MODEL OF DEFORMABLE RIGID BODY WITH DANGEROUS VOLUME

L. Sosnovskiy[1], S. Sherbakov[2]
[1] *S&P Group TRIBOFATIGUE Ltd., Gomel, Belarus*
[2] *Department of Mechanics and Mathematics, Belarusian State University, Minsk, Belarus*

Variety of practically important systems such as toothings, wheel/ rail etc. work in conditions of complex stress-strain state conditioned by local contact and general non-contact volume deformation. Such mechanical systems are called active systems [1] and for them special analysis of surface and subsurface deformation and damage is needed.

Generally the analysis of interaction and damage of elements of active systems is based on statistical model of deformable rigid body with dangerous volume that contains the criteria for limitation of dangerous volumes and the general procedure of their calculation [1].

Definition of forms and sizes of dangerous volumes requires the knowledge of function of distribution of fatigue limits for corresponding element of a system and the stress state in considered areas of interacting bodies. Dangerous volumes are finite three-dimensional areas where the stresses exceeding the minimum values of fatigue limit ($\sigma_{-1\min}$) distribution are present (figure 1).
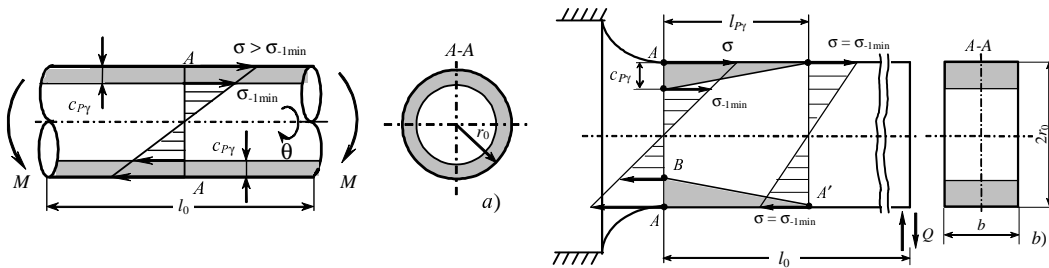

Figure 1. Scheme of formation of dangerous volume for pure bending of a shaft (a)
and console bending of a thick plate (b).

Active system differs from a bent shaft because all six independent components stress tensor are usually nonzero. Generally the limiting state according to the criterion of contact fatigue (formation of microcracks) in some point of an active system may be reached by several various tensor components. Thus fatigue limit for an active system is defined for every independent tensor component as an extreme value of its distribution under the action of limiting load $F_{*\lim}$. For a homogeneous isotropic deformable rigid body limiting normal and tangential stresses $\sigma_n^{(*\lim)}$ and $\sigma_\tau^{(*\lim)}$ and also limiting main stress $\sigma_1^{(*\lim)}$ and limiting intensity of stresses $\sigma_{\text{int}}^{(*\lim)}$ are defined in the following way [2]:

(1)
$$\sigma_n^{(*\lim)} = \max_{dV,i}\left(\left|\sigma_{ii}\left(F_{*\lim}, dV\right)\right|\right), i{=}x,y,z, \ \sigma_\tau^{(*\lim)} = \max_{dV,i,j}\left(\left|\sigma_{ij}\left(F_{*\lim}, dV\right)\right|\right), i,j{=}x,y,z, \ i \neq j,$$
$$\sigma_1^{(*\lim)} = \max_{dV}\left(\left|\sigma_1\left(F_{*\lim}, dV\right)\right|\right), \sigma_{\text{int}}^{(*\lim)} = \max_{dV}\left(\sigma_{\text{int}}\left(F_{*\lim}, dV\right)\right),$$

where dV - elementary volume of the loaded body.

Limiting stresses $\sigma_{ij}^{(\pm*\lim)}$, $i, j = x, y, z$, $\sigma_i^{(\pm*\lim)}$, $i = 1, 2, 3$, $\sigma_{\text{int}}^{(*\lim)}$ are defined similarly for the general case of rigid body.

