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Blood as Complex Fluid, Flow of Suspensions

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Blood is the most complicated fluid. While flowing, it interacts with vessel walls both mechanically and chemically. Still, however, descriptions of blood in the framework of suspensions theory is incomplete. In this paper current problems with blood modelling will be presented and the physiology of blood composition and hemorheology will be studied. Finally, the most popular constitutive models of blood and the range of their applicability will be discussed.

Key words: *Blood modelling, rheological parameters of blood, viscoelasticity of blood*

1. Introduction

Mathematical and numerical models together with computer simulations play an important role in biology and medicine. Research in blood flow has a direct impact on our improved understanding and management of human health.

Close examination of blood is supposed to become one of the major mathematical challenges of the next decade. Blood, like other biological fluids, is a "mysterious" one. It means that due to its vital functions in living organisms, blood has a highly complicated structure which changes depending on health and conditions of life. From the physical point of view, blood is a viscoelastic, complex fluid. The term "complex fluid" usually stands for a non-Newtonian fluid, which means that the shear stress and rate of strain are not directly proportional. Various cells in blood (typically making up 45% of the blood's volume) make of it a suspension of particles [28, 31] what results in the non-Newtonian characteristic. When blood starts moving, the particles (or cells) interact with plasma and among one another.

Many fundamental issues concerning blood, like blood rheology and modelling, still need to be studied. The rheological parameters of blood can be used in diagnosis of clinical disorders, in maintaining nonbiological fluids which have rheological properties comparable to blood. What is more, the knowledge of the rheological parameters is necessary in mathematical modelling of blood circulation because of formulating blood flow equations.

Rheological analysis and modelling of blood is still incomplete. Blood is a highly concentrated, complex suspension of polydisperse cells. The cells are flexible, chemically and electrostatically active. They are suspended in an electrolyte fluid (plasma) of critical pH in which there are numerous active proteins and organic substances. The modelling of complex suspensions of flexible particles presents a difficult task for scientists and engineers. The boundaries determine the flow of homogeneous fluid, whereas the flow of a multi-component's fluid is additionally affected by individual, suspended particles interacting with one another and with the boundaries of the flow. To describe the rheology of a dilute suspension within the reach of analytical and computational methods there are already well established theories (derived from Einstein work [23]). Computations and analyses have shown that the response of a solitary liquid drop (in a dilute suspension) involves deformations in shape by stretching, contracting or shearing. The nature and extent of deformations can be determined by the intensity and type of the flow. The reaction of a drop depends on two parameters. The first is a measure of the balance between the shear force acting on a drop which tends to deform the drop and the tension on the surface of the drop, which keeps the drop together. The second parameter is the ratio between the viscosities of the drop and the suspending fluid.

The physical mechanisms which determine the dynamics of a concentrated suspension's flow are very complicated. It is known, for instance, that during a flow in a tube, the particles tend to migrate towards the centerline of a channel yielding the core annulus type of a flow with the majority of particles suspended in the fastest moving flow near the centerline.

Due to the complexity of a concentrated suspension's motions, no comprehensive theory has been developed to describe the flow of a general multicomponent system, including blood. Furthermore, the convoluted and fragmented shapes of the fluid's interfaces prevent the application of classical numerical approaches, such as finite-difference or finite-element methods.

Therefore till now exact descriptions of blood using the concentrated suspension theory have not existed. The main difficulties in blood modelling as the frame of the theory can be summarized as follows:

- blood is a concentrated suspension, outside the range of applicability of theory of dilute suspensions [45, 23],
- in case of such a concentration, forces between particles should be taken under consideration,
- forces are not known; what is more, particles change their shape in reaction to the fluid's forces,
- the nature of red blood cells' membranes and their deformation in response to stress/strain interaction is much less established,
- red blood cells continuously deform.

For these reasons researchers are forced to seek simplified models to be able to construct constitutive relations for blood.

In this review, I shall discuss problems with blood modelling. First, blood composition and physiology will be discussed briefly. Next, we will go through hemorheology of blood and its determinants. Finally, the most popular constitutive models of blood will be shortly presented.

2. Blood Composition and Physiology

2.1. General Information

The first requirement in blood studying is gaining general information of its physiology. In this chapter a brief outline of blood's composition and circulatory system is presented.

Human blood is a liquid tissue which makes up about 1/13 of the total body mass and accounts for 5 to 6 litres in an average adult male [27, 29].

