MULTIDIMENSIONAL LOTKA - VOLTERRA SYSTEMS FOR CARCINOGENESIS MUTATIONS

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Abstract

In the paper $(n+1)$ dimensional models describing carcinogenesis mutations are studied. The models are formulated on the basis of the Lotka–Volterra systems (food–chains and competition systems) with linear diffusion. We study the properties of the systems without diffusion (ODE systems), and with the Neumann boundary conditions as well as the Dirichlet ones. It occurs that the behaviour of solutions to the systems without diffusion and with the Neumann boundary conditions is similar, i.e. does not depend on diffusion coefficients, but strongly depends on the type of model. On the other hand, in the case of the Dirichlet boundary conditions this behaviour is related to the magnitude of diffusion coefficients. For sufficiently large diffusion coefficients it is similar for every model, i.e. the trivial solution which is unstable for zero diffusion gains stability.

1. The models

Depending on the environmental conditions we study the folowing systems of equations (compare [1] for detailed explanation):

$$\begin{cases} \frac{\partial y_0}{\partial t} &= d_0\Delta y_0 + a_0y_0(1-y_0) - \mu_1y_0y_1 \\ \frac{\partial y_i}{\partial t} &= d_i\Delta y_i + a_iy_i(1-y_i) + \eta_iy_iy_{i-1} - \mu_{i+1}y_iy_{i+1},\ i=1,\dots,n-1\ , \\ \frac{\partial y_n}{\partial t} &= d_n\Delta y_n + y_n + \eta_ny_ny_{n-1} \end{cases} \tag{1}$$

$$\begin{cases} \frac{\partial y_0}{\partial t} &= d_0\Delta y_0 + a_0y_0(1-y_0) - \mu_1y_0y_1 \\ \frac{\partial y_i}{\partial t} &= d_i\Delta y_i + a_iy_i(1-y_i) + \eta_iy_iy_{i-1} - \mu_{i+1}y_iy_{i+1},\ i=1,\dots,n-1\ , \\ \frac{\partial y_n}{\partial t} &= d_n\Delta y_n + y_n - \eta_ny_ny_{n-1} \end{cases} \tag{2}$$

$$\begin{cases} \frac{\partial y_0}{\partial t} &= d_0\Delta y_0 + a_0y_0(1-y_0) - \mu_1y_0y_1 \\ \frac{\partial y_i}{\partial t} &= d_i\Delta y_i + a_iy_i(1-y_i) + \eta_iy_iy_{i-1} - \mu_{i+1}y_iy_{i+1},\ i=1,\dots,n-1\ , \\ \frac{\partial y_n}{\partial t} &= d_n\Delta y_n - y_n + \eta_ny_ny_{n-1} \end{cases} \tag{3}$$

with non-negative coefficients and non-negative initial functions $y_i(0,\omega) \geq 0$, y_i sufficiently smooth and $\omega \in \mathbf{\Omega} \subset \mathbb{R}$, $\mathbf{\Omega}$ is the open interval in $\mathbb{R}$ (for simplicity), or $\mathbf{\Omega}$ is open, convex, with smooth boundary bd $\mathbf{\Omega}$ in three-variable space $\mathbb{R}^3$. We study these systems with the homogenous Neumann (zero-flux) or Dirichlet (zero) boundary conditions.

2. Results

We start our analysis with the case without diffusion. In this case, in favourable conditions, that is for Eqs. (1), there is always unrestricted tumour growth and without any treatment the patient cannot survive. In the competitive conditions the dynamics can be similar to those obtained from Eqs. (1) but can be also different from it. If for every $t \geq 0$ there is $y_{n-1} < \frac{1}{\eta_n}$, that is the number of pre-malignant cells always stays at the level smaller that the threshold value $\frac{1}{\eta_n}$, then we observe unrestricted tumour growth. On the other hand, if y_{n-1} is bounded above this threshold, then the solution is attracted by the critical point $\overline{y^n}$ with $y_n = 0$. In this system depending on the model parameters, we can also expect bi-stable behaviour, as in the typical competitive Lotka–Volterra system, compare e.g. [3]. The most stable behaviour we get for unfavourable conditions, described by Eqs. (3). For this model if the positive critical point $\bar{y}$ exists, then it is globally attractive. If not, then we expect that one of the semi-trivial critical points is attractive, compare also the analysis for $n = 2$ in [2].

We also considered the influence of spatial arrangement due to diffusivity of cells. It occurs that for the case with diffusion the behaviour of the systems strongly depends on the boundary conditions. In the case of the Dirichlet boundary conditions the qualitative behaviour of solutions to every studied system for sufficiently large diffusion is the same — every solution tends to the trivial one for t tending to ∞. On the other hand, the solution to the systems with the Neumann boundary conditions strongly depends on the system. In the author's opinion this suggest that either diffusion coefficients cannot be large or the Neumann boundary conditions better reflect the real process. In fact, even in the case of malignant cells occurrence we do not expect extinction of all cellular populations.

It should be marked that in every considered case there is no possibility to recovery, because the semi-trivial critical point which describes a healthy organism is always unstable. Therefore, we should try to target tumour cells paralelly increasing the competition coefficient η_n.

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