EXTENSION OF ISOTROPIC MULLINS MODELS TO ANISOTROPIC STRESS-SOFTENING MODELS

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1. Abstract

The Mullins effect in rubber-like materials is inherently anisotropic. For example, in Pawelski [1] homogeneous plain strain compression experiment, after loading from a virgin state and unloading, the block is rotated by 90 degrees and compressed; it was found that the non-virgin material behaves almost like a virgin one, which indicates that stress softening in the first direction has hardly any influence on stress softening in the direction orthogonal to the first. This anisotropic behaviour can also be found in other experiments [2] [3]. However, most models developed in the past, for Mullins effect, are isotropic. Nevertheless, recently, Shariff [4] developed a constitutive equation that characterizes anisotropic stress softening. Shariff's [4] theory compares well with several anisotropic experimental data and is consistent with expected behaviour. In this paper, based on Shariff's [4] theory, we proposed a model that can easily extend some well known isotropic models [5] [6] to describe incompressible anisotropic behaviour of Mullins phenomenon; hence indicating the generality of the proposed model. The quasi-static constitutive equation is purely phenomenological and does not take account the underlying physical structure of the material; hence it can be applied to any incompressible material exhibiting anisotropic stress softening induced by strain. We treat the virgin undeformed material as isotropic and are not concerned with hysteresis, residual strain and, thermal and viscoelastic effects.

We use a principal axes technique to facilitate our anisotropic modelling. This type of technique is also used by Shariff [7] to derive a novel constitutive equation for an incompressible transversely isotropic hyperelastic solid. In the proposed model, a set of damage parameters which depend on the history of the principal-direction line elements is proposed. Together with this, we introduce a general concept of damage function to facilitate the analysis of anisotropic stress softening. The effect of shearing on stress softening materials is described via shear-history parameters; they are the maximum and minimum values of the cosine of the angle between two principal-direction line elements throughout the history of the deformation. The damage and shear-history parameters are introduced into the constitutive equation via symmetric, second order, damage and shear-history structural tensors, respectively. The damage tensor is positive definite. The "free" energy is expressed as a function of principal stretches and invariants of the dyadic products of the principal directions of the right stretch tensor and the two structural tensors. In this communication, we only consider a class of free energy functions that is a subset of a wider class of free energy functions proposed in this paper. Energy dissipation is shown via the Clausius-Duhem inequality.

The generality of the proposed damage function allows us to easily extend some existing well known isotropic models to model anisotropic behaviour of Mullins effect. In order to demonstrate the capabilities of the proposed theory, results are given for several types of homogeneous deformations. For some of these deformations, we show that the non-virgin stress free configurations have certain types of anisotropy which are consistent with previous conjectures [8]. We also show that our theoretical results compare well, qualitatively and quantitatively, with published experimental data.

2. References

[1] H. Pawelski (2001). Softening behaviour of elastomeric media after loading in changing directions. In: D. Besdo, R.H. Schuster & J. Ihlemann (eds), *Constitutive Models for Rubber II*. Lisse: A.A. Balkema, 27-36.

[2] A.H. Muhr, J. Gough and I.H. Gregory (1999). Experimental determination of model for liquid silicone rubber. In: A. Muhr & A. Dorfmann (eds), *Constitutive Models for Rubber*. Rotterdam: A.A. Balkema, 181-187.

[3] J. Gough (2000). Stress-strain behaviour of rubber, *PhD Thesis Queen Mary and Westfield College, University of London* 255 pp.

[4] M.H.B.M. Shariff (2006). An anisotropic model of the Mullins effect, *J. of Engng. Maths.*, **56**, 415-435.

[5] R.W. Ogden and D.G. Roxburgh (1999). A pseudo-elastic model for the Mullins effect in filled rubber, *Proc. R. Soc. Lond.*, **A 455**, 2861-2877.

[6] M.F. Beatty and S. Krishnaswamy (2000). A theory of stress-softening in incompressible isotropic materials, *J. Mech. Phys. Solids*, **48**, 1931-1965.

[7] M.H.B.M. Shariff (2008). Nonlinear transversely isotropic elastic solids: an alternative representation, *Q. J. Mech. Appl. Math.*, DOI:10.1093/qjmam/hbm028, 1-21.

[8] C.O. Horgan, R.W. Ogden and G.Saccomandi (2004). A theory of stress softening of elastomers based on finite chain extensibility, *Proc. R. Soc. London*, **A 460**, 1737-1754.