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CODED ULTRASONOGRAPHY

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Abstract - The issue of maximizing penetration depth with concurrent retaining or enhancement of image resolution constitutes one of the time invariant challenges in ultrasound imaging. Concerns about potential and undesirable side effects set limits on the possibility of overcoming the frequency dependent attenuation effects by increasing peak acoustic amplitudes of the waves probing the tissue. To overcome this limitation a pulse compression technique employing 16 bits Complementary Golay Code (CGS) was implemented at 4 MHz. In comparison with other, earlier proposed, coded excitation schemes, such as chirp, pseudo-random chirp and Barker codes, the CGS allowed virtually side lobe free operation. Experimental data indicate that the quality - resolution, signal penetration and contrast dynamics - of CGS images is better than the one obtain for standard ultrasonography using short burst excitation.

1. Introduction

Maximum range / tissue penetration - and fine range resolution which are the two most important considerations in ultrasonographic imaging are contradicting demands.

Sound absorption in the tissue increases approximately linearly with frequency thus limiting the resolution in investigating of the deep structures. On the other hand the possible biological effects related to the insonification limit the probing peak power. The following relationship [1, 3,8]

$$\frac{\max\ range}{range\ resolution} \leq \frac{burst\ interval}{burstwidth} = \frac{peak\ power}{average\ power}$$

stresses the limit that is imposed by the peak maximum power on the ratio of the penetration depth to the required range resolution.

This limitation can be overcome by using long wide band transmitting sequences and compression techniques on the receiver side. To this end different processing systems were proposed in both, Non Destructive Testing (NDT) and in medical imaging. Basically all use coded transmitted signals and employ correlation and averaging on reception of the echoes. Consequently the high peak transmitted power is no more required – the gain in SNR results from the compression of the echoes. Extensive comparison of the standard radio frequency sine wave bursts compared to the random noise transmission and the subsequent compression using polarity coincidence correlator was done by Bilgutay et al [1]. They showed the SNR ratio enhancement especially when integration time of the correlator was made arbitrarily long. However the requirements of the real time medical scanning do not permit such extensive integration time and the attained SNR final gain depends on the length of the transmitted sequence and the efficacy of the compression algorithm.

There are several papers in literature concerning similar boundary-condition problem of signal compression in medical diagnostic imaging. Cohen [2] analysed the principles of pulse compressions in radar system concentrating on the behaviour of the linear frequency modulation and binary phase modulation, using Barker, pseudorandom and Golay codes. Suppression of the side lobes after pulse compression was also extensively examined and it was pointed out that using the pulse compression leads to: a) improvement of the detection performance for a given peak power; b) mutual interference reduction; c) increase in system operational flexibility.

The improvement of the SNR in medical ultrasonic imaging was clearly demonstrated among others by Haider and al [5], O'Donnell [10] and Misaridis et al [7].

This paper is organized as following. R.f. transmitted ultrasonic signals with phase modulated according to the Barker and Golay binary codes shortly described and next the experimental in vitro results comparing the echoes from the tissue phantom for burst two period transmission and Complementary Golay Codes at center frequency 4MHz are demonstrated

2. The Barker Codes

The most widely used binary codes are the Barker sequences. They are optimum in the sense that the autocorrelation function peak is N and the side lobe level falls between +1 and-1, where N is the number of subpulses (elements). The compression ratio is proportional to the length of the code, however no Barker code larger than 13 elements has been found. For code length 13 "+ + + + + - - + + - + - +" the Peak Side Lobe level. PSL=10 log (maximum side-lobe power / peak response power) is equal to –22.3 dB and Integrated Side Lobe level. ISL=10 log (totai power in the side lobes / peak response power) is equal to –11.5 dB. In this work "+" refers to a pulse amplitude of "1" or positive phase and "-" designates a pulse amplitude of "-1" or negative phase.

Table 1. The Known Barker Codes.