Blood performs two major functions:

1. transporting through the body:
    - oxygen and carbon dioxide,
    - food molecules (glucose, lipids, amino acids),
    - ions (e.g., Na^+, Ca^{2+}, HCO_3^-),
    - wastes (e.g., urea),
    - hormones,
    - heat.

2. defending the body against infections and other foreign materials.

When blood is centrifuged, it separates into 2 portions (Fig. 1). Plasma is a fluid part of blood and it consists of about 90% of water, 7% of protein and small amounts of organic and inorganic molecules as well as dissolved gases. It behaves like a Newtonian, viscous fluid with viscosity of about 20% higher than that of water. The second phase consists of cells, primarily red blood cells, which make up over 50% of the volume of blood. Red cells, or erythrocytes, contain hemoglobin and carry oxygen throughout the body. Platelets are small cells that are involved in blood clotting. All of these cells have finite life spans ranging from 1 day to a month and are replenished by the bone marrow. Cells arise and die all the time, so their numbers vary constantly.

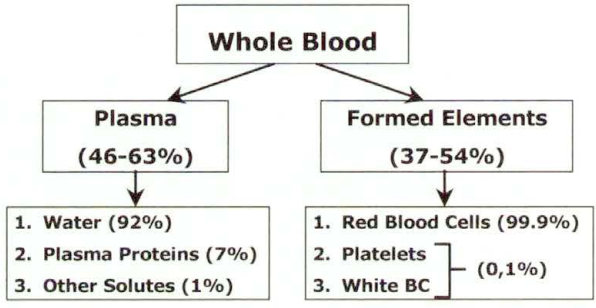

FIGURE 1. Scheme of blood composition

The blood is transported to all living cells in a body by a network of blood vessels. Their structure enables an exchange of blood plasma and dissolved molecules between blood and surrounding tissues. Blood flows away from the heart passing through a series of vessels progressively smaller in diameter: from arteries to arterioles and then to capillaries. Blood returns to the heart through a series of vessels progressively larger in diameter: from capillaries to venules and to veins. Capillaries are the simplest-structured vessels which permeate the entire body in a form of a fine mesh. The structure provides room for blood and allows the transfer of interstitial fluid.

The complex behaviour of blood and its interaction with the vascular walls play an important role in the physiology of blood circulation. Blood interacts both mechanically and chemically with the vessel walls which can get deformed under the blood pressure.

2.2. Composition of Blood

Blood is composed of fluid plasma, solids (erythrocytes, leukocytes, platelets), and other elements either carried to or away from cells. Microscopic view of blood and its solids is presented in Fig. 2.

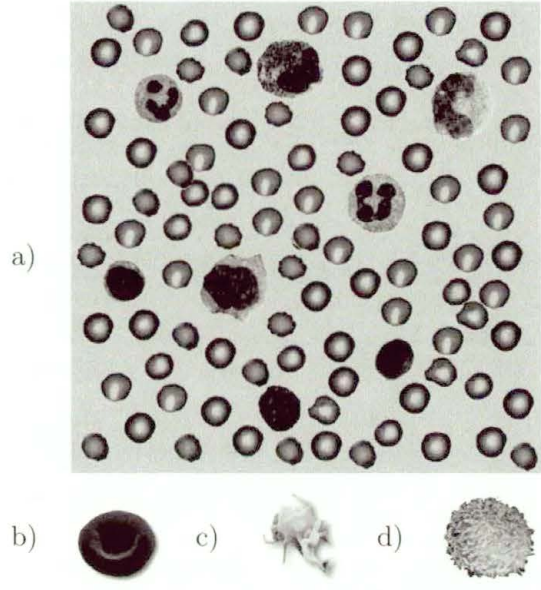

Figure 2. Microscopic view of a) whole blood, b) red blood cells c) platelet and d) white blood cell [68, 69, 70].

2.2.1. Plasma. Blood cells are suspended in straw-coloured plasma (liquid part of blood—Fig. 1). Plasma is a mixture of water, sugar, fat, protein, potassium and calcium salts. It contains also many chemicals which aid blood to form clots necessary to stop bleeding. Water makes up more than 92% of plasma. Water of plasma is freely exchangeable with that of body cells and other extra cellular fluids and is available to maintain the normal state of hydration in all body tissues.

Plasma is a complex solution which transports materials needed by cells and materials which must be removed from cells:

- various ions (Na^+, Ca^{2+}, HCO_3^-, etc.),
- glucose and traces of other sugars,
- amino acids,

- other organic acids,
- cholesterol and other lipids,
- hormones,
- urea and other wastes.

