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NONLINEAR PROPERTIES OF TISSUES

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Abstract - In the paper the investigation results of the dependence of a finite amplitude beam propagation on the medium linear and non-linear properties are presented. The influence of such parameters as medium density, acoustic velocity, weak-signal attenuation coefficient and its frequency-dependence index as well as acoustic non-linearity parameter were considered. The analysis was carried out on the basis of numerical simulations of the spatial harmonic distributions of two-dimensional (2D) beam propagating in two-layer structures: water – standard liquid and water–soft tissue. The efficient software (2D numerical solver), describing the nonlinear propagation of the ultrasonic wave radiated by the axially symmetrical source in the multi-layer attenuating medium for given boundary conditions, was used as the tool to predict the behavior of the beam. The algorithm of the above numerical solver, proposed by the second author, is based on the frequency-domain numerical solution of the non-linear wave equation for finite amplitude sound beams. That equation accounts for the effects of diffraction, non-linearity and thermo-viscous absorption on the propagation of sound beams. The diffraction is included within the parabolic approximation, non-linearity and absorption – within the quasi-plane wave approximation. The analysis of the investigated numerical simulations for several boundary conditions made it possible to note that the attenuating coefficient and its frequency-dependence index have the dominant influence on the spatial harmonic distributions of the beam.

1. Introduction

Recently the Tissue Harmonic Imaging (THI) technique is widely used in clinical practice. The main aim of the THI technique (utilizing acoustic non-linear propagation effects) is to achieve the best possible resolution of the image of tested biological structures in order to improve the image quality. Therefore wider and wider THI technique applications in medical diagnosis have increased the importance of the tissue non-linear properties investigations. The non-linearity parameter B/A in an attenuating medium is a basic parameter to determine the degree of acoustic waveform distortions.

2. Boundary conditions

The drive levels of medical pulse-echo equipments are high enough to generate high amplitude ultrasonic fields. The transmitted waveforms are distorted during propagation in attenuating medium resulting from the native generation of harmonics of the initial frequency components radiated by the transducer. This distortion of the pulse shape in the time domain indicates that the waveform contains additional frequencies. Therefore the propagation of finite amplitude ultrasound waves cannot be described by the linear wave equation (when waves travel at a constant velocity).

In this case compressional phases of the wave travel faster than rarefaction ones. Therefore in order to model the non-linear propagation of a wave in the near-field of an ultrasonic transducer the non-linear wave equation for scalar acoustic potential [3] has been used as a mathematical model. This equation accounts for diffraction, non-linearity and thermo-viscous absorption in a forming beam. The diffraction is regarded by parabolic approximation, non-linearity and absorption - by the quasi-plane wave approximation.

$$\Delta\phi - \partial_{tt}\phi - 2A\partial_t\phi = q\beta\,\partial_t(\partial_t\phi)^2 + 2q\,\partial_t L[\phi], \quad (1)$$

where $$L[\phi] \equiv \frac{1}{2}\left[(\nabla\phi)^2 - (\partial_t\phi)^2\right], \quad \mathbf{v} = \nabla\phi, \quad \beta = \frac{\gamma+1}{2},$$

$$A\phi = A(t) \otimes \phi(\mathbf{x},t), \quad A(t) = F^{-1}[a(\omega)].$$

$p(\mathbf{x},t) \equiv -\partial_t\phi(\mathbf{x},t)$ - the normalized, in relation to the initial amplitude, acoustic pressure waveform at the field point with coordinates (x, y, z); $\phi(\mathbf{x},t)$ - normalized acoustic potential at that point; $\mathbf{v}$ - vector of the partial velocity field; A – absorption operator; t – normalized time (in relation to the propagating pulse repetition frequency $\omega_0 \equiv 2\pi f_0$); $q \equiv p_0/\rho_0 c_0^2$ - acoustic Mach number; $\gamma = 1+B/A$ – index of an adiabatic curve; B/A - non-linearity parameter; $\mathbf{x}$ – the normalized space coordinate (x, y, z) vector (in relation to $k_0 = 2\pi f_0/c_0$); ρ_0, c_0 - equilibrium density and acoustic velocity, $\omega = n\cdot\omega_0$ – frequency of the n-th spectral component.

