INDIRECT TREFFTZ SOLUTIONS FOR PLANE PIEZOELECTRICITY BY STROH FORMALISM AND COLLOCATION TECHNIQUE

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1. Introduction

The coupled field analysis of piezoelectric materials requires solution of continuum mechanics and continnum electrodynamics equations [1,3]. Practically, the process of solving the boundary – value problems is realized using the numerical methods. The most popular are: the finite element method (FEM) and the boundary element method (BEM). The FEM requires the whole region discretization; in the BEM, in many cases, only the boundary is discretized. Hence, in these both methods the mesh of finite and boundary elements is required. Recently, there can be noticed a development of the meshless methods, which do not need the time-consuming mesh generation process. One of the meshless methods is the indirect Trefftz collocation method [1,4,6,7].

In the Trefftz method, the solution of the boundary-value problem is approximated by the series of the T-complete functions [4,7]. These functions satisfy the system of the governing equations, i.e. the homogenous system of the elliptic differential equations of the linear piezoelectricity. The piezoelectric materials are modelled as: homogenous, anisotropic linear–elastic and linear – dielectric [3]. Even for the transversal isotropic ceramic piezoelectric material, the form of the partial differential operator makes the determination of the T-complete functions quite complicated. The quite similar problem exists, when the fundamental solution is being determined. The Stroh formalism is a powerful and elegant analytic technique for anisotropic elasticity, which is expanded to the linear piezoelectricity in this case. The Stroh formalism allows to obtain both the fundamental solution and the T-complete functions [3,5].

2. The Stroh formalism and the T-complete functions

Since piezoelectric materials are anisotropic, the determination of the fundamental solutions and the T-complete functions are rather complicated, even for the transversal isotropic model of the material. In the Stroh formalism, it is assumed, that the field of the generalized displacements (the mechanical displacements and the electric potential) has a form of product of the unknown complex vector and the analytic complex function [3]. Then, the formalism requires the solution of the special eigenvalue problem with respect to the material constants of the piezoelectric material. The general solution, which is the base of the T-complete functions set, can be expressed by using the eigenvalues, eigenvectors (of the special eigenvalue problem) and arbitrary complex vector and arbitrary analytic complex function. The orientation of the polarization direction is also taken into account using this formalism. The eigenvalues and eigenvectors, related to these constants, are specially transformed according to the polarization direction.

3. The collocation technique

When the set of the T-complete functions is determined, the solution of the boundary-value problem can be approximated by the superposition of these functions [1,4,7]. The superposition of the T-complete functions satisfies the governing equations, but does not satisfy the mechanical and electric boundary conditions. This problem leads to the minimization problem of the boundary residuals [1]. The unknowns are the coefficients of the superposition of the T-complete functions, which describe the wanted mechanical and electric fields. The collocation method assumes that the residuals vanish at the boundary points. The resulting system of algebraic equations is usually solved by using the least square method [1,6].

4. Least square method and regularization technique

The indirect Trefftz collocation approach usually requires the solution of the overdetermined system of equations, which determines the unknown coefficients of the superposition of the T-complete functions [1,6]. The matrix of the system of equations is usually nearly singular and ill-conditioned [4]. For a system of equations with these properties, a singular value decomposition (SVD) solver is one of the most popular solution. The SVD allows to regularize the solution with the minimal norm [2]. In numerical computations, the nearly singular and the ill-conditioned matrix has no rank exactly equal to the mathematical rank. The numerical rank is smaller than the mathematical rank, because of small nonzero singular values. When the matrix has very small nonzero singular values, then a norm of the solution is very large. To remove this effect, the least singular values must be neglected, so the new solution is called the truncated singular value decomposition solution [2]. The truncation number is a regularization parameter in this method. In the present work the L – curve method is used for to determine the optimal truncation number.

5. Numerical examples

The indirect Trefftz collocation method program for plane boundary – value problem of linear piezoelectricity is developed. Results for some simple boundary – value problems are compared to analytical and BEM solutions. The numerical examples demonstrate a good agreement of the Trefftz method solutions with the exact and BEM solutions.

6. Conclusions

The point-collocation technique has the simplest algorithm among the others Trefftz methods and is therefore the most computationally – efficient approach [4]. This is also truly meshless boundary method and no integration is carried out in this technique. The Trefftz method uses regular functions, this is an important advantage when the indirect Trefftz method is compared with other boundary methods [7]. The necessity of regularization is some kind of drawback, but the SVD method ensures accurate and stable results.

7. References

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