Total volume and concentration of plasma is important in the regulation of blood pressure. Sodium ion is the major solute in plasma. Its concentration determines the amount of plasma water, and thus blood volume.

2.2.2. Red blood cells—erythrocytes. Red blood cells (RBCs), are the most abundant blood cells; 1 μL of male blood contains 4.5–6.3 million RBCs and 1 μL of female blood contains 4.2–5.5 million RBCs.

RBC it's a membrane filled with a solution of hemoglobin and various salts. It's shape is similar to flattened biconcave disc (closed torus) with a depressed center, about 2.5×10^{-6} m thick and 7.5×10^{-6} m in diameter, [6]. The depressed center provides increased surface area for the diffusion of gases. The membrane is composed of chemically complex lipids, proteins, and carbohydrates in a highly organized structure. RBCs carry the oxygen from the lungs to all parts of a body and then return carbon dioxide from our body to the lungs.

RBC creates hemoglobin until it accounts for some 90% of the dry weight of the cell. Hemoglobin is also responsible for making red blood cells red. The viscosity of RBC's interior fluid is five to ten times greater than that of exterior fluid. RBC in quiescent plasma tends to form aggregates known as rouleaux.

An extraordinary distortion of a red cell occurs in its passage through minute blood vessels, many of which have a diameter smaller than that of a red cell. When the deforming stress is removed, the cell springs back to its original shape. The red cell readily tolerates bending and folding, but, if an appreciable stretching of the membrane occurs, the cell is damaged or destroyed. Healthy red cells behave like liquid drops because membranes of red cells are equally elastic and flexible. Sick red cells, for example deformed ones in *sickle cell anemia* lose their elastic properties and may clog small blood vessels.

RBCs are produced continuously in our bone marrow from stem cells. They never divide. After $\approx$120 days, a RBC cell membrane ruptures, or the damage is detected by phagocytic cells in liver and spleen. Most of the iron in their hemoglobin is reclaimed for reuse.

2.2.3. White blood cells—lymphocytes. White blood cells (WBCs) are clear, round cells that are bigger than red blood cells. They have a nuclei and mitochondria which enable them to move around. WBCs are capable of squeezing through pores in capillary walls in order to reach sites of infection. This aids WBCs in their participation in the immune response of the body.

White blood cells produce proteins called antibodies that help our bodies fight infections caused by bacteria, viruses, and foreign proteins. A typical μL of blood contains 6000–9000 WBCs (1% volume). Most of the WBCs in a body at a given moment are in the connective tissue or in organs of the lymphatic system. They remain viable only during the last 18–36 hours before they also are removed.

WBCs can be classified on the basis of the appearance of granules when viewed under the light microscope (Fig. 3) and function as follows:

1. Granulocytes protect body from infection and are represented by:
   - basophils,
   - eosinophils,
   - neutrophils.
2. Agranulocytes are a part of immune system and are represented by:
   - lymphocytes,
   - monocytes.

2.2.4. Platelets (thrombocytes). They are the smallest formed elements and actually are fragments of large bone marrow cells. Their shape is flat-

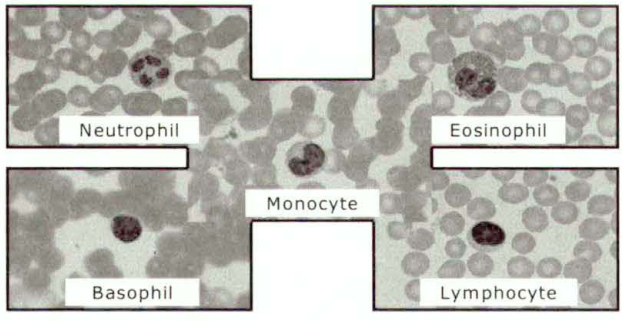

FIGURE 3. Microscopic view of various kind of white blood cells.

tened, disc-like, and the characteristic size of a cell is about 1 μm by 4 μm (1/3 size of RBC).

Platelets are continuously replaced. Each platelet circulates for 9–12 days before being removed by splenic phagocytes. They contain no nuclei but still are capable of moving and functioning in blood clotting. They act as a participant in the vascular clotting system.

When blood vessels are cut or damaged, the loss of blood from the system must be stopped before a shock or possible death. This is accomplished by solidification of blood, a process called coagulation or clotting.

3. Rheological Parameters of Blood

The heart pumps energy into the blood with each beat. Portions of this energy are either dissipated or stored as blood cells rearrange, orient and deform. This behavior is indirectly expressed by the rheological parameters of blood viscosity and elasticity coefficients.