Code Length	Code Elements	PSL (dB)	ISL (dB)
2	+	-6.0	-3.0
3	+ -, (+ +)	-9.5	-6.5
4	+ + -, (+ - +)	-12.0	-6.0
5	+ + + - +	-14.0	-8.0
7	+ + + - - + -	-16.9	-9.1
11	+ + + - - - + - - + -	-20.8	-10.8
13	+ + + + + - - + + - + - +	-22.3	-11.5

where: *PSL* - Peak side-lobe level. *PSL*=10 log (maximum side-lobe power / peak response power), *ISL* - Integrated side-lobe level. *ISL*=10 log (total power in the side lobes / peak response power).

Fig.1 compares transmission and matched filtering of a 7 periods sine burst for the uncoded and Barker coded sequence. The total energy in the five periods sequence is five times greater than the one from the single period excitation. After compression the length of the output is equal to 2N-1, 13 in our example, and the peak amplitude is N times greater than the amplitude of the transmitted sequence. Integrated side-lobe level (ISL) increases by -9 dB. The amplitude of the center pulse is seven times as large as a single pulse and represents 17 dB SNR improvement for both cases.

In general the range lobe level decreases with code length and much longer code length >1000 is required for the 60 dB dynamic range of the ultrasonographic images. Barker codes longer than 13 are not known, however there are combined Barker Codes where much larger pulse compression ratio is achievable [2].

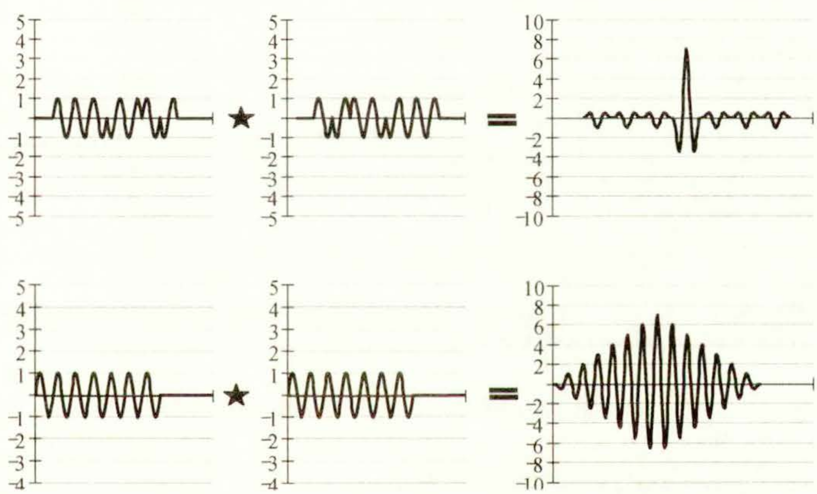

Fig.1. Comparison of 7 cycles uncoded (bottom) and 7 bits Barker coded (top) pulse sequence and resulting compressed outputs, * represents convolution operator.

3. The Golay codes

Golay complementary sequences are pairs of binary codes, belonging to a bigger family of signals called complementary pairs, which consist of two codes of the same length *N* whose auto-correlation functions have side-lobes equal in magnitude but opposite in sign, [4]. Summing them up results in a composite auto-correlation function with a peak of 2*N* and zero side-lobes. Fig. 2 illustrates the principle of the side-lobe-canceling for a pair of signed of length equal to 8 bits each.