Then the conditions for limitation of dangerous volumes are

(2)
$$V_{(ij)} = \left\{dV / \sigma_{ij} \geq \sigma_k^{(*\lim)}, dV \subset V_k\right\}, \quad i,j = x, y, z, \quad k = \begin{cases} n, \text{ при } i = j, \\ \tau, \text{ при } i \neq j, \end{cases}$$

(3) $V_{(i)} = \{dV / \sigma_i \geq \sigma_1^{(*\lim)}, dV \subset V_k\}$, $i = 1, 2, 3$, $V_{\text{int}} = \{dV / \sigma_{\text{int}} \geq \sigma_{\text{int}}^{(*\lim)}, dV \subset V_k\}, V_T = \bigcup_{i,j} V_{ij}$

where V_k - working volume of a deformable rigid body.

Corresponding measures of damage are

(4) $\omega_{ij} = V_{ij} / V_k$, $\omega_T = V_T / V_k$.

Since dangerous volumes may have arbitrary and complex form and their analytical definition is difficult then they are calculated using Monte-Carlo method.

Figure 5 shows the example of calculation of dangerous volumes for the case of non-conforming elliptical Hertzian contact for the following initial data: $\sigma_n^{(*\lim)} = 0.3 p_0$, $\sigma_\tau^{(*\lim)} = 0.09 p_0$ (p_0 is the maximum contact pressure in the center of contact), friction coefficient $f = 0,05$, ratio between smaller b and bigger a semi-axes of contact ellipse $b/a = 0.813$ [2]. It is visible from the given figures that the greatest by size are $V_{(zz)}$, $V_{(xz)}$ and $V_{(yz)}$ dangerous volumes correspond to the greatest stresses $\sigma_{zz}^{(n)}$, $\sigma_{xz}^{(n)}$ and $\sigma_{yz}^{(n)}$.
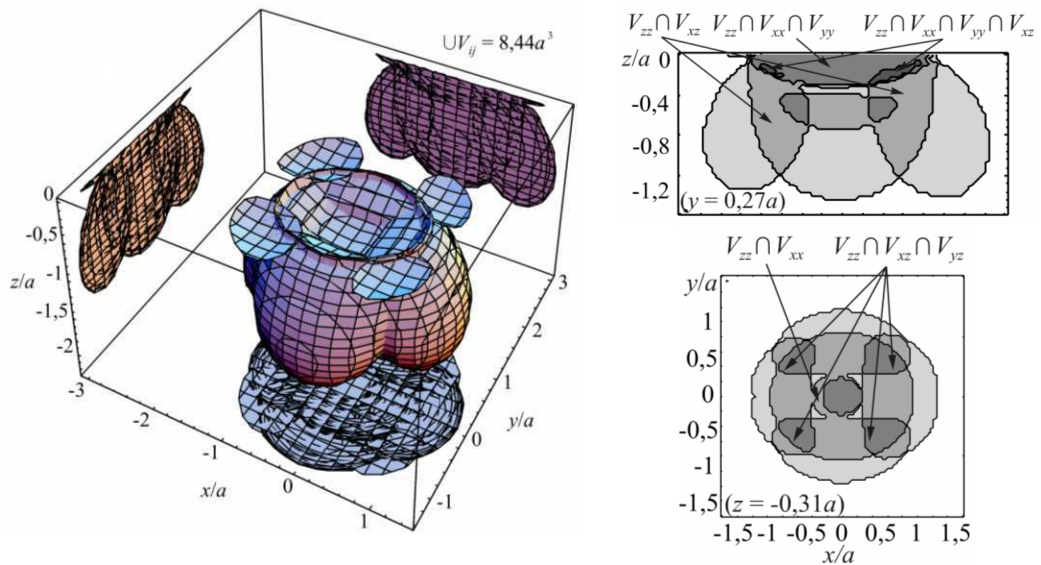

Figure 2. Union of dangerous volumes and its sections

Since dangerous volumes are the measures of damage of deformable bodies then while analyzing figure 2 it is possible to specify concrete areas where the origin and development of both surface and internal cracks is possible. It is obvious, that occurrence of irreversible damages (primary cracks) have higher probability where corresponding dangerous volumes intersect.

References

[1]. L.A. Sosnovskiy (2004). Tribo-Fatigue. Wear-fatigue damage and its prediction (Foundations of engineering mechanics), Springer.
[2]. L. A. Sosnovskiy, S.S. Sherbakov (2007). Vibro-impact in rolling contact, J. of Sound and Vibration, 308, 489–503.