The simplest physical interpretation of the rheological parameters can be as follows: viscosity is an assessment of the rate of energy dissipation due to cell deformation and sliding; elasticity is an assessment of the elastic storage of energy primarily in the kinetic deformability of the red blood cells.

The viscosity and elasticity determine the pressure required to produce blood flow. Due to correlations between the whole blood viscosity and arterial diseases, stroke, hypertension, diabetes, smoking and aging, the hemorheology has been of great interest in the fields of biomedical engineering and medical researches. Hemorheological properties of blood include the whole blood viscosity, plasma viscosity, hematocrit, RBC deformability and aggregation, and fibrinogen concentration in plasma.

3.1. Viscosity and Viscoelasticity

Viscosity is a measure of flow resistance depending on internal friction when one layer of fluid moves in relation to another layer. Viscoelasticity is the tendency to respond to stress as if the material were a combination of elastic solid and viscous fluid. This feature, possessed to some degree by all plastics, says that materials which have solid-like characteristics such as elasticity, strength and form stability also have liquid-like characteristics like flow depending on time, temperature, rate and amount of loading. The experimental value of the viscosity coefficient of a fluid is obtained from the

ratio of shearing stress to shearing rate. If the flow is constant in time, then the ratio of shear stress to shear rate is the viscosity of the fluid. When flows change in time, some liquids generally demonstrate both a viscous and an elastic effect; such liquids are called viscoelastic [25, 46, 58]. To determine the parameters of fluid, methods based on the relation between shear stress and time rate of shear strain (or shear rate) are employed [25, 46, 58].

Whole blood is both viscous and elastic while blood plasma normally exhibits viscosity only [38]. The viscoelasticity behavior of blood results mainly from red blood cells deformability and their ability to aggregate. The viscous and elastic properties of blood can be measured by use of standard rheometry or by using BioProfiler [64]. It should be noted, that the first who has measured the viscoelastic properties that control the pulsatile flow of blood was G.B. Thursto in 1972 [56].

3.2. Determinants of Whole Blood Viscosity

There are four main factors which influence the rheological parameters of blood: (1) plasma viscosity, (2) hematocrit, (3) RBC deformability and aggregation, and (4) temperature. Especially, the hematocrit and RBC aggregations, mainly contribute to the non-Newtonian characteristics of shear-thinning viscosity and yield stress. Below we describe them in detail.

3.2.1. Plasma Viscosity. Since blood is a suspension of various cells in plasma, the plasma viscosity affects blood viscosity and viscoelasticity, particularly at high shear rates. Studies have shown that normal plasma is a Newtonian fluid [28], Therefore, its viscosity is independent of shear rate [20, 21]. The viscosity coefficient of plasma is $\mu = 1.2 \times 10^{-3}$ Pa s.

3.2.2. Hematocrit. The rheological properties of suspensions highly correlate with concentrations of suspended particles. In blood, the most numerous suspended particles are red blood cells (RBC). Therefore hematocrit is the most important factor which effects the whole blood viscosity [7, 18, 28, 55].

Hematocrit is defined as a volume percentage of red blood cells in the whole blood. Hematocrit's average value is 46 (in the range of 40–54) for men and 42 (in the range of 37–47) for women. It can be determined by centrifuging a blood sample so that all formed elements come out of the suspension.

The effect of hematocrit in blood viscosities has been well documented in literature. In general, the higher the hematocrit, the greater the value of the whole blood viscosity [17, 20, 31]. Fig. 4 presents the influence of the hematocrit concentration on viscosity and viscoelasticity of blood.

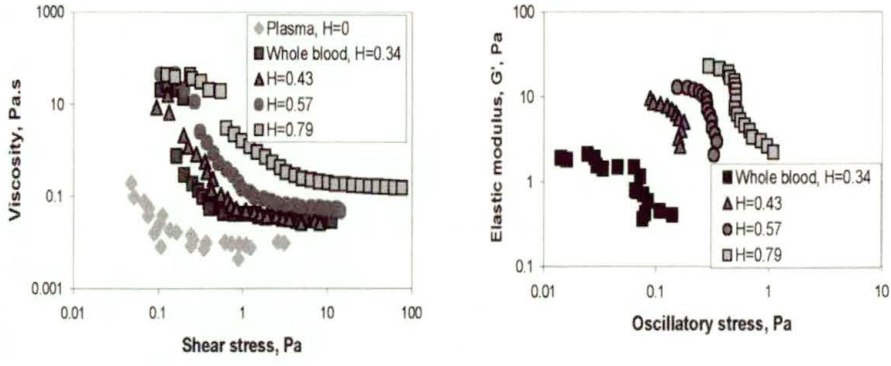

FIGURE 4. The influence of blood cell concentration (hematocrit H) on viscosity and viscoelasticity of blood (after [64]).

