ORTHOTROPIC MODEL OF CANCELLOUS BONE. APPLICATION TO SIMULATION OF ADAPTIVE REMODELLING

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1. Introduction

A numerical model that allows to simulate the process of anisotropic remodelling of cancellous bone is presented. The bone is treated as continuum with linear elastic orthotropic mechanical properties. Elastic constants and relative density are explicitly known functions of geometric parameters of microstructure. The parameters are nonuniformly distributed in the bone volume. The remodelling rule is an optimization problem in which the "cost" functional is a time rate of a certain global measure of bone quality at a given load state. Instantaneous rates of the parameters are supposed to minimize the functional. The numerically predicted evolution of the parameters is obtained from the time integration of the results of the instantaneous optimization problem.

2. Methods

Cancellous bone is a macroscopically continuous medium that exhibits orthotropic elastic properties within the physiological range of small deformations. Macroscopic mechanical properties are directly related to geometry and mechanical properties of trabecular microstructure. The latter are subject to evolution — this is the way bone adapts to changing (in the long time scale) mechanical conditions. For modelling of the evolution it is crucial to know (i) the mechanism of tissue changes and (ii) the way mechanical properties change along with the changes in microstructure.

Most constitutive models known for cancellous bone do not directly define the dependence between material constants and microstructure characteristics. In the following research, cancellous bone will be modelled with the use of the parametric constitutive model described by the author in [2]. In this model, macroscopic elastic constants are tabularized functions of certain geometric parameters $\{\mu_p\}$ — thicknesses of trabecular bars/plates and orientation angles of principal directions of orthotropy,

$$\sigma_{ij} = D_{ijkl}\,\varepsilon_{kl}\,, \qquad D_{ijkl} = D_{ijkl}(\mu_p), \quad p = 1, N_p\,. \tag{1}$$

The functions are derived numerically for a family of idealized, repeatable bone-like microstructures.

Bone remodelling is understood as evolution of trabecular microstructure within the prescribed occupied domain Ω in a way ensuring the fastest possible improvement of bone quality at given loading conditions and at certain limitations resulting from bone physiology. In particular, it will be assumed here after [4] that bone quality is identified with the total strain energy accumulated at a given load and corresponding displacement field $u_i(\mathbf{x})$,

$$G[u_m, \mu_p] = \int_\Omega \frac{1}{2}\, u_{i,j}\, D_{ijkl}(\mu_p)\, u_{k,l}\, \mathrm{d}\Omega \tag{2}$$

and thus the evolution of parameters $\dot{\mu}_p$ tends to minimize the rate of this functional

$$\Psi = \dot{G} = \int_\Omega \left[\frac{1}{2}\, u_{i,j} \left(\frac{\partial D_{ijkl}}{\partial \mu_p}\,\dot{\mu}_p\right) u_{k,l} + u_{i,j}\, D_{ijkl}(\mu_p)\, \dot{u}_{k,l}\right] \mathrm{d}\Omega. \tag{3}$$

Employing the finite element discretization and introducing an incremental time integration procedure, we can replace the functional Ψ with its incremental approximate at the time interval $[t_n\ ,\ t_{n+1}]$,

$$\Psi(\mathbf{q}_n, \Delta\mathbf{q}, \mathbf{m}_n, \Delta\mathbf{m}) = \Delta G = \frac{1}{2}\,\mathbf{q}_{n+1}^T\,\mathbf{K}(\mathbf{m}_{n+1})\,\mathbf{q}_{n+1} - \frac{1}{2}\,\mathbf{q}_n^T\,\mathbf{K}(\mathbf{m}_n)\,\mathbf{q}_n\ , \tag{4}$$

where $\mathbf{q}$ denotes nodal displacement vector, $\mathbf{m}$ is an array of parameter values μ_p at element integration points, and $\mathbf{K}$ is the stiffness matrix. Ψ is going to be minimized at the time interval with respect to the increment $\Delta\mathbf{m}$.

The minimization problem is subject to constraints: the equilibrium equation $\mathbf{K}_{n+1}\mathbf{q}_{n+1} = \mathbf{f}_{n+1}$, prescribed total mass, and physiological constraints on μ_p and $\dot{\mu}_p$. See details in [3].

The procedure has been implemented in an author's finite element code featuring design sensitivity analysis. Optimization at each time step is performed with the use of the HOPDM routine [1].

3. Results

Figure 1 presents results of computer simulation of mass and anisotropy distribution in a human femur (2D model). Initially bone is assumed uniformly filled with isotropic material. Application of three staggered load cases corresponding to real every day activities stimulates the remodelling process which finally leads to a distribution closely resembling patterns observed in natural bones.

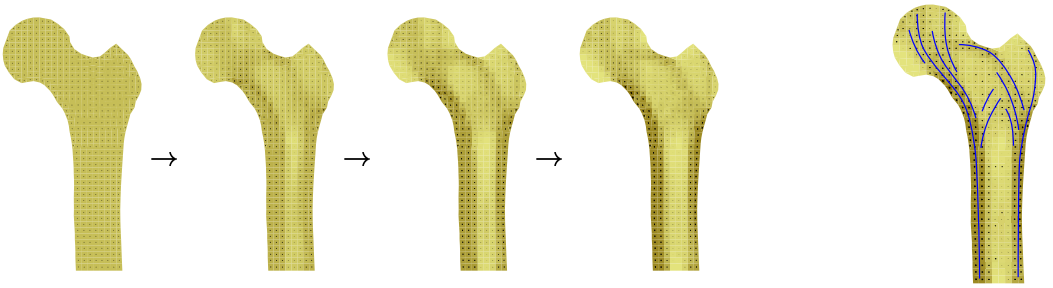

Figure 1. Evolution of mass and anisotropy in a F.E. model of a human femur.

4. References

[1] J. Gondzio (1995). HOPDM (version 2.12) – A fast LP solver based on a primal-dual interior point method. *European Journal of Operational Research*, **85**, 221–225.

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[4] T. Lekszycki (1999). Optimality conditions in modeling of bone adaptation phenomenon. *Journal of Theoretical and Applied Mechanics*, **37**, 607–623.