BOUNDARY VALUE PROBLEMS IN THE TWO-TEMPERATURE THEORY OF THERMOELASTICITY OF BINARY MIXTURES

M. Svanadze
Ilia Chavchavadze State University, Tbilisi, Georgia

1. Introduction

The nonlinear theory of thermoelasticity of mixtures of two- or many-component solids was developed by Green and Steel [1]. A linear variant of this theory (the diffusion model) was proposed by Steel [2]. The theory of thermoelasticity of binary mixtures (the shift model) was constructed by Iesan [3]. In [1-3], the mixture components are assumed to have the same temperature value.

The linear and nonlinear theory of thermoelasticity of binary mixtures with components having different temperature values were respectively constructed by Khoroshun and Soltanov [4] and Iesan [5]. Fundamental solutions of steady oscillation (vibration) equations of the two-temperature linear theory of mixtures are constructed in terms of elementary functions in [6].

In this paper, the boundary value problems (BVPs) of steady vibration of the two-temperature linear theory of thermoelasticity of binary mixtures are investigated by means of the boundary integral equation method (potential method [7, 8]). The Sommerfeld-Kupradze type radiation conditions are established. The uniqueness and existence theorems of solutions of the BVPs are proved using the potential method and the theory of multidimensional singular integral equations.

2. Basic boundary value problems

The system of equations of steady vibration in the two-temperature linear theory of thermoelasticity of binary mixtures is written as [4, 5]

$$
\begin{aligned}
&a_1\Delta u + b_1 \operatorname{grad}\operatorname{div} u + c\Delta w + d \operatorname{grad}\operatorname{div} w + \omega^2\rho_1 u - \alpha(u-w) - \alpha_{11}\operatorname{grad}\theta_1 - \alpha_{12}\operatorname{grad}\theta_2 = 0,\\
&c\Delta u + d \operatorname{grad}\operatorname{div} u + a_2\Delta w + b_2 \operatorname{grad}\operatorname{div} w + \omega^2\rho_2 w + \alpha(u-w) - \alpha_{21}\operatorname{grad}\theta_1 - \alpha_{22}\operatorname{grad}\theta_2 = 0,\\
&(a_{11}\Delta + i\omega m_{11})\theta_1 + (a_{12}\Delta + i\omega m_{12})\theta_2 + i\omega \operatorname{div}(\beta_{11} u + \beta_{21} w) = 0,\\
&(a_{21}\Delta + i\omega m_{21})\theta_1 + (a_{22}\Delta + i\omega m_{22})\theta_2 + i\omega \operatorname{div}(\beta_{12} u + \beta_{22} w) = 0,
\end{aligned}
\tag{1}
$$

where $u=(u_1,u_2,u_3)$ and $w=(w_1,w_2,w_3)$ are the partial displacements, θ_1 and θ_2 are the temperature variations of each component, $a_j, b_j, c, d, a_{lj}, \alpha_{lj}, \beta_{lj}, m_{lj}$ $(l, j=1,2)$ are thermoelastic constants of the mixture, $\alpha \ge 0$, ω is the oscillation frequency, ρ_1 and ρ_2 are the partial densities.

Let $x=(x_1,x_2,x_3)$ be the point of the Euclidean three-dimensional space E^3. Let S be the closed surface surrounding the finite domain Ω^+ in E^3. $S \in C^{2,\nu}$, $0<\nu\le 1$, $\overline{\Omega}^+ = \Omega^+ \cup S$, $\Omega^- = E^3 \setminus \overline{\Omega}^+$. A vector function U is called *regular* in Ω^- (or Ω^+) if $U_l \in C^2(\Omega^-)\cap C^1(\overline{\Omega}^-)$ (or $U_l \in C^2(\Omega^+)\cap C^1(\overline{\Omega}^+)$), $U_l(x) = \sum_{j=1}^{6} U_{lj}(x)$, $U_{lj} \in C^2(\Omega^-)\cap C^1(\overline{\Omega}^-)$, $(\Delta + k_j^2)U_{lj}(x) = 0$, and

$$
\left(\frac{\partial}{\partial |x|} - ik_j\right)U_{lj}(x) = e^{ik_j|x|}o(|x|^{-1}),
\tag{2}
$$

for $|x| >> 1$, where k_j is the wave number, $l=1,2,...,8$, $j=1,2,...,6$, $|x| = (x_1^2+x_2^2+x_3^2)^{1/2}$. Equalities in (2) is the Sommerfeld-Kupradze type *radiation conditions* in the two-temperature theory of thermoelasticity of binary mixture.

Problem $(I)_f^+$: Find a regular solution to system (1) for $x \in \Omega^+$ that satisfies the boundary

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condition $\lim\limits_{\Omega^+ \ni x \to z \in S} U(x) \equiv \{U(z)\}^+ = f(z)$.

Problem $(I)_f^-$: Find a regular solution to system (1) for $x \in \Omega^-$ that satisfies the boundary condition $\lim\limits_{\Omega^- \ni x \to z \in S} U(x) \equiv \{U(z)\}^- = f(z)$, where f is the known vector function on S.

4. Uniqueness and Existence Theorems

Theorem 1. Exterior BVP $(I)_f^-$ admits at most one regular solution.

Theorem 2. Interior homogeneous BVP $(I)_0^+$ has a non-trivial solution $U = (u, w, 0, 0)$ in the class of regular vectors, where the vector $V = (u, w)$ is a solution to the system

$$a_1 \Delta u + b_1 \operatorname{grad}\operatorname{div} u + c\Delta w + d \operatorname{grad}\operatorname{div} w + \omega^2 \rho_1 u - \alpha(u - w) = 0,$$

(3) $$c\Delta u + d \operatorname{grad}\operatorname{div} u + a_2 \Delta w + b_2 \operatorname{grad}\operatorname{div} w + \omega^2 \rho_2 w + \alpha(u - w) = 0,$$

$$\beta_{11} \operatorname{div} u + \beta_{12} \operatorname{div} w = 0, \quad \beta_{21} \operatorname{div} u + \beta_{22} \operatorname{div} w = 0, \quad \text{for} \quad x \in \Omega^+$$

satisfying the boundary condition

(4) $$\{V(z)\}^+ = 0;$$

the problems $(I)_0^+$ and (3), (4) have the same eigenfrequencies.

Theorem 3. If $S \in C^{2,\nu}$, $f \in C^{1,\nu'}(S)$, $0 < \nu' \le \nu \le 1$, then a regular solution of the problem $(I)_f^-$ exists, is unique, and is represented by sum $U(x) = Z^{(2)}(x, g) + a' Z^{(1)}(x, g)$ for $x \in \Omega^-$, where $Z^{(1)}(x, g)$ and $Z^{(2)}(x, g)$ are the single-layer and double-layer potentials, respectively, $a' = a_1' + i a_2'$; a_1' and a_2' are the real numbers, $a_1' > 0$, $a_2' < 0$, and g is a solution of the singular integral equation $-\frac{1}{2} g(z) + Z^{(2)}(z, g) + a' Z^{(1)}(z, g) = f(z)$ for $z \in S$, which is always solvable for an arbitrary vector f.

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