Universal methods of the precise solutions of the non-linear equations does not exist. So a numerical approach to this problem have been developed by the second author [4]. A frequency-domain numerical solution of this equation for periodic signals was a basis to work out the powerful software applied (2D numerical solver). The increase in PC computing power made it possible to model non-linear beams very fast.

The 2D numerical solver enables to calculate the spatial field distribution of the axially symmetrical ultrasonic beam radiated by a circular source in a multi-layer attenuating medium for various boundary conditions. The boundary condition parameters are the input data of the solver and include:

1. shape (plane or focused) and size (diameter) of the 2D radiating source,
2. waveform, frequency, length and repetition period of the transmitted acoustic pulse,
3. apodisation function (initial radial pressure distribution on the source surface),
4. number of media layers and values of their linear acoustic parameters (density,

acoustic velocity, attenuation coefficient and its frequency-dependence index),
5. non-linearity parameter of the every layer.

So the solver made it possible to realize fast simulations of the 2D beams in space and time and was used as a tool to predict the behavior of the acoustic beam for various boundary conditions. The additional advantage of this solver is an adaptability to work in the PC operating system environment WINDOWS.

In this paper the results of numerical simulations of the dependence of beam harmonic distributions on the media linear and non-linear parameters are presented. Several numerical simulations for acoustic pulse waves radiated in water as well as in two-layer media, containing water and liquid or water and biological tissue, have been done for this purpose. There were considered cases, when circular sources with a diameter of Ø = 10÷30 mm (plane or focused with focal distance F = 8 cm) were radiated long (8-cycle) and short (4-cycle) acoustic pulse waves with the carrier frequency of f_0 = 1, 2, 3 MHz and initial pressure amplitude of p_0 = 0.4 MPa. The constant ratio (equal to 0.4) of the pulse duration to its repetition period for all the cases has been assumed. Numerical simulations were realized for initial acoustic pulse envelope in the form of polynomial function (1) for m = 4. The value of the exponent m is fitted usually to obtain the best matched waveform of the analytical initial pulse to the measured one (see Fig. 1).

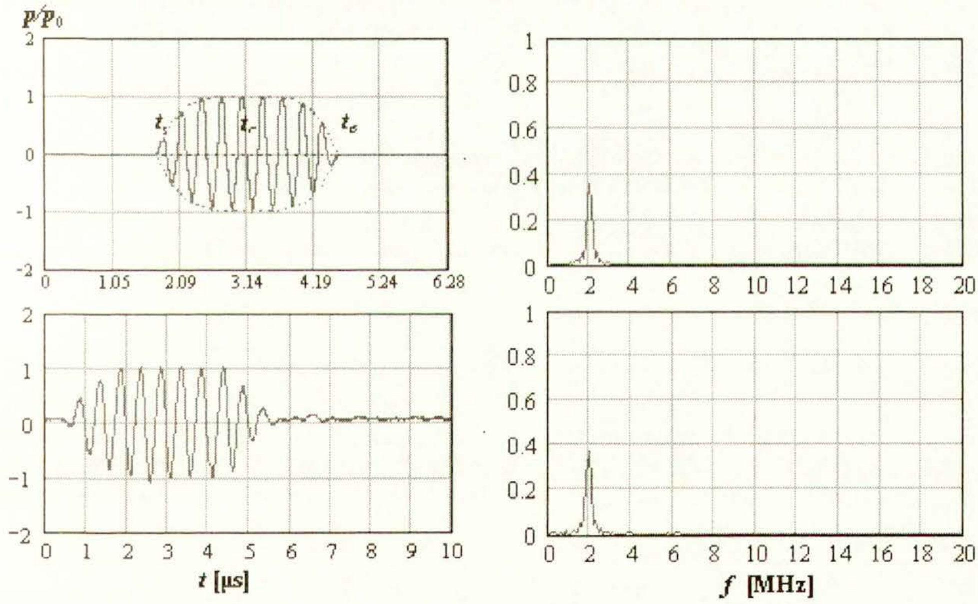

Fig. 1. The initial acoustic waveform containing 8-cycles of sinusoid with the carrier frequency of 2 MHz, normalized with respect to the initial amplitude p_0 (left) and its spectrum (right).

$$f(t) = \left(1 - \left|\frac{t - t_c}{t_e - t_s}\right|^m\right) \sin[\omega(t - t_c)] \qquad \text{for } t_s \le t \le t_e \qquad (1)$$

$$f(t) = 0 \qquad \text{for } t \notin (t_s, t_e)$$

where t_s , t_c , t_e – dimensionless times corresponding to the start, the center and the end of the initial acoustic pulse.

