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ABSTRACT

Nonspecular deformations of a Gaussian beam interacting with a multilayered, planar dielectric structure are described in terms of the complex ray formalism. A 3-D ray transfer matrix of the structure is built and relations between the transfer matrix elements and real beam distortions are derived.

1. BASIC RELATIONS

Let us consider a two-dimensional field generated in a homogeneous medium by an *y*-directed stationary electric (or magnetic) line source of strength b. The harmonic field with the wavenumber k obeys the Helmholz equation (the time dependence $\exp(-i\omega t)$ is assumed and suppressed)

$$(\partial_x^2+\partial_z^2+k^2)V(x,z)=-b\delta(z-iz_F)\delta(x), \tag{1}$$

the solution of which, that is the Green's function with an amplitude b,

$$V(x,z)=bG(x,z-iz_F)=(ib/4)H_0^{(1)}(kr), \qquad r^2=x^2+(z-iz_F)^2 \tag{2}$$

is the zero-order Hankel function of the first kind $H_0^{(1)}$, dependent on the complex distance r between the complex source point $(x_s,z_s)=(0,iz_F)$ and the real observation point (x,z). For observation points placed in the far field $(x^2+z^2 \gg z_F^2)$ and, simultaneously, paraxial $(x^2 \ll z^2+z_F^2)$ region the solution to (1)

$$V(x,z)=ibk^{-1}2^{-1}(2\pi)^{-1/4}\exp(kz_F)\Psi_0(x,z), \qquad (3)$$

resolves into the normalized in power Gaussian beam Ψ_0

$$\Psi_0(x,z)=(2/\pi)^{1/4}v(z)^{-1}w_0{}^{1/2}\exp[-(x/v(z))^2+ikz], \quad v^2(z)=w_0^2(1+iz/z_F), \qquad (4)$$

with the half-width $w_0=(2z_F/k)^{1/2}$ at the spot. In the specified region the field V equals the Gaussian beam Ψ_0 provided

$$b=-2ik(2\pi)^{1/4}w_0{}^{-1/2}\exp(-kz_F). \qquad (5)$$

Consequently, the spectrum of the plane wave representation of the field

$$V(x,z)=ib(4\pi)^{-1}\int_{-\infty}^{+\infty} k_z^{-1}\ \exp(k_z z_F)\ \exp[i(k_z z+k_x x)]\ dk_x \qquad (6)$$

with the imposed paraxial approximation

$$k_z(z-iz_F)\simeq k(z-iz_F)+i(k_x v/2)^2, \qquad (7)$$

takes the Gaussian shape as well

$$V(x,z)=\Psi_0(x,z)=ib(4\pi k)^{-1}\exp[ik(z-iz_F)]\int_{-\infty}^{+\infty}\exp[-(k_x v/2)^2]\exp(ik_x x)\ dk_x. \qquad (8)$$

It appears that the field representations in the wavenumber space (Eqs.(6) and (8)) and in the complex space (Eqs.(2) and (4)) give an appropriate basis of the analysis of the beam-interface interaction in terms of changes of the field spectrum on the one hand, and of the nonspecular deformations of the beam on the other hand.

2. NONSPECULAR DIFFRACTION EFFECTS - REDERIVATION.

A Gaussian beam is incident at an angle θ_i on the plane interface between two linear dielectric media, characterized by their refractive indices n_{in} and n_{out} and wave impedances Z_{in} and Z_{out} (for TM polarization) or admittances (for TE polarization). The coordinate frame (x,z) is bond to the planar structure with z axis along the interface. In the following, the interface can be as well understood as an arbitrary multilayered structure, which divides the two different dielectric media. The incident beam has the form (6) in the frame (x_i, z_i) which is rotated with respect to (x,z) frame by θ_i and whose origin is in the middle of the beam waist. As a result of the beam-interface interaction each spectral component of the incident field in (6) is multiplied by an appropriate interaction function R (reflection or transmission coefficient), what leads to the integral field representation in the paraxial approximation

$$V_r(x_g, z_g)=ib(4\pi k)^{-1}\exp(kz_F)\exp(ikz_g)\int_{-\infty}^{+\infty} R(k_{xg})\exp[-(k_{xg}v_g/2)^2+ik_{xg}x_g)]\, dk_{xg}. \tag{9}$$

Here (x_g, z_g) stands for the frame of coordinates tied to the beam reflected (or transmitted) at the interface according to geometrical-optics (g-o) predictions. The k_{xg} and k_{zg} are the wavevector components in this frame and $v_g=w_0(1+iz_g/z_F)^{1/2}$ denotes the complex width of the beam. Under the assumption that the beam incidence angle θ_i is far from any singularity of the spectrum, $R(k_{xg})$ can be approximated by two first terms of its Taylor series ([1])

$$R(k_{xg})=R_i\exp[\ln(R(k_{xg})/R_i)]\simeq R_i\exp[-ik_{xg}L+ik_{xg}^2F/(2k)]. \tag{10}$$