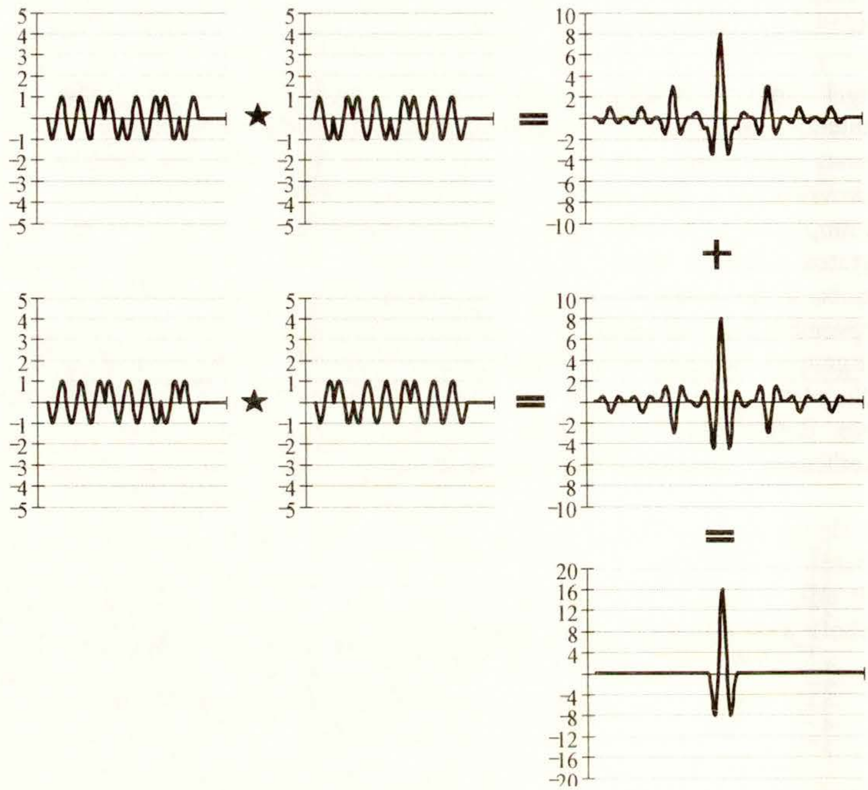

Fig. 2. Principle of side lobe cancellation using pair of Golay complementary sequences of length 8, * represents convolution operator.

There are essentially several algorithms for generating Golay pairs, [11].
Let the variables a_i and b_i (i=1,2,…n) are the elements of two n-long complementary series equal either '+1'or '-1'

$$A = a_1, a_2, ..., a_n;$$
$$B = b_1, b_2, ..., b_n. \tag{1}$$

The ordered pair (A;B) are Golay sequences of length n if and only if their associated polynomials

$$A(x) = a_1 + a_2 x + ... + a_n x^{n-1},$$
$$B(x) = b_1 + b_2 x + ... + b_n x^{n-1}, \tag{2}$$

satisfy the identity

$$A(x)A(x^{-1}) + B(x)B(x^{-1}) = 2n \tag{3}$$

in the Laurent polynomial ring $Z[x, x^{-1}]$.

Let their respectable auto-correlation functions N_A and N_B of the sequences A and B respectively be defined by

$$N_A(j) = \sum_{i \in Z} a_i a_{i+j}$$
$$N_B(j) = \sum_{i \in Z} b_i b_{i+j} \tag{4}$$

where we set $a_k = 0$ if $k \notin (1...n)$. Now condition (3) can be expressed by the sum $N_A + N_B$, and

$$N_A(j) + N_B(j) = \begin{cases} 2N, & j = 0 \\ 0, & j \neq 0 \end{cases} \tag{5}$$

The sum of both autocorrelation function is at j=0 and zeroing otherwise.

The second, recursive method for constructing the Golay's sequences is presented below.

Let the variables ***a***(i) and ***b***(i) be the elements (i=0,1,2,… 2^n-1) of two complementary sequences with elements +1 and –1 of length 2^n

$$a_0(i) = \delta(i)$$
$$b_0(i) = \delta(i) \tag{6}$$

$$a_n(i) = a_{n-1}(i) + b_{n-1}(i - 2^{n-1})$$
$$b_n(i) = a_{n-1}(i) - b_{n-1}(i - 2^{n-1}) \tag{7}$$

where $\delta(i)$ is the Kronecker delta function.

Equation (7) shows that in each step the new elements of the sequences are produced by concatenation of elements $a_n(i)$ and $b_n(i)$ of the length ***n***.

Example:

Let ***n***=1, than ***i*** takes values 0 and 1.

$a_1(0) = a_0(0) + b_0(-1) = 1;$

$b_1(0) = a_0(0) - b_0(-1) = 1;$

$a_1(1) = a_0(1) + b_0(0) = 1;$

$b_1(1) = a_0(1) - b_0(0) = -1.$

As a final results two complementary sequences of the length 2^n are obtained:

a_1={1, 1};

b_1={1, -1}.