The radial acoustic pressure distribution on the source radiating surface (apodisation function), as the next input parameter of the numerical solver, is matched analytically by searching the initial function that settles the radial pressure distribution in the nearest vicinity of the radiating surface as close as possible to the measured one at the same distance. Fig. 2 presents the initial apodisation function described by the formula $f(r) = \left|1 - (r / a_t)^8\right|$, used for numerical simulations, and the radial distribution computed very close to the radiating source. Here a_t is the source radius. This function was the best approximation of the measurement results [2].

A variety of simulations of the non-linear beam propagation in water, as in the reference material with known linear acoustic parameters (density, acoustic velocity, attenuation coefficient and its frequency-dependence index) and non-linearity parameter B/A = 5.2, for several boundary conditions have been done [1].

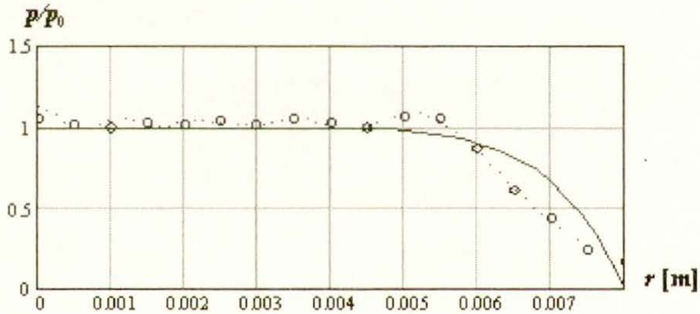

Fig. 2. The analytical apodisation function (normalized in respect to the initial acoustic pressure amplitude p_0) of the plane source of radius 15 mm (solid line), the computed radial pressure distribution at the distance z = 2 mm from the radiating surface (dashed line) and the measurements results (points).

The obtained results were compared with the measurement results for the same boundary conditions. The correlation between the calculated and the experimental harmonic distributions was excellent. As an example, Fig. 3 presents the axial harmonic distributions in water, calculated and measured, for the circular plane transducer with the diameter of 15 mm radiated the waveform (see Fig. 1) of pulse with the carrier frequency of 3 MHz and initial acoustic pressure amplitude p_0 = 0.4 MPa. The normalized radial pressure distribution in the nearest vicinity of the transducer surface is shown in Fig. 2.

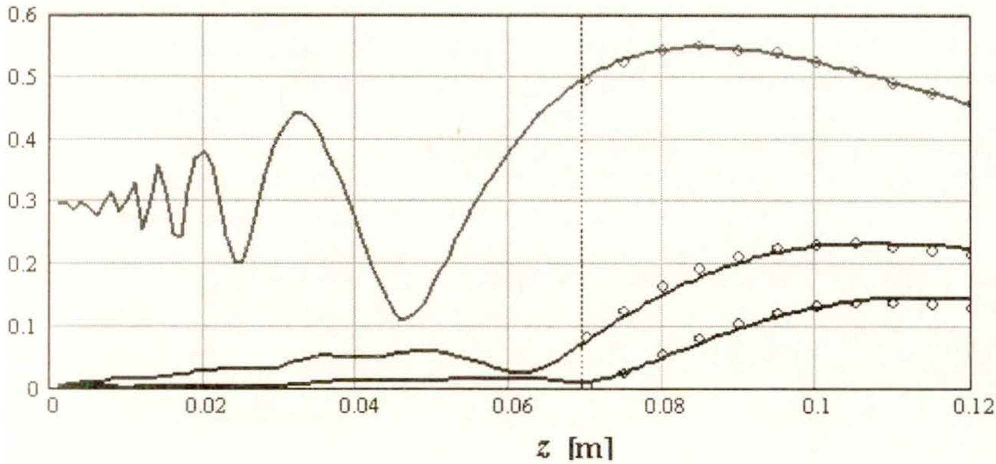

Fig. 3. The 1-st (top), the 2-nd (middle) and the 3-rd (bottom) axial harmonic distribution in water, calculated (solid lines) by the 2D numerical solver and measured by the PVDF hydrophone (points) for the circular plane transducer with the diameter of 15 mm, radiating acoustic waveform, shown in Fig. 1, with initial acoustic pressure amplitude $p_0 = 0.4$ MPa.

