LONG-TERM MICRODAMAGING OF COMPOSITES WITH TRANSVERSALLY-ISOTROPIC COMPONENTS FOR LIMITED FUNCTION OF DURABILITY

L. Nazarenko[1], L. Khoroshun[2], W.H. Müller[1] and R. Wille[1]
[1] LKM TU Berlin, Berlin, Germany
[2] S.P. Timoshenko Institute of Mechanics of NAS of Ukraine, Kiev, Ukraine

1. Introduction

One of the important problems of mechanics of composites is the investigation of stresses under elevated loads. Such a loading is associated with the accumulation of damage which finally leads to the fracture of the material. A survey of theoretical papers dealing with microcracked elastic materials is presented in [1]. A study of materials weakened by periodically or randomly distributed microcracks was performed in [2] by using homogenization methods. In [3] a stochastic model of short-term microdamages of the material was proposed and then applied to the case of anisotropic composites [4]. In the present paper a stochastic model is developed for investigation of long-term microdamages of discrete-fibers composites with transversally-isotropic components.

2. Mechanical model. General relation

We consider a representative volume V of a composite. Under homogeneous loading the stresses and strains appearing in the representative volume will form statistically homogeneous random fields satisfying the ergodicity condition and we can replace the operation of averaging over the representative volume by the operation of averaging over an ensemble of realizations. Then the macroscopic stresses $\langle \sigma_{ij} \rangle$ and strains $\langle \varepsilon_{kl} \rangle$ of such a material will be related by Hooke's law:

(1) $$\langle \sigma_{ij} \rangle = \lambda^*_{ijkl} \langle \varepsilon_{kl} \rangle , \quad (i,\ j, k, l = 1, 2, 3\,).$$

Here, λ^*_{ijkl} is the tensor of effective elastic constants, which can be determined by the method of conditional moments [5]. The effective elastic moduli of the composite are functions depending on the elastic moduli of the components $\lambda^{[1]}_{ijkl}$, $\lambda^{[2]}_{ijkl}$, the volume contents of the inclusions c_1, the porosity of the components p_1, p_2, and the shape of the inclusions s

(2) $$\lambda^*_{ijkl} = \lambda^*_{ijkl}\left(\lambda^{[1]}_{mnpr}, \lambda^{[2]}_{mnpr}, c_1, p_1, p_2, s\right), \quad s = s_1/s_2\,, \quad (m, n,\ p, r = 1, 2, 3\,),$$

where s_1, s_2 are semi-axes of spheroids. Knowing the effective elastic moduli and the macrostresses or macrostrains of such a composite it is possible to calculate stresses $\langle \sigma^r_{ij} \rangle$ averaged over the skeletons of components using the relations obtained in [5]. As a the fracture criterion we consider generelized Huber-von Mises criterion for a transversally-isotropic material

(3) $$J^r_\sigma = \sqrt{\sigma^{r'}_{ij}\sigma^{r'}_{ij} + a_{1r}\left(\sigma^{r\,'}_{33}\right)^2 + a_{2r}\left(\sigma^{r\,'}_{11} + \sigma^{r\,'}_{22}\right)\sigma^{r\,'}_{33} + a_{3r}\left(\left(\sigma^{r\,'}_{13}\right)^2 + \left(\sigma^{r\,'}_{23}\right)^2\right)} = k_r\,, \quad (r = 1, 2)\,,$$

where a_{1r}, a_{2r}, a_{3r} are dimensionless constants, $\sigma^{r'}_{ij}$ is the deviator of the stresses averaged over an undamaged part of the material of the r-th component, and k_r is the limiting value of the material strength, which is a random function of coordinates. If the invariant J^r_σ does not achieve its limiting value k_r in some microvolume, then according to the long-term failure criterion, failure will occur after some time τ^r_k,which depends on how close J^r_σ is to k_r.This dependence can be represented by

(4) $$\tau_k^r = \varphi\,(J_\sigma^r, k_r)\,.$$

The one-point distribution function $F(k_r)$ of the ultimate strength k_r in a microvolume of the undamaged part of the material can be approximated by a Weibull distribution function:

(5) $$F(k_r) = \begin{cases} 0 & , k_r < k_{0r} \\ 1 - \exp\left(-m_r (k_r - k_{0r})^{\alpha_r}\right), & k_r \ge k_{0r} \end{cases},$$

k_{0r} is the minimum value of the ultimate microstrength of the material of the r-th component, m_r and α_r are constants determined by fitting experimental microstrength scatter curves. If the stresses $\langle \sigma_{ij}^r \rangle$ are known, then the distribution function $F(k_r)$ determines the relative content of the destroyed microvolumes in the undamaged part of the material of the r-th component. If the destroyed microvolumes are modeled by pores, it is possible to write down the balance porosity equation

(6) $$p_r = p_{0r} + (1 - p_{0r}) F(k_r)\,,$$

where p_{0r} denotes the initial porosity of the material of the r-th component. If the stresses or strains act for some time t, then, according to (4), microvolumes with the following values of k_r will fail in this time τ_k^r, which can be represent by a fractional power law

(7) $$t \ge \tau_k^r = \varphi\,(J_\sigma^r, k_r)\,,\ \ \varphi\,(J_\sigma^r, k_r) = \tau_{0r}\left((k_r - J_\sigma^r)/(J_\sigma^r - \gamma_r k_r)\right)^{n_r},\ \ (\gamma_r k_r \le J_\sigma^r \le k_r,\ \gamma_r < 1)\,,$$

where τ_{0r}, n_r and γ_r are determined by fitting experimental long-term strength curves.

Transforming (7) we arrive at the inequality

(8) $$k_r \le J_\sigma^r \psi\,(\bar{t}_r)\,,\quad \psi\,(\bar{t}_r) = \left(1 + \bar{t}_r^{1/n_r}\right)/\left(1 + \gamma_r \bar{t}_r^{1/n_r}\right),\quad (\bar{t}_r = t/\tau_{0r})\,.$$

In this case the function $F\,[J_\sigma^r \psi\,(\bar{t}_r)]$ defines the relative fraction of destroyed microvolumes in the part of the material which is undamaged prior to loading at the time $\bar{t}_r$ .Then for given macrostresses or macrodeformations, the equation of balance of destroyed microvolumes or porosity under long-term damage can be represented by

(9) $$p_r = p_{0r} + (1 - p_{0r}) F\,[J_\sigma^r \psi\,(\bar{t}_r)]\,.$$

On the basis of the above approach we investigate the stress-strain state of transversally-isotropic composite material under matrix microdamages.

6. References

[1] M. Kachanov. Effective elastic properties of cracked solids: critical review of some basic concepts. *Appl. Mech. Rev.* 1992. 45(8): 304-335, (1992).

[2] B. Gambin and J.J. Telega. Effective properties of elastic solids with randomly distributed microcracks. *Mechanics Research Communications.* 27(6): 697-706, (2000).

[3] L.P. Khoroshun. Principles of the micromechanics of material damage. 1. Short-term damage. *International Applied Mechanics.* 34(4): 1035 – 1041, (1999).

[4] L.P. Khoroshun and L.V. Nazarenko. Deformation and microdamaging of discretely-fibrous composites with transversally-isotropic components. *International Applied Mechanics.* 39(6): 696-703, (2003).

[5] L.P. Khoroshun, B.P. Maslov, E.N. Shikula, and L.V. Nazarenko. *Statistical mechanic and effective properties of materials [in Russian]*. Naukova Dumka, Kiev, (1993).