Once these operations are performed recursively for $\boldsymbol{n}$=2,3,4,.., the following complementary sequences are obtained:

a_2={1, 1, 1, -1};

b_2={1, 1, -1, 1}.

a_3={1, 1, 1, -1, 1, 1, -1, 1};

b_3={1, 1, 1, -1, -1, -1, 1, -1}.

a_4={1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1};

b_4={1, 1, 1, -1, 1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, 1}.

Similar method of generating the complementary code pairs, differing only in the applied mathematical formalism has been described by Mendieta and al [6]. This method can be applied to sequences of length n to obtain another code pair of length $2n$

$$\left[\frac{A}{B}\right] \rightarrow \left[\frac{A \oplus B}{A \oplus \overline{B}}\right] \tag{8}$$

where $\overline{B}$ is the inverse of B and $\oplus$ indicates concatenation of functions. This procedure may be iterated in the following way

$$\left[\frac{A}{B}\right] \rightarrow \left[\frac{A \oplus B}{A \oplus \overline{B}}\right] \rightarrow \left[\frac{(A \oplus B) \oplus (A \oplus \overline{B})}{(A \oplus B) \oplus \overline{(A \oplus \overline{B})}}\right]$$

$$\rightarrow \left[\frac{((A \oplus B) \oplus (A \oplus \overline{B})) \oplus ((A \oplus B) \oplus \overline{(A \oplus \overline{B})})}{((A \oplus B) \oplus (A \oplus \overline{B})) \oplus \overline{((A \oplus B) \oplus \overline{(A \oplus \overline{B})})}}\right]$$

For example, starting with the one element Golay pair, the Golay codes of length 2,4,etc. are derived:

$$\left[\frac{1}{1}\right] \rightarrow \left[\frac{1 \quad 1}{1 \quad -1}\right] \rightarrow \left[\frac{1 \quad 1 \quad 1 \quad -1}{1 \quad 1 \quad -1 \quad 1}\right] \rightarrow \left[\frac{1 \quad 1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1 \quad 1}{1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1 \quad 1 \quad -1}\right] \rightarrow etc.$$

The length of the sequences $\boldsymbol{n}$ is the input element. The next step is generation n-long sequences a_i and b_i with elements '+1' and '-1'. After that their auto-correlative functions N_A and N_B are calculated. At the end the condition (5) is verified and if it is true, the pair of sequences are written to the file 'Golay sequences'. Thus all possible sequences are obtained, which can be called Golay pairs. In the table 1 the number of Golay sequences for different length up to 20 bits are shown.

Table 2. The number of Golay sequences for different length

Long sequences, *n*	2	4	8	10	16	20
Number of Golay pairs	2	8	48	32	384	272

Although these codes may seem to represent the ideal solution to the side lobe suppression problem, but in practice tissue and especially blood is moving between the two transmits, so perfect cancellation between the two firings will not be achieved.

Table 3 shows a few codes of Golay. The plus sign refers to a pulse amplitude of "1"; the minus designates a pulse amplitude of "-1".

Table 3. The Golay Codes

Code length	X (X*)	Y(Y*)	Amplitude of the main peak
2	+ + (+ +)	+ - (- +)	4
4	+ + - + (+ - + +)	+ + + - (- + + +)	8
8	+ + + - + + - +(+ - + + - + + +)	+ + + - - - + - (- + - - - + + +)	16
16	- - + - - - - + - + + + - + - - (- - + - + + + - + - - - - + - -)	- - + - - - - + + - - - + - + + (+ + - + - - - + + - - - - + - -)	32

where X* and Y* are code sequences applied for compression using matched filters

4. Experimental Results

The aim of this experiment was to investigate the special features of Golay codes behaviour, their advantages in comparison with ordinary sine-like signal, Barker phased modulated sine sequences and chirp signal. The block diagram of the experimental setup is shown below in the Fig. 3.