The simulation model may thus be used to predict the measurements of the nonlinear beam propagation in water as well as in two-layer (water-liquid or water-soft tissue) structures with known linear acoustic parameters [5, 6] and values of the non-linear parameter (see Table 1).

Table 1. Ultrasonic properties at 25°C of the tested materials

Material	Density (kg/m^3)	Acoustic velocity (m/s)	Attenuation coefficient (Np/m · Hz^b)	*B*/*A* (literature)	b
Distilled degassed water	997	1497	$2.8 \cdot 10^{-14}$	5.2	2
Ethylene glycol	1110	1660	$18 \cdot 10^{-14}$	9.9	2
Corn oil	920	1470	$70 \cdot 10^{-14}$	10.5	2
Glycerol	1260	1890	$570 \cdot 10^{-14}$	9.4	2
Porcine blood	1080	1600	$16 \cdot 10^{-7}$	6.2	1.1
Homogenized porcine liver	1060	1550	$78 \cdot 10^{-7}$	6.6	1

b – attenuation coefficient frequency-dependence index

Using the mentioned 2D numerical solver, a variety of simulations of the higher harmonic distributions in two-layer structures (containing layers of various thickness of water and corn oil, water and ethylene glycol, water and glycerol, water and homogenized porcine liver, water and porcine blood) for various boundary conditions has been carried out.

The thickness of the water layer L always has been chosen as the distance from the radiating source surface in water, at which the 2-nd and the highest harmonics have started to grow rapidly. The analysis of the dependence of higher harmonic distributions in water on such boundary condition parameters as the radiating source shape and size as well as the propagating pulse waveform, frequency and initial pressure amplitude was presented in the paper [1].

Several numerical simulations of harmonic distributions in two-layer structures water-representative liquid or water-soft tissue for various values of the material non-linearity parameter have been realized. In calculations the 3 values of the material non-linearity parameter B/A have been assumed: published in literature, 10% higher and 10% lower. The frequency-dependence index was assumed to be for liquids $b = 2$, for soft tissue 1÷1.4.

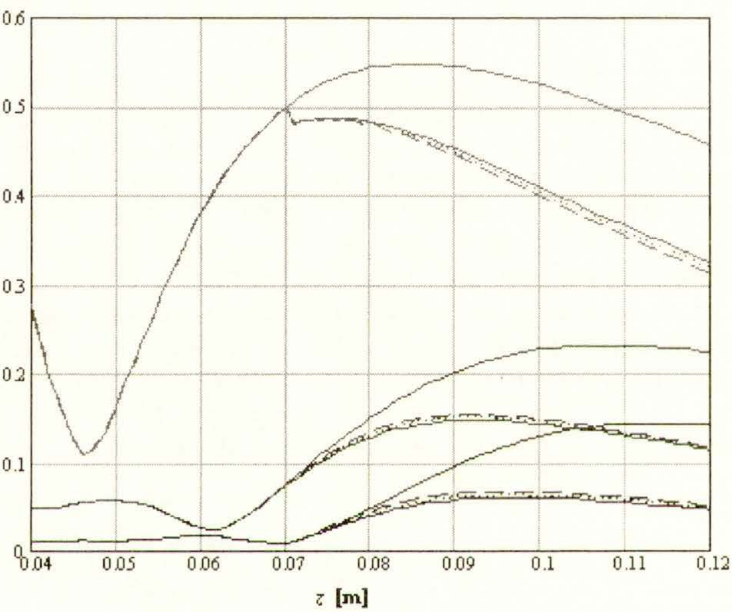

Fig. 4. The 1-st (4 top curves), the 2-nd (4 middle curves) and the 3-rd (4 bottom curves) axial harmonic distributions in water (thick solid lines) and in two-layer structure: 7 cm of water + 5 cm of corn oil (only the range for z = 4 - 12 cm is shown) calculated for the boundary condition as in Fig. 3, when the corn oil non-linearity parameter value equal to 10.5 (dotted lines), 10% lower (thin solid lines), and 10% higher (dashed lines).