3.2.3. RBC Deformability. Deformability describes the structural response of a body or cell to applied forces. The effect of RBC deformability in influencing general fluidity of the whole blood is clearly revealed in Fig. 5. This figure shows the relative viscosity of blood at a shear rate $>100\ s^{-1}$, at which particle aggregation is negligible, compared with that of suspensions with rigid spheres and oil-water emulsion.

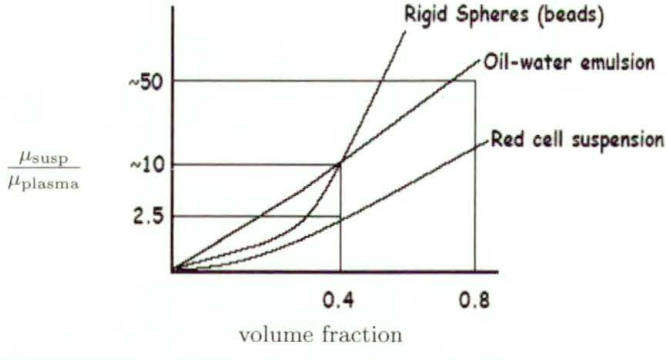

FIGURE 5. Variation of the relative viscosity of blood, oil-water emulsion and suspension with rigid spheres at a shear rate $> 100\,s^{-1}$ after [30].

We can observe that at 50% concentration, the viscosity of a suspension of rigid spheres reaches almost infinity, so the suspension is not able to flow. On the contrary, normal blood remains fluid even at hematocrit's level of 98% on account of the deformability of its RBCs [28].

This blood fluidity is due to the special properties of red blood cells, particularly due to their shape and elastic properties of their membrane. These properties permit tremendous deformations of red cells and consequently blood can flow. In many small blood vessels, the capillary diameters are the same of even smaller then the one of a red cell. In such cases blood flow would be blocked if red cells were not so flexible.

3.2.4. RBC Aggregation. Since red cells do not have a nucleus, they behave like fluid drops [20]. Hence, when a number of red cells clusters together as in the flow of a low shear rate, they stack together, like coins, into aggregates called rouleaux. The extent of aggregation is strongly dependent on the shear rate; the aggregates will break up when the shear rate is increased, qualitatively explaining the decrease in viscosity at increasing shear rates shown in Fig. 6.

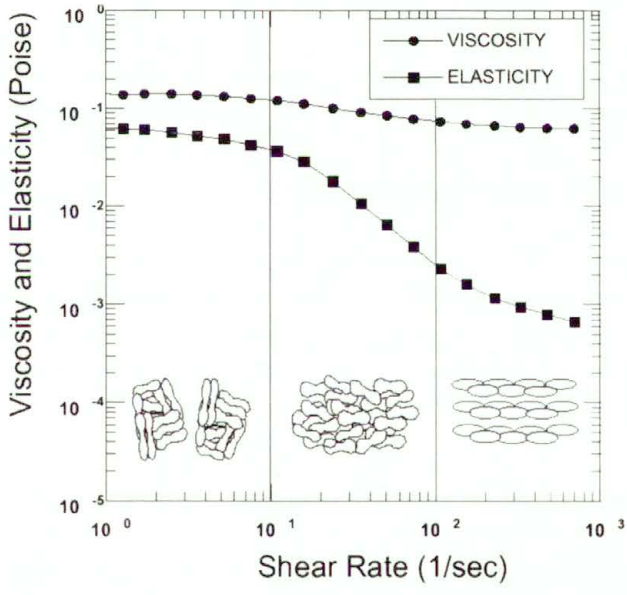

FIGURE 6. The shear rate dependence of normal human blood viscosity and elasticity at 2 Hz and 22°C, after [63].

Figure 6 shows the relationship between blood viscosity and elasticity and rouleaux formation, which can be divided into tree parts regardless of shear stresses [56, 64]. Rouleaux formation of healthy red cells decreases at increasing shear rates. As shear rate increases, blood aggregates tend to be broken up. The collapse disturbs the flow and requires the consumption of energy, which manifests itself in increasing blood viscosity at low shear rates [28]. So we can say that rouleaux formation increases blood viscosity, whereas breaking up rouleaux decreases blood viscosity.

