HYPERBOLIC HEAT CONDUCTION WITH FUZZY PARAMETERS

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We consider a heat conductor for which the heat transfer model [2, 6] with the semi-empirical temperature is used. In real-life problems material coefficients and parameters used in mathematical modelling can be just vague. For instance at 15 K for NaF crystals, where the first and second sound waves have been observed, known measurements of material parameters (cf. [4] and the literature cited there) show large volatilities of observed quantities. Hence to model this one can go to an extended model in which some material coefficients and data are fuzzy. In this paper we assume them in the form of ordered fuzzy numbers appearing in the new model introduced by the second author (W.K.) with two coworkers in 2002 in [7, 9, 10, 11, 12]. Our aim is to investigate waves described by the hyperbolic model of heat conduction [1, 3] to give solutions of the coupled system of fuzzy differential equations. Some defuzzyfications operators [7, 9] are proposed in order to get crisp numerical results.

Fuzzy numbers as a particular case of fuzzy sets were introduced by Zadeh in 1965 [14]. They have entered the applications area such as control theory or economy. In most applications the so called (L, R)-numbers with two shape functions L and R, proposed by Dubois and Prade in 1978 as a restricted class of membership functions, are commonly used together with triangular and trapezoidal fuzzy numbers [5]. Arithmetic operations on fuzzy numbers have been developed with both the Zadeh's extension principle [15, 16] and the α-cut with interval arithmetic methods [5].

The concept of convex fuzzy numbers has been introduced by Nguyen [13] in order to improve calculation and implementation properties of fuzzy numbers. However, the results of multipl operations on the convex fuzzy numbers are leading to the large growth of the fuzziness, and depend on the order of operations since the distributive law, which involves the interaction of addition and multiplication, does not hold there.

Recently introduced and developed main concepts of the space of ordered fuzzy numbers (OFN) by the second author and his coworkers solved several drawbacks of both (L, R) as well as the convex fuzzy numbers. In this approach the concept of membership functions has been weakened by requiring a mere *membership relation*.

By an ordered fuzzy number A we mean an ordered pair (f, g) of functions such that $f, g : [0, 1] \to \mathbf{R}$ are continuous. The new model of fuzzy numbers has a lot of useful mathematical properties, in the particular the arithmetic of ordered fuzzy numbers is similar to that known for real numbers. Moreover, we are getting rid of the main problem in a classical fuzzy numbers - the unbounded increase of inaccuracies with next calculations.

Now, we apply the concept to the heat conduction problem mentioned above. Following [2, 6] a scalar internal state variable β is introduced. For β a kinetic equation is proposed. Then the heat flux q is given in terms of gradients of the absolute temperature θ and β. Taking gradients of the both sides of the kinetic equation one obtains an evolution equation for $g = \nabla\beta$

(1) $$g_{,t} = \gamma\theta_{,x} - \gamma g\,.$$

Throughout this present paper we restrict ourselves to the simplest case where β itself does not appear explicitely in that equation, i.e., we postulate a linear kinetic equation.

In the previous paper [4] we assumed that θ is small enough so the heat capacity c_v and the conductivity κ have been regared as constants (measured at some reference temperature θ_0). In the present paper we assume that they are ordered fuzzy numbers.

In the previous paper we have discussed the case of fuzzy ordinary differential equations (FODE) [8]. Now the governing system becomes a system of fuzzy partial differenatial equations (FPDE) for which numerical calculations are performed. The results are discussed from the point of view of their physical applicablity. Some defuzzyfication operators [9] are proposed, which map fuzzy numbers into reals in order to give an appropriate physical interpetation for the results obtained.

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