DIRECT NUMERICAL COMPUTATION OF THE EFFECTIVE MATERIAL PROPERTIES OF THE MATERIAL WITH RANDOM DISTRIBUTION OF THE MICROCRACKS

G. Mejak
University of Ljubljana, Ljubljana, Slovenia

1. Introduction

In the paper direct numerical computation of the effective material properties of the material with random distribution of the frictionless microcracks is presented. To this end a new numerical method, three level finite element method is introduced. Using it problems with very fine discretization can be solved in real time. This allows computing a large population of effective material properties of materials with random distribution of microcracks. Its statistical quantities then describe probabilistic distribution of effective material properties.

2. Three level finite element method

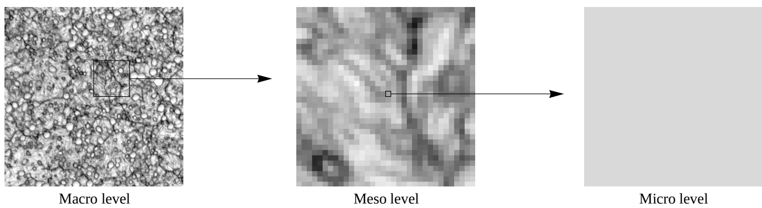

Figure 1. Three level discretization.

The method is described within the context of a linear problem although it works equally fine for nonlinear problems which are solved by iteration of linear subproblems. Figure 1 shows three discretization levels. The macro level is the size of the specimen of the material under consideration and the micro level is the size where the material becomes homogeneous. Between them is a meso level which has already complex structure but is too small for a representative volume. After standard FEM discretization the micro level degrees of freedom are statically condensed to the meso level degrees of freedom which are further condensed to the macro level. Thus at the micro level the basis functions are standard finite element functions, at the meso level the basis functions are linear combinations of the micro level basis functions and at the macro level the basis functions are linear combinations of the meso level basis functions.

Efficiency η of the three level FEM is estimated by the ratio of the number of the floating point operations of the linear solver of the standard FEM and three level FEM. For example, we consider a plane problem which has p^2 micro cells. We group m^2 micro cells into one meso cell and M^2 meso cells into one macro cell which is just the macro structure. Then $p = Mm$. The micro cell is discretized into μ^2 quadrilateral bilinear elements. It turns out that for the optimal number of the meso cells $M = (1 - 1/2\mu)^{2/3}(6 - 1/\mu)p^{1/3}$ the efficiency ratio is of order $\eta = O(p^{2/3})$ for large values of p. For example, for $p = 64$ the three level FEM is 16 times more efficient as the standard FEM. Generalization to multilevel FEM is possible. However, we note that on a single processor computer the optimal number of levels is three.

3. Numerical example

Although a real problem with random distribution of heterogeneities such as in Figure 1 and with random distribution of microcracks can be considered, we restrict ourselves to the plane stress problem for homogeneous isotropic material ($\mu/E = 2/5$) with random distribution of frictionless microcracks, see Figure 2, as this enables comparison with a dilute distribution model [1]. The length of the individual microcracks is $a = 1/64$. Individual microcracks are only in x and y directions but they can be combined into more complex patterns. At the right a bar chart of the tallies of the effective material parameters between their minimal and maximal values for population of 4000 distributions of the microcracks is shown. In particular, the normalized effective Young modulus E_1 varies between 0.857 and 0.902 with the mean 0.881 and standard deviation 0.0061.

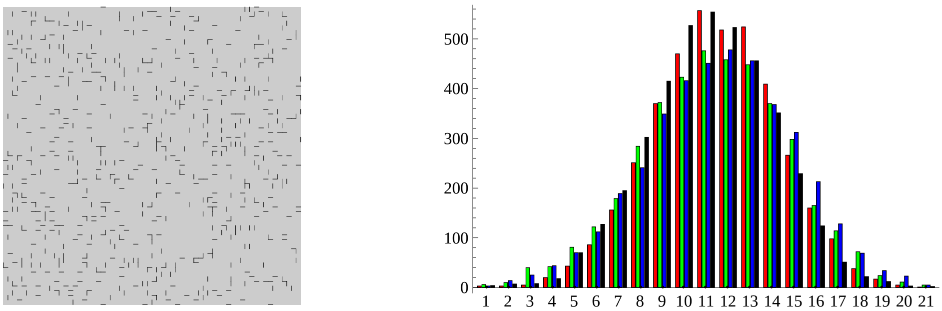

Figure 2. Left: a square domain with random distribution of 640 cracks aligned with the sides of the square. Right: a bar chart of tallies of material parameters E_1(red), E_2(green), μ(blue) and ν_{12}(black).

Comparison with the dilute distribution model is shown in Figure 3. Now all micro cracks are aligned with the y axis. Denoting by $f = Na^2$ the crack density parameter, N is the number of microcracks, the model predicts that for the prescribed unit tensile macrostress in x direction the normalized effective Young and shear moduli are $\bar{E}_1 = (1 + 2\pi f)^{-1}$ and $\bar{\mu} = (1 + 2\pi f\mu/E)^{-1}$. It can be seen that the dilute distribution model is valid up to $f = 0.04$. For the prescribed macrostrain the match is even better. The comparison validates our approach and opens the way to approach many other interesting problems with microcracks.

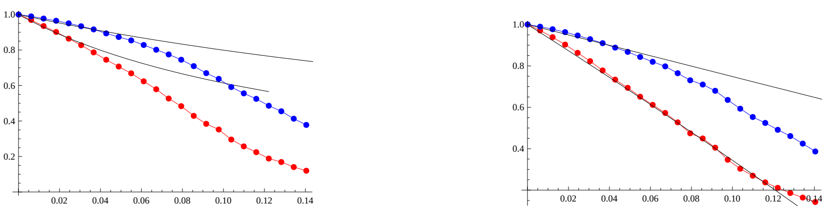

Figure 3. Computed(dotted line) effective moduli and moduli predicted by the dilute model(solid line). Lower/upper curve: normalized Young/shear modulus, prescribed macrostress/macrostrain at right/left.

4. References

[1] M. Hori and S. Nemat-Nasser (1999). *Micromechanics: Overall Properties of Heterogeneous Materials*, 2nd ed. North-Holland, Amsterdam, 73-206.