Rouleaux formation is highly dependent on the concentration of fibrinogen and globulin in plasma. Note that bovine blood does not form rouleaux because of absence of fibrinogen and globulin in plasma [28].

It is important to point out, that forces which disaggregate the cells also produce elastic deformation and orientation of the cells, causing elastic energy to be stored in the cellular microstructure of the blood.

3.2.5. Temperature. As in most fluids, blood viscosity increases as temperature decreases [28, 31]. Typically, blood viscosity increases less than 2% for each °C decrease in temperature [4]. Precise control of the sample temperature is necessary to measure viscosity accurately in vitro.

In blood, reduced RBC deformability and increased plasma viscosity elevate particularly whole blood viscosity at low temperatures [4].

3.3. Yield Stress and Thixotropy

In addition to viscosity, blood also exhibits a yield stress [48, 49]. A fluid with no suspended particles starts moving as soon as an infinitely small amount of force is applied. Such a fluid is called a fluid without yield stress. Examples fluids with no yield stress include water, air, mineral oils, and vegetable oils.

The source of the yield stress in blood is the presence of cells in blood, particularly red cells. When such a huge amount (40–45% by volume) of red cells of 8–10 microns in diameter is suspended in plasma, cohesive forces among the cells are not negligible. The forces existing between particles are van der Waals-London forces and Coulomb forces [6, 44]. So the force needed to start the blood flow is large enough to break up particle-particle links among the cells.

The magnitude of the yield stress of human blood appears to be of the order of 0.05 dyne/cm^2 (or 5 mPa) [28, 51, 53, 61] and is almost independent of temperature in the range of 10–37°C [4].

The phenomenon of thixotropy in a liquid results from the microstructure of the liquid system. Thixotropy may be explained as a consequence of aggregation of suspended particles. If the suspension is at rest, the particle aggregation can form, whereas if the suspension is sheared, the weak physical bonds among particles are ruptured, and the network among them breaks down into separate aggregates, which can disintegrate further into smaller fragments [6]. This effect on blood viscosity has been studied in [34, 35, 55]. At high shear rates, structural change occurs more rapidly than that at low shear rates. Based on the results, it can be concluded that the recovery of quiescent structure requires approximately 50 seconds, while the high shear rate structure is attained in a few seconds. In other words, in order to minimize the effect of the thixotropic characteristic of blood on the viscosity measurement between the shear rates of 500 and 1 s^{-1}, at least 50 seconds should be allowed during the test to have the fully aggregated quiescent state at a shear rate near 1 s^{-1}.

3.4. Clinical Significance of Blood Viscosity and Viscoelasticity

A number of researchers who measured both blood and plasma viscosities, reported that both whole blood viscosity and plasma viscosity were significantly higher in patients with essential hypertension than in healthy people [52, 59, 60]. In the case of diabetics, whole blood viscosity, plasma viscosity, and hematocrit were elevated, whereas RBC deformability was decreased [21]. Other scientists conducted hemorheological studies to determine the relationships between whole blood viscosity and smoking, age, and gender [9, 36, 67]. They found that smoking and aging might cause the elevated blood viscosity. Variation in blood viscoelasticity in healthy population is very small. Thus, changes due to disease or surgical intervention can be readily identified, making blood viscoelasticity an useful clinical parameter.

Now extensive basic research on blood viscoelasticity and the factors affecting it have provided a firm foundation for the increasing interest in viscoelasticity among researchers in clinical medicine and physiology. It has been discovered that major shifts in the viscoelasticity of blood are associated with such pathologies as myocardial infarction, peripheral vascular disease, cancer and diabetes [14, 42].

4. Constitutive Models of Blood

For over four decades great attempts have been made to obtain a constitutive relation for blood. While some of these models are empirical, others involve rigorous mathematical derivations. Detailed review of these models can be found in [50, 57, 62, 69]. In this chapter some of the most popular constitutive models of blood: Newtonian, Casson, Herschel-Bulkley and micropolar model will be presented. The mathematical description will be given and range of applicability of the models will be discussed.

In general, viscous liquids can be divided in terms of rheological properties into Newtonian, general non- Newtonian, and viscoplastic fluids [54, 46]. The properties are expressed by constitutive equations. The Newtonian fluid model is the basis for classical fluid mechanics. Gases and liquids like water are Newtonian fluids. Blood, polymers, paint, and food are non-Newtonian.