The sinusoidal signals at the frequencies of 4 MHz were synthesised using Signal Synthesiser (HP8643A, Agilent, USA). This signal was connected to the bipolar modulator driven by the {0,1} sequences from the custom design coder, see Fig.4. The coder preludes programmed logic (EPM7064, Altera[TM], USA) allowed to generate one of different transmitter functions: {1,1} sequence resulting in 2 periods of the sine wave and switched pair of 8 bits Golay codes. The prf of the transmitted signals was set to 1 kHz. After amplification in the power rf amplifier (ENI 3100LA, USA) the transmitter burst were exciting the ultrasonic transducer immerged in water tank or moved over the Tissue phantom (Gs, RMI, USA). The excitation voltage applied to the ultrasonic transducer was equal to 50 Vp-p for all different transmitted sequences in order to keep the I_{SPTP} intensity constant(Mechanical Index constant), [9].

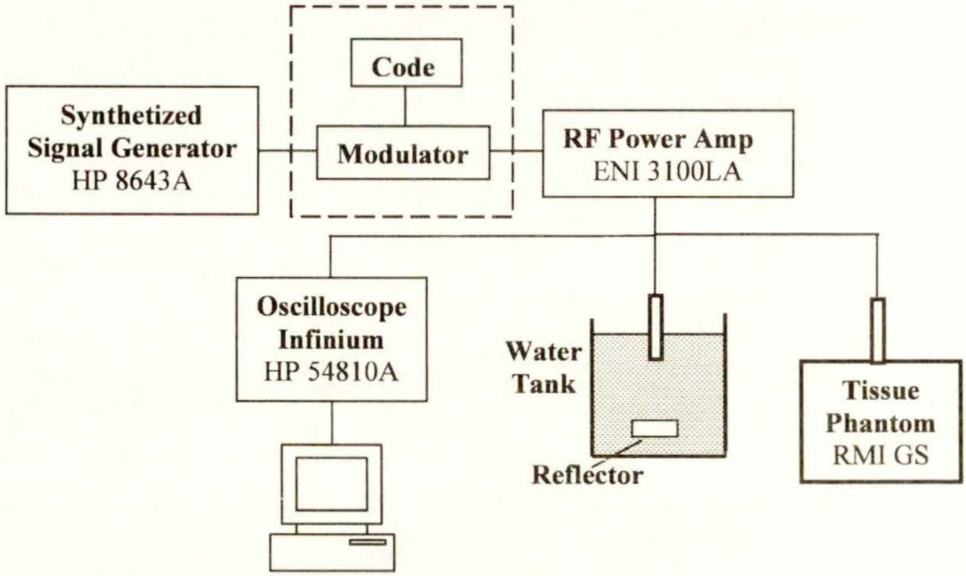

Fig. 3. The experimental setup, for details of the Coder-Modulator block see fig.4

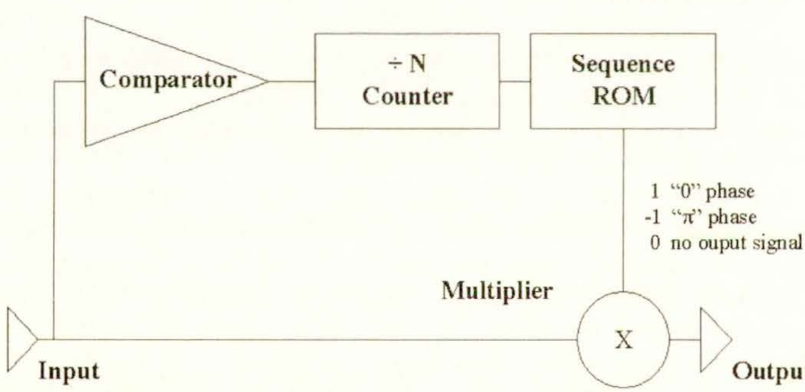

Fig.4 Block diagram of the custom designed Coder/Modulator circuitry

The coder (Fig.4) consists of comparator, PLD logic and analog multiplier. The input signal, sine wave at 0dBm level, is multiplied either by "1" (output in-phase) or "-1" (output out-of-phase) or "0" (no output signal). The PLD logic is a divide-by-N counter, generating cyclic sequences with pulse repetition frequency and sequence ROM, where different codes are stored.

