RAYLEIGH SCATTERING OF ULTRASOUNDS IN CANCELLOUS BONE

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1. Abstract

The paper is devoted to long wave propagation in composite material which is used as a model of cancellous bone. First, homogenized (macroscopic) properties of composite are calculated taking into account the multilevel structure of cortical bone. Next, the macroscopic properties of cancellous bone are calculated treated as two-phase composite built by trabeculae and marrow. Then, we deal with higher order terms in asymptotic expansion method of homogenization theory to take into account effects due to finite characteristic dimension of heterogeneities. Hence the scattering of waves which are long as compared to the dimension is determined.

2. Effective elastic properties of cancellous bone material

The mathematical homogenization of periodic media as well as stochastic media is based on assumption that an inhomogeneous medium behaves as a homogeneous one provided that macroscopic size, L, is infinitely large as compared to size l of its heterogeneities. The macroscopic behavior is described by effective (macroscopic) properties of a “homogenized” material, which are obtained in the case when the small dimensionless parameter $\varepsilon = l/L$ tends to zero.

The first step is to find the effective macroscopic elastic moduli of compact (cortical) bone by using reiterated homogenization method elaborated in [1]. Biological material like compact bone is characterized by several structural levels, cf. [2-4]. In this case three structural levels are of primary importance. At the lowest level, the lamellar structure is considered: collagen fibers are embedded in hydroxyapatite crystals. In a single lamella, all the collagen fibers have the same orientation but the orientation of these fibers can differ between two adjacent lamellae. The second level corresponds to a single osteon and of a part of the interstitial system, an osteon being a set of concentric lamellae, which surround the Haversian canal, cf. Fig. 1.

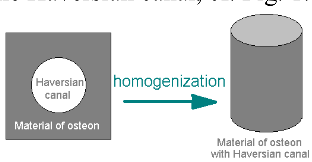

Fig. 1 . The method of calculation of effective material constants of osteon with Haversian canal.

At the highest level, compact bone is examined. The compact bone consists of large number of osteons embedded in the interstitial system. The osteons are packed tightly together, mutually parallel and oriented in the direction of the long axis of the bone, cf. Fig. 2.

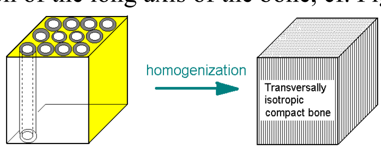

Fig. 2. Third step of calculations.

Consequently, three successive steps allow us to derive the final form of the *macroscopic elastic moduli of compact bone*. These moduli of compact bone are taken for moduli of trabecular component of cancellous bone. At the end the *effective properties of cancellous bone* are obtained as a result of homogenization applied to the two phase structure of the bone composed of fluid (marrow) and solid (compact bone) components.

3. Acoustic waves in cancellous bone

This study deals with long wave phenomena i.e. we assume that the wavelengths are large as compared with the characteristic size of pore dimension in trabecular structure of bone, but not infinitely large. This situation corresponds to wavelengths which are about 10-100 times greater than heterogeneities, i.e. 100< ε <10. It is typical for ultrasounds excitations used in extended bone diagnostic techniques, cf. [5]. Thus, classical homogenization method must be enlarged to take into account higher order effects due to the size of inhomogeneities. It is done by using the formal asymptotic expansion method and preserving the terms proportional to successive powers of small parameter ε. Numerical results show, particularly, explicit dependence of velocities and polarizations on geometrical characteristics of trabecular structure, i.e. direction of anisotropy and volume fraction of marrow, cf. Fig. 3.

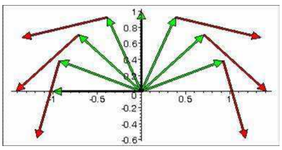

Fig. 3. Polarization vectors – red, wave vectors – green of the wave propagating in the direction along the length of the bone.

3. Results

All scattering effects are analyzed and polarization, dispersion and attenuation coefficients are visibly related to microstructure information about trabecula and marrow distributions. An example of layered structure being of great practical importance for ultrasounds diagnostic is studied in detail.

Aknowledgements

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4. References

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