The complex lateral (L) and focal (F) shifts are related to the first and second derivatives of R with respect to k_{xg},

$$L=i(R'/R) , \qquad F=-ik[R''/R-(R'/R)^2] , \tag{11}$$

evaluated at the direction of the beam propagation $k_{xg}=0$. Incorporation of (10) into (9) leads to the field representation in terms of a Gaussian beam displaced by the complex shifts L and F along transverse and propagation directions:

$$V_r(x_g, z_g)=R_1 \exp(ikF)\Psi_0(x_g-L, z_g-F), \tag{12}$$

which, in the paraxial region, can be described also as the Green's function

$$V_r(x_g, z_g)=bR_1 \exp(ikF)G(x_g-L, z_g-F-iz_F) \tag{13}$$

with the shifted source location $(x_s, z_s)=(L, F+iz_F)$. As it was predicted by Tamir [1] and confirmed by Nasalski, Tamir, Lin [2] and Nasalski [3], the deformation of the Gaussian beam by the complex shifts L and F is equivalent to the composite displacement of the beam by two real space translations δ_x, δ_z of the beam waist center, followed by a rotation of the beam axis by an angle δ_θ and a beam waist magnification by a factor

$$\nu=w_{0r}/w_0, \tag{14}$$

where w_{0r} is the reflected beam half-width. It is worth noting that the previous analyses ([1], [2], [3]) used some additional assumptions or approximations to prove this geometrical interpretation of the beam distortion. However, an explicit form of the real deformations $\delta_x, \delta_z, \delta_\theta$ and ν can be derived in the paraxial region without any further approximations, that is from the field representation (12). To this end, let us presume that, in general, the actual reflected beam has a new waist width w_{0r} different from w_0, and a new reference frame (x_r, z_r), translated and rotated with respect to the g-o reflected frame (x_g, z_g)

$$x_r=(x_g-\delta_x)\cos\delta_\theta-(z_g-\delta_z)\sin\delta_\theta, \qquad z_r=(x_g-\delta_x)\sin\delta_\theta+(z_g-\delta_z)\cos\delta_\theta, \tag{15}$$

and the x- and z- wavevector components in this frame are

$$k_{xr}=k_{xg}\cos\delta_\theta-k_{zg}\sin\delta_\theta, \qquad k_{zr}=k_{xg}\sin\delta_\theta+k_{zg}\cos\delta_\theta . \tag{16}$$

Therefore, the field representation (13) should have the equivalent form in the new frame:

$$V_r(x_r, z_r) = bR_rP_rG(x_r, z_r - i\nu^2 z_F), \qquad (17)$$

with R_rP_r denoting corrected reflection coefficient [3]. Comparison of plane wave spectral representations (6) of the fields (14) and (17), together with the equality $k_{zg}^{-1}\, dk_{xg} = k_{zr}^{-1}\, dk_{xr}$, yields

$$\exp[ik_{xg}(x_g - L) + ik_{zg}(z_g - F - iz_F)] = \exp[ik_{xr}x_r + ik_{zr}(z_r - i\nu^2 z_F)] \; . \qquad (18)$$

The equality of the real and imaginary components of the exponent terms in (18) gives the following form of the real deformations

$$\delta_x = L_R, \qquad \delta_z = F_R, \qquad (19)$$

$$\nu^2 = (1 + F_I/z_F)/\cos\delta_\theta, \qquad \sin\delta_\theta = L_I/(z_F\nu^2), \qquad (20)$$

where L_R, L_I, F_R and F_I denote real and imaginary parts of complex shifts L and F, respectively. Comparison of terms standing in front of the spectral integrals leads to

$$R_rP_r = R_i\nu^{-1/2}\exp(-kz_F\nu^2 + ik\delta_z). \qquad (21)$$

The real shifts in (19) are the same as obtained by Tamir [1], in spite of slight changes in ν^2 and δ_θ, which are of order of $\delta_\theta{}^2$ and disappear for small δ_θ. However, the results (19-21) follow exact solution of the Helmholz equation (1) and are obtained without additional approximations necessary in the previous approach. The field expressions (17, 19, 20) seem to be numerically less accurate than those obtained in [3] and [4], where the complex shifts were evaluated around the angle of the actual, instead of geometrical, beam axis, and by accounting for higher order terms in the Taylor expansion (10). On the other hand, the method presented here has the advantage of giving much simpler analytical expressions. Nevertheless, within the range of the assumed approximations the two methods give equivalent results with slightly different definitions of complex shifts. It is also worth stressing that the spectral approach presented here as well as in [4] and [5] presumes weak interaction of the beam with lateral or evanescent waves sustained by the structure. In the opposite cases a direct field evaluation in the physical space is more appropriate [2], [3].

