

AGGREGATION MULTILEVEL ITERATIVE SOLVER BASED ON SPARSE MATRICES TECHNIQUE

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1. Introduction

An efficient iterative method for solution of large-scale linear equation sets with sparse positive definite matrices is considered. These problems appear in static analysis of finite element problems of structural and solid mechanics.

Realistic design models often produce ill-conditioned large-scale problems. This fact essentially restricts the application area of iterative methods, especially to problems of structural mechanics.

The main idea of this research is to develop an iterative method stable against ill-conditioning, which allows us to compete with sparse direct finite element solvers during the calculation stage of the analysis. This investigation continues the previous researches of author [3 – 6]. A combination of two powerful ideas – the aggregation multilevel preconditioning for preconditioned conjugate gradient method AMIS [2 - 5] and the sparse incomplete Cholesky factorization preconditioning [6] is the basis of the present research.

2. Aggregation multilevel preconditioning for preconditioned conjugate gradient method.

A lot of problems of structural mechanics are ill-conditioned; the respective models have bar substructures and specific finite elements (rigid links, compatible nodes and so on). This fact forces us to reject the multi-grid approach and to prefer to use the preconditioned conjugate gradient (PCG) method with aggregation multilevel preconditioning. This approach combines advantages of both PCG and multilevel methods and allows us to create an iterative approach stable against ill-conditioning. The aggregation approach [2-5] has a clear mechanical interpretation, creates a coarse level model due to imposed local rigid links, allows us to analyze bar structures, continuous structures and combined ones. It also takes into account special finite elements. The application of element-by-element technique to creation of the coarse level stiffness matrix, a restriction-prolongation procedure [3-5], and implementation of the sparse direct solver to keep a relatively large size of coarse level model (till 100 000 – 200 000 equations) [5] allows us to improve the prediction of slow-convergent low modes and accelerate the convergence.

The next important issue is a correction of the interpolated solution vector. The need for it arises when extending the coarse level (aggregated model) solution onto the fine level (finite element model). In previous versions of the AMIS solver, a few steps of the inner iteration procedure (a preconditioned quickest descent method) were applied to damp quickly oscillating residuals. The symmetrical Gauss-Seidel preconditioning as well as incomplete Cholesky factorization by position ICCG0 one were implemented. But in some practical problems a lock of convergence occurs: the coarse level model leads to a fast reduction of the relative norm of residual vector, $\text{err} = \|\mathbf{r}_k\|_2 / \|\mathbf{b}\|_2$, where \mathbf{b} is a load vector, to about 10^{-2} , and then the convergence still slows down.

In the present research we apply a sparse incomplete Cholesky factorization preconditioning [6] to improve the correction abilities of the inner smoothing iterations. The following results

demonstrate an essential improvement of robustness of AMIS_SICPS (an aggregation multilevel iterative solver with sparse incomplete Cholesky preconditioning during smoothing) method.

3. Numerical example

A large-scale design model (multi-storey building) comprises 1 956 634 equations (tab. 1) and has 3 load cases. The efficiency of several solvers is compared. Designations: BSD MFM is a block sparse direct multi-frontal method [6], ICCG0 is an incomplete Cholesky conjugate gradient by position solver, SICCG is a sparse incomplete Cholesky conjugate gradient by value solver [6], AMIS_SICPS is the approach suggested here. The convergence tolerance is $\text{tol} = 10^{-4}$. A PC Pentium IV (RAM 2.0 GB, CPU 2.40 GHz) has been used.

Method	Computation time	Memory storage, MB	Number of iterations
BSD MFM	2 h 20 min	7.2 GB – size of factored matrix	-
ICCG0	3 h 12 min	0.5 GB	8470 / 7944 / 7368
SICCG	27 min 25 s	1.5 GB	562 / 520 / 509
AMIS SICPS	13 min 20 s	1.76 GB	31 / 31 / 30

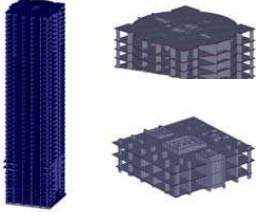


Table 1. Comparison between methods

An essential part of the solution time (about 40%) BSD MFM method is spent for slow input/output disk operations because the size of the factored matrix exceeds the core memory capability significantly. The serial ICCG0 method demonstrates a slow convergence due to ill-conditioning of the design model. The SICCG method has a good stable convergence for this problem (the rejection parameter $\psi = 5 \times 10^{-9}$ and the post-rejection one $\psi = 10^{-6}$). But the best results are demonstrated by the AMIS_SICPS method (the same values for rejection and post-rejection parameters are used).

6. References

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