Question about an appropriate constitutive model for blood is not trivial: it is a concentrated suspension of highly flexible particles in a complex aqueous polymer solution, the plasma, and exhibits a range of non-Newtonian properties. These properties, presented in the previous chapters, are mainly governed by the deformation and aggregation of red blood cells. Other important factors in determining an appropriate constitutive equation for blood—apart from the fluid properties—are the conditions of flow. Since the whole blood is non-Newtonian in nature, blood behaviour depends strongly on the size of blood vessels in relation to dimensions of red blood cells. The dimensions of vessels imply various shear conditions which affect blood viscosity. Therefore in order to apply the appropriate constitutive model for blood, the problem must be restricted to a specific flow area.

For instance, in capillaries where vessel diameters are comparable with that of red blood cells, blood behaves as a shear-thinning fluid and also exhibits viscoelastic properties that can be neglected in large and medium vessels flow. Such properties must be reflected in properly applied constitutive models.

The question of whether blood can be considered as a Newtonian fluid is still standing. The composition of blood would seem to indicate incontestably that it is indeed not a Newtonian fluid. However, in some situations it is sufficient to assume, that blood acts like a Newtonian fluid. So, Newtonian model of blood can be reckoned as the the first approach in blood modelling. It is valid only when the dimension of flow is large enough—in large arteries. In capillary blood flow, the Newtonian fluid model breaks down.

It should be mentioned, that the constitutive models presented below are derived under assumption that blood is a continuum medium. That is, the elements of blood seem to be continuous with each other, with no empty spaces in between. The continuum hypothesis implies too, that every "point" in the fluid represents a fluid element, and that the properties at that point, represent the properties of that fluid element.

4.1. Newtonian Fluid Model

The simplest constitutive equation for the fluid is Newton's law of viscosity [46, 54].

$$\tau = \mu\dot{\gamma}$$

where μ is the Newtonian viscosity and $\dot{\gamma}$ is the shear rate or the rate of strain.

For Newtonian fluid model, when shear stress is plotted against shear rate at a given temperature, the plot shows a straight line with a constant slope that is independent of shear rate (see Fig. 7). This slope is called the viscosity coefficient of the fluid.

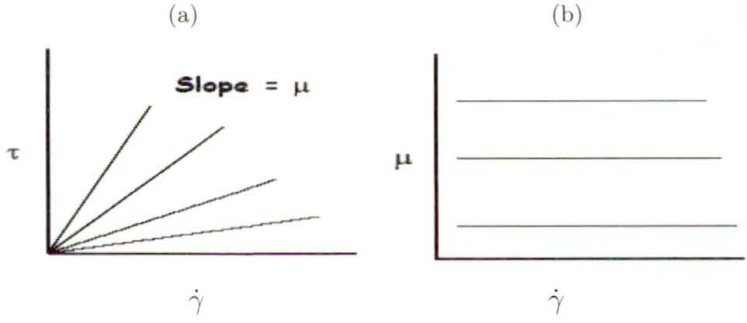

FIGURE 7. Newtonian fluids: a) shear stress vs. shear rate. b) viscosity vs. shear rate.

Plasma is Newtonian fluid with $\mu = 1.2 \times 10^{-3}$ Pa s. The viscosity of blood in Newtonian model is equal: $\mu = 3\text{–}4 \times 10^{-3}$ Pa s. This model is used for blood flow in arteries and large diameter vessels.

4.2. Non-Newtonian Fluid Models

In general, fluids that do not obey the Newtonian relationship between shear stress and shear rate are non-Newtonian [25]. Therefore for non-New-

tonian fluids, the slope of shear stress versus shear rate curve is not constant. The non-Newtonian models presented below can be used to describe blood flow in middle and small blood vessels. The constants which appeared in the models depend on hematocrit and their detailed form can be found in literature [49, 67, 60].

4.2.1. Power law model. One of the most popular is power law model, which can be described by the relation:

$$\tau = m\dot{\gamma}^n.$$

The constant, m, is a measure of the consistency of the fluid: n is a measure of the degree of non-Newtonian behaviour. It is well known that the power-law model does not have the capability to handle the yield stress [25].

4.2.2. Casson Model. The Casson model extends the simple power-law model and is based on a structure model of the interactive behaviour of solid and liquid phases of a two-phase suspension [8]. In contrary to the simple power law, the Casson model can handle both yield stress and shear-thinning characteristics of blood, and can be described as follows [8, 25, 41]:

$$\begin{aligned} \sqrt{\tau} &= \sqrt{\tau_y} + \sqrt{k}\sqrt{\dot{\gamma}}, & \tau \geq \tau_y, \\ \dot{\gamma} &= 0, & \tau \leq \tau_y. \end{aligned}$$

where k is a Casson model constant,
τ = shear stress,
$\dot{\gamma}$ = shear rate,
τ_y = a constant that is interpreted as yield stress.

