INCREMENTAL PLASTIC RESPONSE AND FLOW RULE POSTULATE UNDER GENERAL THREE-DIMENSIONAL CONDITIONS

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1. Introduction

In this contribution we examine the flow rule postulate, a pillar assumption in the framework of the theory of plasticity that regards the direction of plastic strain increment as being independent of loading direction. Recent discrete element method and analytical calculations have pointed out that under three-dimensional (3D) stress conditions the direction of plastic strain increment does depend on the loading direction (Kishino, 2003, Darve and Nicot, 2005). These findings, which have not received much attention, question the validity of flow rule premise. In this respect, classic elastoplastic models based on this postulate will necessarily have shortcomings, especially in ‘true’ triaxial conditions. Through extensive numerical simulations using a particle flow model we verify that the incremental plastic strain response not only depends on the loading direction but also on stress history.

2. Methodology

Firstly, we analyze the incremental behaviour of a cubic assembly of polydispersed spherical particles subject to a series of 3D spherical stress probes ($\Delta\sigma_x$, $\Delta\sigma_y$, $\Delta\sigma_z$) with constant Euclidian norm of 0.1 kPa, see Figure 1a. Prior to the stress probing stage, the specimen was consolidated to 100 kPa, and then sheared along the triaxial extension stress path until the final state, corresponding to a mean stress (p) equal to 100 kPa and deviatoric stress (q) equal to 60 kPa, was achieved.

The plastic strain response under other stress state conditions reached from different stress histories (Figure 2a) prior to probe tests was subsequently investigated. One series of tests comprised of paths moving along the hydrostatic axis to p = 100 kPa, then radiating at various angles in the π-plane at constant value q = 60 kPa. Hence working within a sextant of the π-plane, various radial paths can be obtained starting from triaxial compression (TC) to triaxial extension (TE) passing through various Lode angles. Another series of tests refer to the classical conventional triaxial compression (CTC) and conventional triaxial extension (CTE) tests. In the former, the confining pressure is maintained constant with increasing axial stresses, whereas in the latter, the confining pressure is increased with constant axial stress.

3. Results

A typical strain envelope response generated from the stress probe introduced above is illustrated in Figures 1b-c. The outer (yellow) surface represents the ‘total’ strain incremental response, whereas the inner (orange) surface represents the elastic strain response. The (blue) dots inside are the increments of plastic strains. Differently from what the postulate of plastic flow rule assumes, the increments of plastic strain points do not fall on a single line. In fact, these increments turn out to be a function of both stress increment (probe) direction and stress state. As such, they plot as a series of points which clearly define an oval shaped envelope. Another intriguing fact is that all points fall on a plane very close to π-plane and perpendicular to the Rendulic plane (Figure 1c). Should the plane of plastic strain response coincide with π-plane in the incremental strain space, this would imply null plastic incremental volume change, that is, no dilatancy.

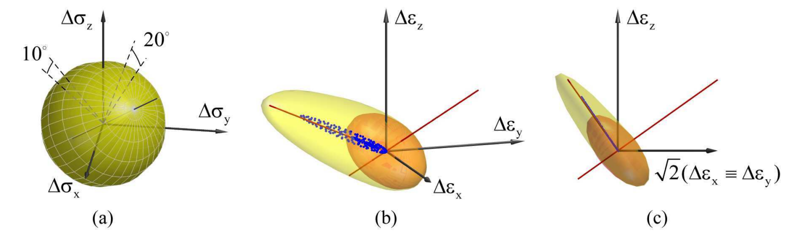

Figure 1. Spherical stress probe (a) Isometric view; (b) Total, elastic, and plastic strain envelope responses; (c) Rendulic plane view. Note: max($\Delta\varepsilon$) = 1.24x10^{-6}.

The results from the series of probe tests under distinct loading histories given in Figure 2a are presented in Figures 2b-c. We found that the incremental plastic strain envelope is symmetric about the direction of the stress path prior to probe only in the TC, TE, CTC and CTE cases. For stress paths along Lode angles equal to 20º and 40º, there is a pronounced deviation of the plastic envelope with respect to the direction of previous stress history. This deviation is dictated by the proximity of the stress probes to the failure surface. As such, along the axisymmetric stress branches, the plastic strain response envelope is bound to be symmetrical about the radial or previous stress path direction as long as the failure envelope is symmetric or no inherent or induced anisotropy exists.

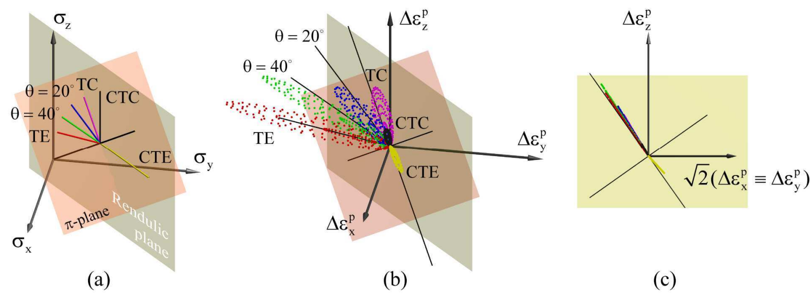

Figure 2. Stress histories prior to stress probe tests (a) Isometric view; (b) Plastic strain envelopes for different stress histories (b) Rendulic plane view.

3. Conclusion

Through discrete element analysis, we showed that plastic strain incremental response is a function of stress probing (loading) direction, as opposed to what the flow rule postulate presumes. For axisymmetric loading cases, the plastic strain envelope was found to be symmetrical about the direction of the stress path prior to probe. For ‘true’ triaxial stress paths a deviation apparently dictated by the proximity of the stress probes to the failure surface was noticed. Intriguingly, for a given spherical stress probe, all plastic response points fell on a unique plane, presumably associated with the zero eigenvector of the tangent constitutive matrix describing the stress increment/plastic strain increment map under plastic flow conditions.

4. References

[1] Y. Kishino (2003). On the incremental non-linearity observed in a numerical model for granular media. *Italian Geotechnical J.*, **3**, 30-38.

[2] F. Darve and F. Nicot (2005). On the flow rule in granular media: phenomenological and multi-scale views (Part II). *Intl. J. Num. Anal. Methods in Geomech.*, **29**, 1411-1432.