CONVERGENCE BEHAVIOUR FOR KPT FINITE ELEMENTS

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1. Abstract

Triangular elements are more versatile than rectangular elements; they can be used for the analysis of plates having various boundary shapes. The use of triangular finite elements for the solution of plate bending under Kirchhoff assumptions is considered. An investigation about the convergence behaviour of two *KPT* elements representing the nature of conformal and non-conformal elements performed.

The KPT elements chosen to show the influence of increase the nodal field variables and the interpolation function order to obtain the continuity requirement for C^1 problems. The numerical examples performed in this work to clarify the advantage of using higher order element in terms of the rate of convergence compared with the lower order element.

2. Introduction

For the well known plate bending governing equation we can apply the first step of the *Galerkin* formulation approach by minimizing the residual *R*.

(1) $$R = \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p_o$$

(2) $$\int_A R \, . Ni \; .dA = 0$$

After substituting element trial solution, the general expression can be written in a matrix form as:
$[K]\{w\} = \{F\}$ Where

(3) $$K_{ij} = \int_A -D\left[\frac{\partial^2 N_i}{\partial x^2}\frac{\partial^2 N_j}{\partial x^2} + v\frac{\partial^2 N_i}{\partial x^2}\frac{\partial^2 N_j}{\partial y^2} + \frac{\partial^2 N_i}{\partial y^2}\frac{\partial^2 N_j}{\partial y^2} + v\frac{\partial^2 N_i}{\partial y^2}\frac{\partial^2 N_j}{\partial x^2} + 2(1-v)\frac{\partial^2 N_i}{\partial x \partial y}\frac{\partial^2 N_j}{\partial x \partial y}\right]dA$$

$$F_i = -\oint_S N_i Q_n ds + \oint_S M_n \frac{\partial N_i}{\partial n} ds - \oint_S N_i \frac{\partial M_{ns}}{\partial s} ds + [N_i M_{ns}]_s - \int_A N_i pdA$$

The parent element will have its own coordinate system (ξ,η), to convert the upper integral forms to this coordinate system we write:

(4) $$dA = Jac.d\xi.d\eta$$ Where *Jac* is the Jacobian determinant.

As we notice the plate bending element still capable to represent the element field variables with satisfactory amount of accuracy even for the elements of non-conforming type. This element is one of the first elements where used for plate bending problems and it shows a good convergence results for three nodes nine DOF KPT element.

Note that no complete polynomial available to represent nine degrees of freedom, the complete cubic polynomial will have ten unknowns (P_3). One of the choices we have is that one of the terms $\xi^2\eta$ or $\xi\eta^2$ is omitted. In this case we use the following polynomial for our displacement interpolation weighing functions.

(5) $$N = a_1 + a_2\xi + a_3\eta + a_4\xi^2 + a_5\xi\eta + a_6\eta^2 + a_7\xi^3 + a_8\xi^2\eta + a_9\eta^3$$

To obtain the nine unknowns (a_i, $i=1, 2\ldots 9$) we need to define the condition of the nodal weighing values (N_i).

Three nodes eighteen DOF KPT element (six degrees of freedom per node) is one of the conformal plate bending elements. The compatibility requirements for *C1* problems require the above six field variables to be continuous at the corner nodes. Here we meet the same complexity that we have seen in the nine DOF triangle element, where also there are no complete polynomials available to represent eighteen DOF. The complete quadric polynomial (P_4) has only fifteen terms.

The following suggested polynomial is complete up to terms of fourth order and contains three terms of fifth order. The last three terms are chosen to force the normal derivative on each side to be cubic in ξ and η, on other hand the parabolic variation of the normal slope is not uniquely defined by the two end nodal values and hence resulted in the non-conformity [4].

$$
\begin{aligned}
N = {} & a_1 + a_2\xi + a_3\eta + a_4\xi^2 + a_5\xi\eta + a_6\eta^2 + a_7\xi^3 + a_8\xi^2\eta + a_9\xi\eta^2 + a_{10}\eta^3 \\
& + a_{11}\xi^4 + a_{12}\xi^3\eta + a_{13}\xi^2\eta^2 + a_{14}\xi\eta^3 + a_{15}\eta^4 + a_{16}(\xi^5 - 5\xi^3\eta^2) \\
& + a_{17}(\xi^2\eta^3 - \xi^3\eta^2) + a_{18}(\eta^5 - 5\xi^2\eta^3)
\end{aligned} \tag{6}
$$

3. Numerical Results

The numerical results for the simply supported and clamped square plates where obtained for deferent number of elements, the plate geometry, physical properties and uniformly distributed load are chosen to be within the *Kirchhoff* assumptions. The plates are *1m x 1m x 13mm, E=200GPa and v=0.3* under uniformly distributed load of *0.1 MPa*. The maximum deflection results of the first element insures the convergence as the number of elements increasing in the simply supported and clamped plates, this is also true for the second element.

The figure shows that in terms of both total degrees of freedom and number of elements the higher order element presents better performance and convergence rate for the simply supported plate and for other cases.

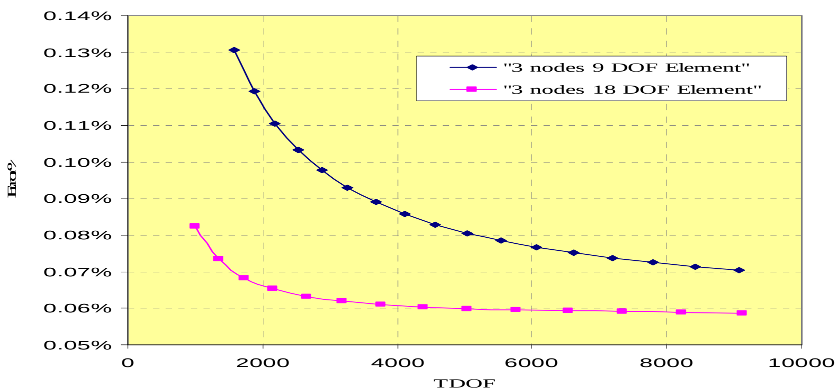

Figure 1. Simply Supported Plate Results (TDOF vs Error %)

4. References

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