4.2.3. Herschel-Bulkley model. The Herschel-Bulkley model extends the simple power-law model to include a yield stress as follows [32, 25]:

$$\begin{aligned} \tau &= m\dot{\gamma}^n + \tau_y, & \tau \geq \tau_y, \\ \dot{\gamma} &= 0, & \tau \leq \tau_y. \end{aligned}$$

τ= shear stress,
$\dot{\gamma}$= shear rate,
τ_y= a constant that is interpreted as yield stress,
m and n = model constants.

The model is capable to describe both yield stress and shear-thinning of blood [25].

4.2.4. Comparison of the experimental data with the non-Newtonian models of blood. To compare the non-Newtonian models of blood an experiment was performed [51]. Viscosity of human blood and bovine blood were measured by using rheometry method. Then, the values were used to fit the coefficients of Casson, Power-law and Herschel-Bulkley models. All the constants which appeared in those constitutive models were determined by using curve fitting experimental data approach. Details of the experiment can be found in [51]. Below, in Table 1, the results of blood viscosity measurements with scanning capillary-tube rheometer (SCTR) are presented.

TABLE 1. Blood viscosity. Experimental data and theoretical prediction based on Power-law, Casson and H-B models, after [51].

Shear rate (s^{-1})	Viscosity (cP)	Viscosity (cP)		
		Power-law	Casson	H-B
300	4.43	4.39	4.49	4.28
150	4.78	4.75	4.84	4.71
90	5.11	5.03	5.18	5.09
45	5.75	5.44	5.85	5.71
30	6.25	5.7	6.38	6.2
13	8.81	6.16	7.7	7.21
7.5	17	6.67	9.7	8.9
3		7.4	14.5	12.8
Lower than 3		8.38 (at 1 s^{-1})	22.5 (at 1.35 s^{-1})	18.55 (at 1.55 s^{-1})

We can observe, that the biggest discrepancies between theoretical prediction and experimental data appeared in small shear rate. For high shear rates, experimental data and those predicted by theoretical models are very close.

4.3. Micropolar Fluid Model

The micropolar fluid model—proposed by Eringen in 1966 [3] is an extension of classical fluid dynamics model. It is based on the assumption of a continuous medium, but takes into account microrotation $\mathbf{w}$ of the molecules, different from the local vorticity of the flow. The occurrence of the microrotation vector, which differs from the stream flow vorticity vector $\mathbf{w} \neq \mathrm{rot}\mathbf{V}$

and from the angular velocity $\mathbf{w} \neq 1/2\,\mathrm{rot}\mathbf{V}$ results in the formation of antisymmetric stresses and coupled stresses. Therefore in micropolar model of fluid description we need two constitutive equations: for shear stress—as in classical continuum medium, and the second—for couple stress, [3].

In last decades numerous papers appeared in which the blood has been modelled as micropolar fluid [35]. For instance steady and pulsatile blood flow was considered in [1], values for the micropolar material coefficients were determined in vitro for blood by Bugliarello and Sevilla [4], the phenomena of pulsatile blood flow were considered in [6] with respect to an investigation of the hydraulic impedance of blood vessels. The comparison of experimental data with the theoretical prediction for the blood flow parameters obtained by use of micropolar fluid shows, that this model is suitable for middle shear rates and small vessels flows.

5. Conclusions

Blood is a very complex fluid: homogeneous at macroscopic length scale but possesses a very complicated structure over a microscopic length scale. One of the primary difficulties in physical rather than empirical approach is the fact that blood is a highly complex and concentrated suspension the content of which varies each time and depends on living and health conditions.

The results presented above show how complex the blood structure is. They also indicate that blood modelling is far from being at a satisfactory level. Many open questions concerning blood modelling still arise. Current research shows that blood flow modelling in small vessels is a serious problem. The assumption that blood behaves like a Newtonian fluid fails in the case of small vessels. Mathematicians try to model blood flow in capillaries and small vessels by using non-Newtonian models. But there is still a gap in our understanding of all quantitative aspects of such flows.

Problems presented this review indicate main research directions on blood modelling in the near future. Obtained results can be helpful in our understanding of vascular diseases and in medical diagnosis and therapy.

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