CONFORMAL CONTACT BETWEEN A PUNCH AND A LAYER WITH THIN COATING

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Consider conformal contact between a double layered foundation and a rigid punch in the case of plane strain. The foundation consists of a viscoelastic aging layer of arbitrary thickness H and a thin viscoelastic aging coating of variable thickness $h(x)$ whose surface follows a complex surface of the punch. The lower border of the foundation is in the state of smooth or ideal contact with the underlying rigid base (smooth or ideal contact is achieved between the layers). Suppose that, starting from an instant τ_0, the smooth rigid punch with a complex shape of its surface is indented into the conformal surface of the coated viscoelastic layer with force $P(t)$ applied with eccentricity $e(t)$. The contact region is independent of time, and the contact line length is $2a$. The viscoelastic coating of variable thickness is produced at an instant $\tau_1 \leq \tau_0$ and homogeneously ages thenceforth. The lower viscoelastic layer of arbitrary thickness is produced at an instant $\tau_2 \leq \tau_0$ and also ages homogeneously.

For the problem stated above, the mixed integral equation and the additional conditions in the plane-strain case have the form [1] ($t \geq \tau_0$)

$$(1) \qquad (\mathbf{I}-\mathbf{V}_1)\frac{\theta q(x,t)h(x)}{E_1(t-\tau_1)} + (\mathbf{I}-\mathbf{V}_2)\mathbf{F}\frac{2(1-\nu_2^2)q(x,t)}{\pi E_2(t-\tau_2)} = \delta(t)+\alpha(t)x, \quad x\in[-a,a],$$

$$(2) \qquad \int_{-a}^{a} q(\xi,t)\,d\xi = P(t), \qquad \int_{-a}^{a} \xi q(\xi,t)\,d\xi = M(t).$$

Here, $q(x,t)$ is contact pressure under the punch; $M(t) = e(t)P(t)$ is the moment of the applied force $P(t)$; $E_1(t)$ and $E_2(t)$ are instant elastic strain moduli of the coating and the lower layer, respectively; τ_1 and τ_2 are the instants at which the coating and the lower layer are produced; θ is a dimensionless coefficient that depends on the properties of the contact between the coating and the lower layer; **I** is the identity operator; $\mathbf{V}_k$ ($k = 1,\, 2$) are Volterra integral operators with tensile creep kernels $K_k(t,\tau)$; **F** is a Fredholm integral operator with a known kernel of the plane contact problem, $k_{\rm pl}[(x-\xi)/H]$ [1,2]; $\delta(t)$ is the punch settlement and $\alpha(t)$ is its tilt angle. Note that conformal contact is a generalization of interaction between bodies with plane surfaces.

Given the applied force and the eccentricity, it is required to determine the contact pressure under the punch, its settlement, and its tilt angle.

A solution of equation (1) with the additional conditions (2) can be found by a generalized projection method used for solving mixed integral equations [3–6]. The structure of the solution for contact pressures has the form

$$q(x,t) = \frac{1}{h(x)}[z_0(t)P_0(x) + z_1(t)P_1(x) + \ldots],$$

where $z_k(t)$ is a function of time t, and $P_k(x)$ are polynomials of some special form ($k = 0,\, 1,\, \ldots$). Thus, it is possible to have an explicit dependence of the solution on the coating thickness $h(x)$. This fact allows us to find effective analytical solutions for bases with coatings whose thickness is specified by functions of a complex structure, in particular, rapidly oscillating functions. In such situations, effective analytical solutions can hardly be found by other known methods. It should be noted that in the case under consideration, contact pressures, the settlement, and the tilt angle of the punch are proportional to the indenting force. This property is observed only in solutions of the conformal contact problem.

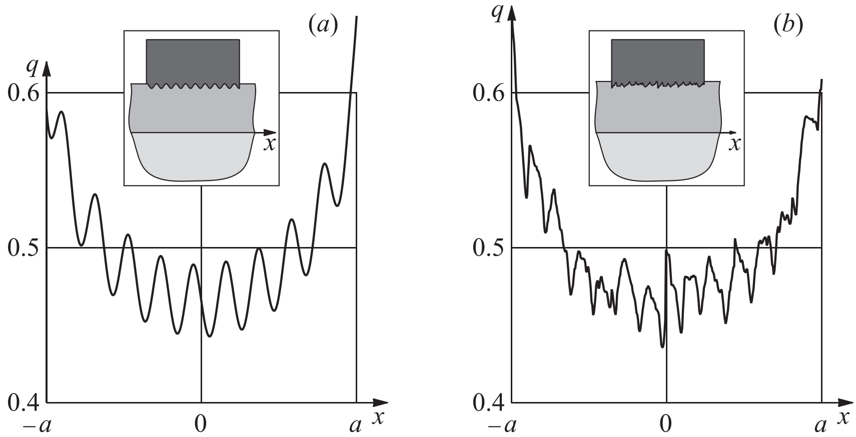

Figure. Contact pressures for $P(t)/a \equiv 1$ N/m^2, $e(t) \equiv 0$ at the initial instant for different surface profiles.

The settlement and the tilt angle of the punch are also obtained in terms of explicit analytic formulas. Obviously, for the constant force and moment, the settlement and the tilt angle tend to some asymptotic values.

Figure represents numerical results for two cases of coated bases: (*a*) the coating thickness is described by an oscillating function; (*b*) the real surface profile is given as determined from experimental data. It can be seen that the solutions obtained above take into account all specific features of the surface profile.

A similar problem can be formulated in the axisymmetric case, and its solution can be obtained by the same method.

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