Fig. 4 shows the axial harmonic distributions in two-layer structure: 7 cm of water + 5 cm of corn oil simulated for the circular plane source of diameter 15 mm, radiated acoustic pulse with carrier frequency of 3 MHz and initial pressure amplitude p_0 = 0.4 MPa, when the non-linearity parameter $(B/A)_{co}$ of the corn oil was assumed to be equal 9.5; 10.5; 11.5. The amplitude differences between harmonic distributions for these 3 different values of $(B/A)_{co}$ are very small, about few hundredth, nevertheless the availability of high speed digital oscilloscopes providing for a sufficiently accurate acquisition data (at least 10^{-4}) ensure the high enough precision to distinguish the measured harmonic distributions of the tested material among the family of simulated ones when assumed non-linearity parameter values changed with the small step, less than $\pm 5\%$.

In the case of strongly attenuating media, for example in two-layer structure: water-glycerol, when the glycerol attenuation coefficient is 8 times higher with respect to the corn oil (for the same boundary conditions) the amplitude difference between harmonic distributions (when the 3 different values of the glycerol non-linear parameter are assumed to be equal to $(B/A)_{gc}$= 9.5, 10% higher and 10% lower) are negligible. In this case to distinguish the amplitude differences in simulated harmonic distributions for these 3 assumed values of $(B/A)_{gc}$ it is necessary to change boundary conditions: to decrease the radiated frequency or to increase the source diameter or to drive the source by higher voltage. Fig. 5 demonstrates the simulated harmonic distributions in two-layers: 7 cm of water + 5 cm of glycerol for the circular plane source of diameter 15 mm, radiated acoustic pulse with carrier frequency of 3 MHz and initial pressure amplitude p_0 = 0.4 MPa, when the non-linearity parameter $(B/A)_{gc}$ of the glycerol was assumed to be equal 8.5; 9.5; 10.5.

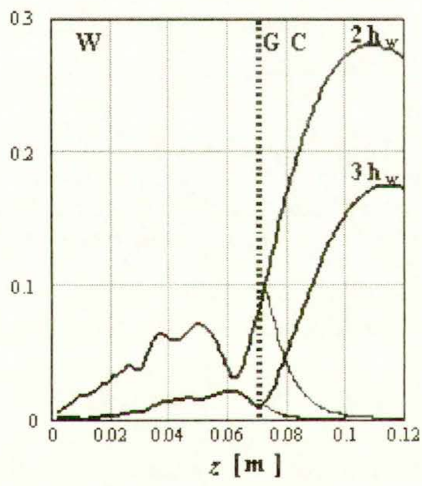

Fig. 5. The simulated 2-nd ($2h_w$) and the 3-rd ($3h_w$) harmonic axial distributions in water and in two-layer structure: 7 cm of water + 5 cm of glycerol calculated for the boundary conditions as in Fig. 3, when the value of glycerol non-linearity parameter was assumed to be equal to 9.5 (dotted lines), 10% lower (thin solid lines), and 10% higher (dashed lines).

The Fig. 5 shows the evident being in line of the 3 simulated harmonic curves for various values of the non-linearity parameter $(B/A)_{gc}$. In this case the boundary conditions should be changed to catch the differences in the amplitude harmonic distributions for various assumed values of the glycerol non-linearity parameter. In the Fig. 6 the 2-nd and the 3-rd harmonic distribution simulation for the circular plane transducer with the diameter of 30 mm, radiating acoustic pulse with the carrier frequency of 1 MHz and initial acoustic pressure amplitude p_0 = 0.4 MPa are presented. In this situation it was possible to distinguish the simulated higher harmonic distributions for various non-linear parameter values and to compare them with the measured harmonic distributions. These observations were the basis to establish the new numerical method for determination of the acoustic non-linearity parameter value in liquids and biological structures.