3. TRANSFER MATRIX FOR THE NONSPECULAR DIFFRACTION.

According to the theory of the nonspecular beam-interface interaction described above the beam waist is nonspecularly shifted in longitudinal and transverse directions by the focal δ_z and lateral δ_x displacements, respectively, the direction of the beam propagation is rotated by the angular shift δ_θ, and the beam-waist width is expanded by the factor ν. Besides the expansion factor ν, the deformation parameters δ_z, δ_x and δ_θ provide direct geometrical interpretation of the beam distortions within he frame of the regular geometrical optics valid in the plane wave limit ($kw_0 \to \infty$). The position (x_j) and normalized slope ($x_j' = n \cdot dx_j/dz_g$, n - refractive index at the observation plane) coordinates of the ray before (j=1) and after (j=2) nonspecular interaction are described by the relations:

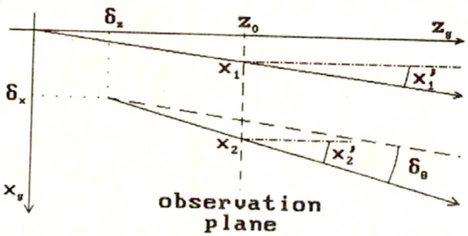

Fig. 1. Input-output ray geometry.

$$x_2 = x_1 + \delta_x + (z_0 - \delta_z) \cdot \delta_\theta - \delta_z x_1' \qquad (22a)$$

$$x_2' = x_1' + \delta_\theta \cdot n \ , \qquad (22b)$$

in which z_0 is the distance between the waist of the incident beam and the interface (see Fig. 1), and $n_{in} = n_{out} = n$, the refraction coefficient in the observation plane. Up to the terms linear in δ_z, δ_x and δ_θ the above equations lead to the matrix formulation of the nonspecular deformation of the beam

$$\eta_2 = T\eta_1 \tag{23a}$$

in which

$$\eta_j = \begin{bmatrix} x_j \\ x_j' \\ 1 \end{bmatrix}, \quad j=1,2 \tag{23b}$$

and

$$T = \begin{bmatrix} A & B & a \\ C & D & c \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1, & [-\delta_z + i(1-\nu^2)z_F]/n & , z_0\delta_\theta + \delta_x \\ 0, & 1 & , \delta_\theta \cdot n \\ 0, & 0 & , 1 \end{bmatrix} \tag{23c}$$

is an augmented ABCD ray transfer matrix. In (23c) the term $i(1-\nu^2)z_F$ indicates a complex change of the position of the source point of the outgoing field (measured at $z=z_0$) and accounts for the nonspecular change of the beam waist. In order to check the correctness of the equations (23) the pure formal canonical approach is the most appropriate to apply. At this point, instead of the real, the discussion must concern the complex rays, complex slopes and complex transfer matrices. The ray coordinates $(x_j,\ x_j')$ are then in general complex canonical coordinates of the ray which satisfy the Hamilton equations ([6]). The evolution of these coordinates is described by the eikonal

$$\rho(x_1,x_2) = (2B)^{-1}[Ax_1^2 + Dx_2^2 - 2x_1x_2 + 2ax_1 + 2(cB-Da)x_2], \tag{24}$$

which determines all the elements of the transfer matrix T, and, this time takes into account a beam waist expansion factor ν as well. The Huygens-Fresnel integral

$$V_2(x_2) = -ik_0/(2\pi B)^{1/2}\int_{-\infty}^{+\infty} V_1(x_1)\ \exp(ik_0\rho(x_1,x_2))\ dx_1, \tag{25}$$

in which k_0 is a vacuum wavenumber, transforms the input Gaussian field, which is a beam diffracted according to the geometrical optics and ignoring dependence of the reflection (or transmission) coefficient on the angle of incidence,

$$V_1(x_1)=R_1\cdot(2/\pi)^{1/4}v_1^{-1}w_0^{1/2}\exp(-(x_1/v_1)^2+ikz_0) \tag{26}$$

into the Gaussian output field

$$V_2(x_2)=d_2(2/\pi)^{1/4}v_2^{-1}w_0^{1/2}\exp(-(x_2+cq_2/n-a)^2/v_2^2+ikz_0) \tag{27}$$

characterized by the complex width v_2 and the constant d_2 (independent on x_1, x_1'):

$$d_2=R_1\cdot(Cq_1/n+D)^{-1/2}\exp[(-ik_0/2)(c^2q_2/n+a^2D/B-2ac). \tag{28}$$

The explicit form of q_2 and v_2 is given by the well known ABCD transformation of the input beam parameter v_1

$$q_2/n=[Aq_1/n+B]/[Cq_1/n+D]\ ,\quad q_j=-ik_0nv_j^2/2\ ,\quad j=1,2, \tag{29}$$

what confirms correctness of the eq.(23).

4. CONCLUSIONS

We have applied a 3x3 ray transfer matrix formalism for the description of nonspecular diffraction effects. Starting from the Green's function approach, we have shown how transfer matrix elements depend on beam deformation parameters.

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