The first part of the experiment was to compare the effective overall axial resolution for both transmission modes, sine burst and CGS.

Rectangular shape, 2 mm thick perplex sheet acting as a reflector was mounted in the water tank at the axial distance equal to 6 cm. This distance corresponds to the focal distance of the 4 MHz transducer used for the experiments.

The RF echoes data were acquired using digital oscilloscope, with a sampling rate 40 ns (25 MHz). Next, the collected digital data were processed off-line and displayed on the oscilloscope. The processing included amplification, pulse compression for 8 bits Golay sequences, and envelope detection.

Fig. 5 shows the RF echoes for two periods sine burst transmission (top), and time compressed complementary Golay series (bottom), respectively. It can be seen that both echoes from the front and rear surface of the perplex reflector are basically identical is shape, however the amplitude of the compressed echoes is approximately 5 times larger than that received from the sine burst transmission. This value is close to the theoretically predicted gain for 8 bits CGS compared to that achievable with the 2 cycles sine burst. Ideally, the expected gain should be 8, however, the finite bandwidth of the pulse-echo transducer slightly elongates the pulse durations and lowers the effective gain.

In Fig. 6 the envelopes of the detected echoes shown in Fig 5 are depicted. To facilitate the comparison the amplitudes of the envelopes were normalized and it can be seen that their shape is virtually identical. That indicates that rather elaborated reception/compression algorithm for Golay series does not modify or disturb the received signal (here sine burst echoes are considered to be the reference ones).

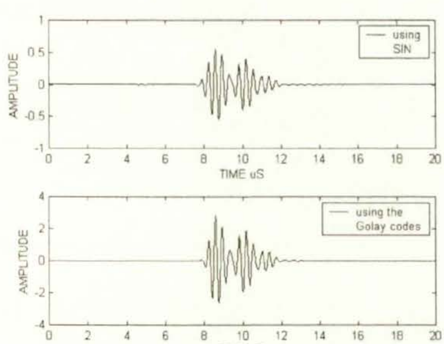

Fig. 5 Echoes from the reflector - plastic sheet, thickness 2 mm. two periods sine burst (top) and Golay codes (bottom)

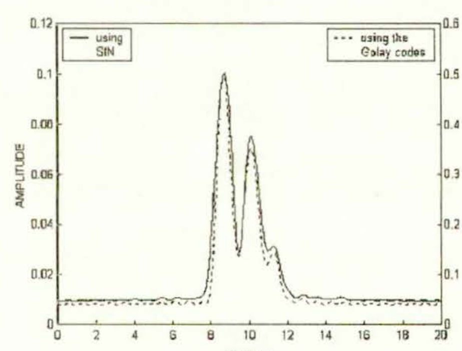

Fig. 6. Comparison of the echoe envelopes obtained using sine-like signal and sequences of Golay codes..

Two images of a tissue phantom RMI 415GX with attenuation of 0.7 dB/[MHz× cm] are shown in Fig. 7. It consists of the nylon wires of 0.374mm in diameter, positioned every 1 cm axially. Additional wires are placed at a 30 degree angle at the top of the phantom. Also some wires are placed in depth 3 cm with decreasing distance down from 3mm to 0.5mm.

The two-cycle pulse of the frequency 4MHz and the pair of the Golay codes of the length 16 bits and at the same frequency were used. The peak pressure levels of the excitation signals at the transducer were set as low as possible to visually detect the echoes received using burst transmission slightly larger than the noise level. The same peak pressure was used for coded transmission.