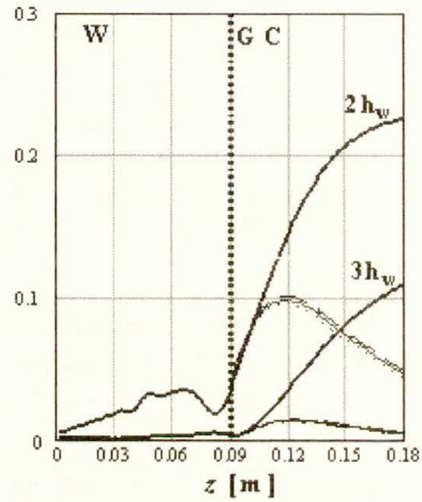

Fig. 6. The simulated 2-nd ($2h_w$) and 3-rd ($3h_w$) harmonic axial distributions in water and in two-layer structure: 9cm of water + 9 cm of glycerol when the plane circular source of diameter 30 mm was radiated ultrasonic pulse with carrier frequency of 1 MHz and initial pressure amplitude $p_0 =$ 0.4 MPa. The 3 assumed values of glycerol non-linear parameter were equal to 8.5; 9.5; 10.5.

Fig. 7 demonstrates the dependence of the non-linear acoustic beam propagation in strongly attenuating media on the attenuation coefficient frequency-dependence index b. The higher is the index b value the lower is the influence of the medium non-linear parameter value on the amplitude of higher harmonic distributions.

So for the precise determination of the acoustic non-linearity parameter of tested liquids or soft tissues on the basis of our proposed comparative method [2] the boundary conditions should be chosen with great care.

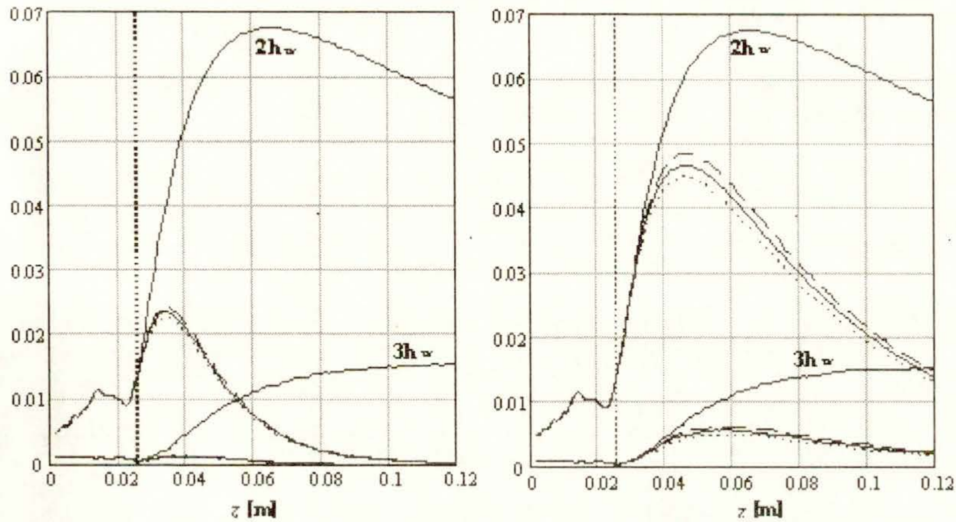

Fig. 7. The simulated 2-nd ($2h_w$) and 3-rd ($3h_w$) harmonic axial distributions in water and in two-layer structure: 2.5cm of water + 9.5 cm of homogenized porcine liver when the plane circular source of diameter 15 mm was radiated ultrasonic pulse with carrier frequency of 1 MHz and initial pressure amplitude $p_0 =$ 0.4 MPa. The 3 assumed values of the porcine liver non-linear parameter were equal to 5.9(dotted lines); 6.6 (thin solid lines); 7.3 (dashed lines) while the attenuation coefficient frequency-dependence index $b = 1.1$ (left) and $b = 1$ (right).

3. Conclusions

The proposed 2D numerical solver, allowing to predict the behavior of the axially symmetrical nonlinear acoustic beam, propagating in layered attenuating media, made it possible to establish a new comparative method for determination of the acoustic non-linearity parameter in liquids and biological structures [2] on the basis of the analysis of obtained numerical simulation results and of their comparison with the measurement results.

The realized numerical investigations referred to the several simulations of harmonic distributions of the non-linear beam propagated in water and two-layer structures, containing water and standard liquids or water and representative soft tissues, for various boundary conditions. The dependence of the non-linear ultrasound beam propagation on such boundary condition parameters as the geometry of the system, the nature of the waveform and the medium non-linear properties has been analyzed.

The analysis of the investigated numerical simulations for various boundary conditions made it possible to note that the attenuating coefficient and its frequency-dependence index have the dominant influence on the spatial harmonic distributions of the beam.

4. Acknowledgements

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