MODELLING OF WAVE PROPAGATION IN SPATIAL FRAME ELEMENTS – NUMERICAL SIMULATIONS AND EXPERIMENTAL WORKS

J. Chróścielewski, M. Rucka, K. Wilde and W. Witkowski
Gdansk University of Technology, Gdańsk, Poland

1. Introduction

Wave propagation in structures is a subject of intensive investigation. One of the possibilities of the wave propagation modelling is the Spectral Element Method (SEM), developed by Patera [1] in 1984 in the context of fluid dynamics. The main idea of the SEM is use of one high-order polynomial for each domain [2].

In this study a spectral frame N–node finite element appropriate for analysis of wave propagation phenomena in engineering structures build from spatial frames is presented. The element is elaborated in linear range. Each node of the element is endowed with six engineering dofs. The kinematical assumptions of Timoshenko beam theory are employed. Associated with the formulation of the element, is the temporal integration scheme. Special emphasis is put on the accuracy and efficiency of the time integration to ensure reasonable simulation times. The algorithm uses accelerations as the primary variables and the mass matrix of an element is integrated using Lobatto quadrature rule. Consequently, it can be recast in a form of the pseudo diagonal matrix and substantial efficiency in computation times can be gained.

2. Formulation

The time integration scheme does not use the stiffness matrix. On the local element level, under the assumptions of classic Timoshenko beam theory, the element load vector $\mathbf{r}$ and mass matrix $\mathbf{M}$ are derived. The damping matrix $\mathbf{C}$ is formulated under the hypothesis of proportional damping. Then, it is possible to find the element inertia force vector $\mathbf{b}$ and damping force vector $\mathbf{c}$. While transforming the above matrices and vectors to the global frame coordinates, the transformation of the internal nodes is omitted, leaving them in the local frame coordinates. This is justified by the fact the wave propagation in local frame (along element axis) is of the primary interest. By writing the dynamical equilibrium condition, the following equation is obtained with respect to increment of acceleration $\delta\ddot{\mathbf{q}}$

$$[\mathbf{M}+\Delta t\gamma\mathbf{C}]\delta\ddot{\mathbf{q}}=\mathbf{p}_{n+1}-\mathbf{b}_{n+1}^{(i)}-\mathbf{c}_{n+1}^{(i)}-\mathbf{r}(\mathbf{q}_{n+1}^{(i)}+(\Delta t)^2\beta\delta\ddot{\mathbf{q}}),\quad \beta=1/4,\quad \gamma=1/2 \tag{1}$$

where $\mathbf{p}$ is the element external load vector, n denotes the time step, i is the label for iteration and Δt is the time step. Simple iteration method is then used to obtain correction of $\delta\ddot{\mathbf{q}}$

$$\delta\ddot{\mathbf{q}}=[\mathbf{M}+\Delta t\gamma\mathbf{C}]^{-1}\left(\mathbf{p}_{n+1}-\mathbf{b}_{n+1}^{(i)}-\mathbf{c}_{n+1}^{(i)}-\mathbf{r}(\mathbf{q}_{n+1}^{(i)})\right). \tag{2}$$

Obviously, if $\mathbf{M}$ and $\mathbf{C}$ are pseudo-diagonal the time integration scheme becomes efficient. The iterative process **Błąd! Nie można odnaleźć źródła odwołania.** is terminated when equilibrium condition

$$\mathbf{j}_{n+1}^{(i+1)}=\mathbf{p}_{n+1}-\mathbf{b}_{n+1}^{(i+1)}-\mathbf{c}_{n+1}^{(i+1)}-\mathbf{r}(\mathbf{q}_{n+1}^{(i+1)})\quad\rightarrow\quad\mathbf{0}. \tag{3}$$

is satisfied in some sense.

3. Experiments and numerical simulations for rod

The investigations were carried out for a steel rod [3] of the length 1 m, height 8 mm and width 8 mm (Fig. 1). The boundary condition was assumed as pinned-pinned. The experimentally determined material properties were found to be: Young's modulus E = 195 GPa and mass density ρ = 7563 kg/m^3. The rod was subjected to a dynamic load applied in the half of the rod length. The response was recorded at the same point as the load. The measurements were made using piezoelectric plate transducers Noliac CMAP11. The excitation signal was chosen as sine wave of frequency 40 kHz modulated by the Hanning window.

The spectral element with 101 nodes was applied for modelling of the rod. The time step was assumed as 10^{-8} s. The minimum number of nodes for proper response modelling is 101 (above 12 nodes per wavelength). The comparison with the experimental results is given in Fig. 1. It is noted, that both numerical simulations are in good agreement with the experimental data.

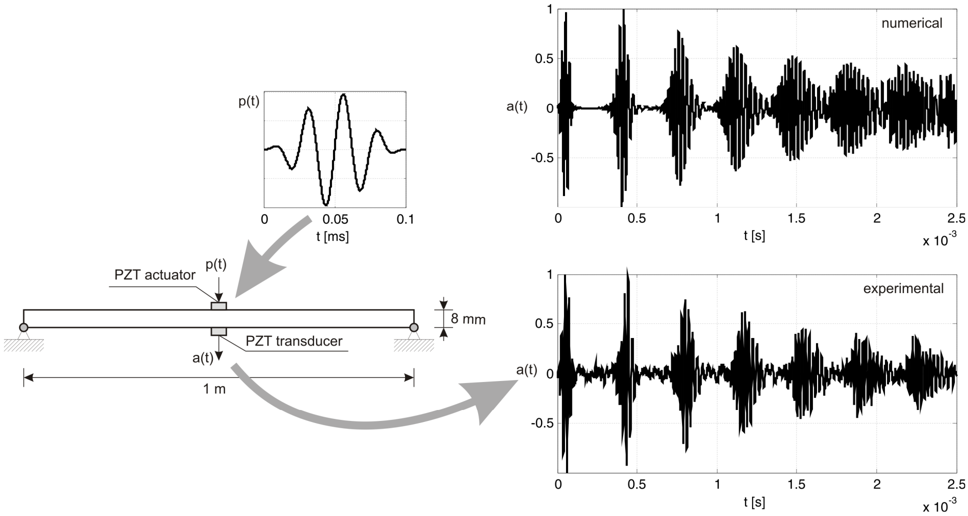

Figure 1. Steel rod, comparison of experimental and numerical solutions

4. Conclusions

The study on modelling of wave propagation in frame elements leads to the following conclusions and suggestions:

a) The integration of the equations of wave propagation can be efficiently conducted due to the pseudo-diagonal mass matrix

b) Application of GLL nodes in both natural and geometric coordinates requires 12 nodes per wavelength.

5. References

[1] T. Patera (1984). *A spectral element method for fluid dynamics: laminar flow in a channel expansion*. Journal of Computational Physics **54,** 468-488.

[2] C. Pozrikidis (2005) *Introduction to Finite and Spectral Element Methods using MATLAB®*. Chapman & Hall/CRC.

[3] M. Rucka, W. Witkowski, K. Wilde, J. Chróścielewski (2007) Wave propagation in steel truss girder for structural health monitoring. III ECCOMAS Thematic Conference on Smart