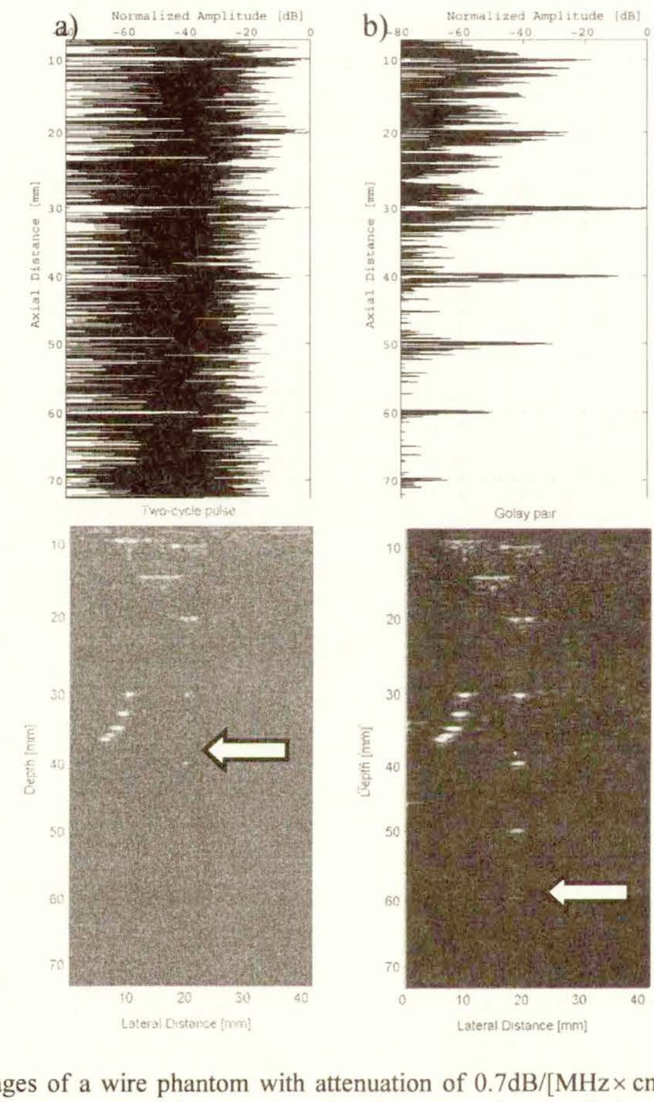

Fig. 7. Images of a wire phantom with attenuation of 0.7dB/[MHz× cm]. Left image was obtained using two periods burst and on the right one using the Golay codes of length 16 bits. On the top of the graph, the central RF-lines are plotted, respectively. Arrows show the region of the phantom, where the phantom objects are no more visible for conventional brief pulse excitation and clearly displayed using Golay coded transmission

The resulting images are shown in figure 7a and 7b. For quantitative comparisons, the RF-lines are also shown. The SNR gain when moving from burst to coded transmission is evident. Applying conventional pulses results in penetration reaching hardly 4 cm. The scan distance obtained with Golay coded transmission extends down to 6 cm (lowest visible white dot at the image), fig.7b. The respective RF- echo lines shown below each image the central RF-lines are plotted confirm the outstanding quality of the signal received, when the Golay coded transmission was used.

These two images clearly demonstrate that abdominal ultrasound imaging can benefit from Golay sequences yielding a higher SNR and therefore deeper penetration, while maintaining both axial and lateral resolution. The range resolution that can be achieved is always higher to that of a conventional system. The main disadvantage of Golay pairs is that it requires two transmitting events for every line that decreases the frame rate by half.

5. Conclusions

Some methods for calculating the pairs of Golay sequences of different length which can be used in ultrasonography were described. Transmission of long coded sequences and compression of the received echoes by means of the matched filtering allow to obtain axial resolution better or similar that obtained using burst transmission but with considerably higher amplitude. Using Golay sequences allows to improve the SNR that play the main role in ultrasonography imaging. For example, SNR is equal 24.1dB when Golay sequences were used and 8.4dB for sin burst excitation at the depth of 30mm, improving the contrast, resolution. This makes it possible to explore the signals with lower amplitude that in its turn is very important since it decreases the patients exposure to ultrasonics. Another important reason of using Golay sequences is the fact that they allow using of higher frequencies, improving the resolution imaging.

References

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