THE NEW FRACTURE CRITERION FOR MIXED-MODE CRACK. THE *MK* CRITERION.

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1. Introduction

In formulating the fracture criteria an effort is usually made to refer to the physical phenomena associated with the damage processes of the material in front of the crack tip. The complexity of the damage mechanisms involved makes it necessary to introduce some assumptions that take into account the most important features of the damage process and make the analysis viable. There are many approaches based on the linear elastic fracture mechanics (LEFM). Some of them use the stress or strain energy calculated at a finite distance from the notch tip. In the case of the *S*-criterion [1] the strain energy density factor is assumed as a fracture criterion and calculated at a point located at a certain constant radius $r = r^{\mathrm{C}}$ from the crack tip being independent of the geometry and loading conditions. A strain energy density parameter is also the basis in formulation of the *T*-criterion [2], but, opposite to the *S*-criterion, this parameter consists of two energy components: distortional, T_{D}, and dilatation, T_{V}, ones. The *T*-criterion postulates that the crack propagates along the direction determined by the maximum of total energy density, which is also the maximum of the dilatational strain-energy density evaluated on the locus of constant distortional strain energy density what corresponds to Mises elastic-plastic boundary. This criterion uses the variable radius of the elastic-plastic boundary but in the case of brittle materials it tends to *S*-criterion since the size of plastic zone is then very small and the boundary can be assume as a circle with a constant radius. However, in the vicinity of the crack tip other damage processes may exist, e.g. microcracks, especially, in the case of brittle materials. They may be accompanied by a small plastic region [3]. Then the LEFM solutions used in *T*- and *S*-criterions are not fully correct because the stress or strain energy are calculated at some finite distance from the crack tip. In many cases this distance appears to be too small for the LEFM to be used properly to formulate some fracture criteria because it is not located on the boundary of the damage zone.

2. The formulation of the new criterion

Let's assume the strain energy density components, T_{D} and T_{V}, and a condition $T_{\mathrm{V}}/T_{\mathrm{D}}\big|_{\max} \to \theta_{\mathrm{pr}}$, cf. [2], to determine the plane of the fracture. This condition means that the fracture appears in the direction where the ratio between the dilatation, T_{V} (corresponding to the decohesion mechanism of the fracture process), and distortional, T_{D} (corresponding to the plastic deformations) components of the strain energy density achieves its maximum value. It means that the crack will propagate in direction where the dissipation energy is the smallest corresponding to decohesion. Now, we can assume that the angle of the crack fracture, θ_{pr}, will follow the minimum value the distortional component of the strain energy density, T_{D}, calculated on the locus $T_{\mathrm{V}}(r,\theta) = T_{\mathrm{V}}^{\mathrm{C}} = \mathrm{const.}$, cf. [4], associated with the maximum fracture toughness, σ_C:

(1) $$\left.\frac{T_{\mathrm{V}}}{T_{\mathrm{D}}}\right|_{\max} \to \theta_{\mathrm{pr}} \quad \text{at} \quad T_{\mathrm{V}} = T_{\mathrm{V}}^{\mathrm{C}} = \mathrm{const.} \quad \to \quad T_{\mathrm{D}}\left(r(T_{\mathrm{V}}^{\mathrm{c}},\theta)\right)\Big|_{\min} \to \theta_{\mathrm{pr}},$$

where

(2) $$r = r_{\mathrm{V}}\big|_{T_{\mathrm{V}}=\mathrm{const}} = \frac{1}{2\pi}\left(\frac{2K_{\mathrm{I}}\cos\frac{\theta}{2} - 2K_{\mathrm{II}}\sin\frac{\theta}{2}}{\sqrt{T_{\mathrm{V}}^{\mathrm{c}}\frac{6E}{(1-2\nu)(1+f)^2}} - T} \right)^2,$$

T-term is the second (constant) term of the series representations of the local stress and displacement, while $f=0$ for the plane stress and $f=\nu$ for the plane strain. The critical value which determines the crack initiation assumes the radius of the decohesion zone along the fracture direction to be derived from Eq. (2) as:

(3) $$r\big|_{\theta=\theta_{\mathrm{pr}}} \geq r_{\mathrm{cr}} \;\;\rightarrow\;\; K_{\mathrm{I}}\cos\frac{\theta}{2} - K_{\mathrm{II}}\sin\frac{\theta}{2} \geq K_{\mathrm{cr}},$$

where r_{cr} corresponds to an uniaxial test.

3. The results

In Table 1 the angles of crack propagation θ_{pr} for various inclinations of the main crack, α, in the uniaxial tension test and various relations between the loading and maximum fracture toughness are shown. It is interesting to point out the difference between the angles of the crack propagation for the same α and different loading values. It results from the effect of the *T*-term included into the solution that simultaneously introduces a correction of the contour *r*, cf. Eq. 2, on which the components of the stress tensor are calculated. In the case of a singular solution when *T* = 0 the relationship σ/σ_C appears to be unimportant for the fracture direction because the components of the stress tensor are always proportional. Accounting for the *T*-term causes, however, that any change of the ratio σ/σ_C affects the resulting angle of the crack propagation and, the stress tensor components are not proportional any more. There is also important to point out that the assumption that the radius defining the decohesion zone is a constant value causes the relationship σ/σ_C to be meaningless.

	α [°]				
σ/σ_C	**15**	**30**	**45**	**60**	**80**
Singular solut.	-85.7	-72.7	-59.6	-45.1	-19.3
0.1	-86.7	-73.7	-59.6	-43.1	-15.5
0.2	-87.7	-74.7	-59.6	-40.6	-13.5
0.4	-89.7	-76.7	-59.6	-37.6	-11.5
0.6	-91.2	-78.7	-59.6	-35.6	-11.0
0.8	-92.7	-80.2	-59.6	-34.1	-10.5
1	-93.7	-81.2	-59.6	-33.1	-10.0

Table 1. The angle of the crack propagation based on the **MK** – criterion.

4. References

[1] G.C Sih (1973). Some basic problems in fracture mechanics and new concepts, *Engng Fracture Mechanics*, **5**, 365–377.

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