - \dot{\psi} - \dot{\vartheta} \eta - \dot{\eta} \vartheta - \frac{1}{\rho} \text{div} \mathbf{q} = 0,$$

where ρ_{Ref} is reference density, $\boldsymbol{\tau}$ is Kirchhoff stress tensor, \mathbf{d} is symmetric part of the spatial velocity gradient, ρ is actual density and \mathbf{q} denotes the vectorial heat flux.

Assuming moreover that

$$(3) \quad \psi = \hat{\psi}(\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}),$$

where \mathbf{e} is spatial strain tensor, \mathbf{F} is deformation gradient and $\boldsymbol{\mu}$ denotes the internal state vector, one can obtain the following local form of the second law of thermodynamics

$$(4) \quad -\frac{\partial \hat{\psi}}{\partial \boldsymbol{\mu}} \cdot \mathbf{L} \boldsymbol{\mu} - \frac{1}{\rho \vartheta} \mathbf{q} \cdot \text{grad} \vartheta \geq 0,$$

and its important consequence that

$$(5) \quad \eta = -\frac{\partial \hat{\psi}}{\partial \vartheta},$$

where $L_{\mathbf{v}}$ denotes Lie derivative, where \mathbf{v} denotes velocity field. Eq. (2) can be then rewritten to the form

$$(6) \quad \rho \vartheta \dot{\eta} = -\operatorname{div} \mathbf{q} - \rho \frac{\partial \hat{\psi}}{\partial \boldsymbol{\mu}} \cdot L_{\mathbf{v}} \boldsymbol{\mu}.$$

Assuming that the internal state vector has two components, namely

$$(7) \quad \boldsymbol{\mu} = (\in^p, \boldsymbol{\xi}),$$

where \in^p is the equivalent viscoplastic deformation and $\boldsymbol{\xi}$ is microdamage tensor and taking the time derivative of the Eq. (5), we have from Eq. (6) the fundamental temperature evolution law [1]

$$(8) \quad \rho c_p \dot{\vartheta} = -\operatorname{div} \mathbf{q} + \vartheta \frac{\rho}{\rho_{Ref}} \frac{\partial \boldsymbol{\tau}}{\partial \vartheta} : \mathbf{d} + \rho \chi^* \boldsymbol{\tau} : \mathbf{d}^p + \rho \chi^{**} \mathbf{K} : L_{\mathbf{v}} \boldsymbol{\xi},$$

where the specific heat

$$(9) \quad c_p = -\vartheta \frac{\partial^2 \hat{\psi}}{\partial \vartheta^2},$$

and the irreversibility coefficients χ^* and χ^{**} are determined by

$$(10) \quad \chi^* = - \left(\frac{\partial \hat{\psi}}{\partial \in^p} - \vartheta \frac{\partial^2 \hat{\psi}}{\partial \vartheta \partial \in^p} \right) \sqrt{\frac{2}{3}} \frac{1}{\boldsymbol{\tau} : \mathbf{P}},$$

$$\chi^{**} = - \left(\frac{\partial \hat{\psi}}{\partial \boldsymbol{\xi}} - \vartheta \frac{\partial^2 \hat{\psi}}{\partial \vartheta \partial \boldsymbol{\xi}} \right) \frac{1}{\mathbf{K}}.$$

If one puts $\mathbf{q} = \mathbf{0}$, and taking the crucial assumption that $\mathbf{K} = L_{\mathbf{v}} \boldsymbol{\xi}$, the final form of the temperature evolution law in adiabatic anisotropic process is obtained

$$(11) \quad \dot{\vartheta} = \vartheta \frac{1}{c_p \rho_{Ref}} \frac{\partial \boldsymbol{\tau}}{\partial \vartheta} : \mathbf{d} + \frac{\chi^*}{c_p} \boldsymbol{\tau} : \mathbf{d}^p + \frac{\chi^{**}}{c_p} L_{\mathbf{v}} \boldsymbol{\xi} : L_{\mathbf{v}} \boldsymbol{\xi}.$$

The last term in Eq. (11) governs the influence of the anisotropy on the temperature field in material structure.

Instructive numerical examples will be presented.

3. References

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